SYDE 556/750

# Simulating Neurobiological Systems
# Lecture 5: Feed-Forward Transformation

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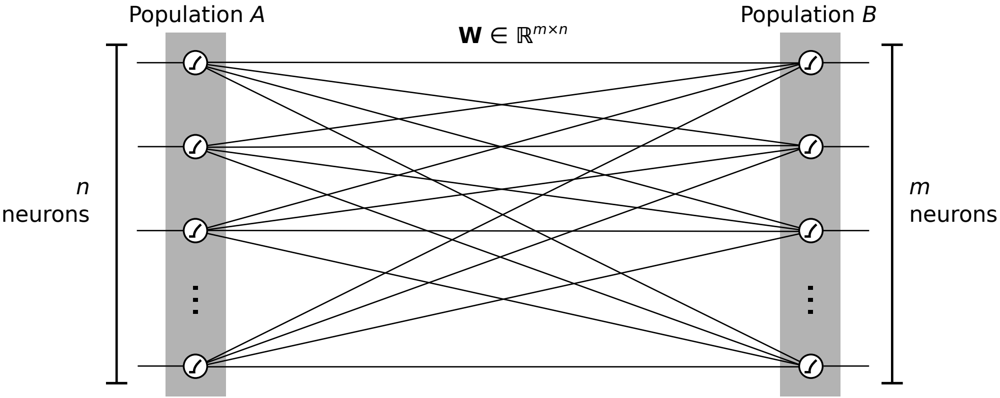
October 4 & 6, 2021

- Slide design: Andreas Stöckel
- Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

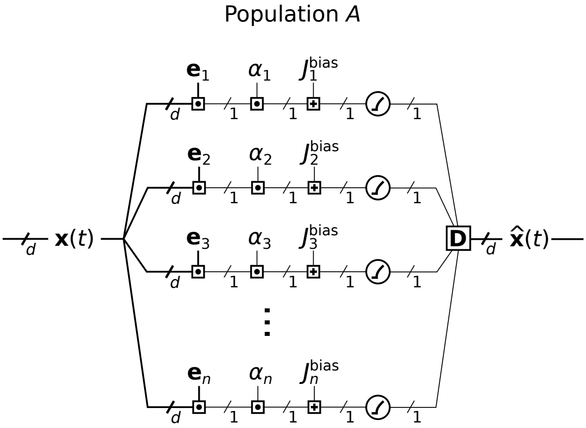
UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

# NEF Principle 2: Transformation




**NEF Principle 2 – Transformation**
Connections between populations describe *transformations* of neural representations. Transformations are functions of the variables represented by neural populations.

# A Tale of Two Populations (I)



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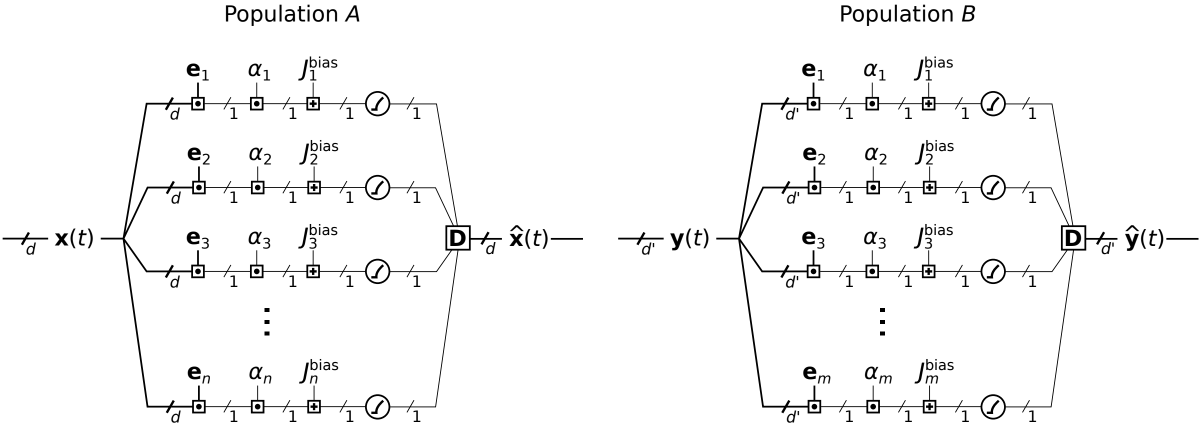
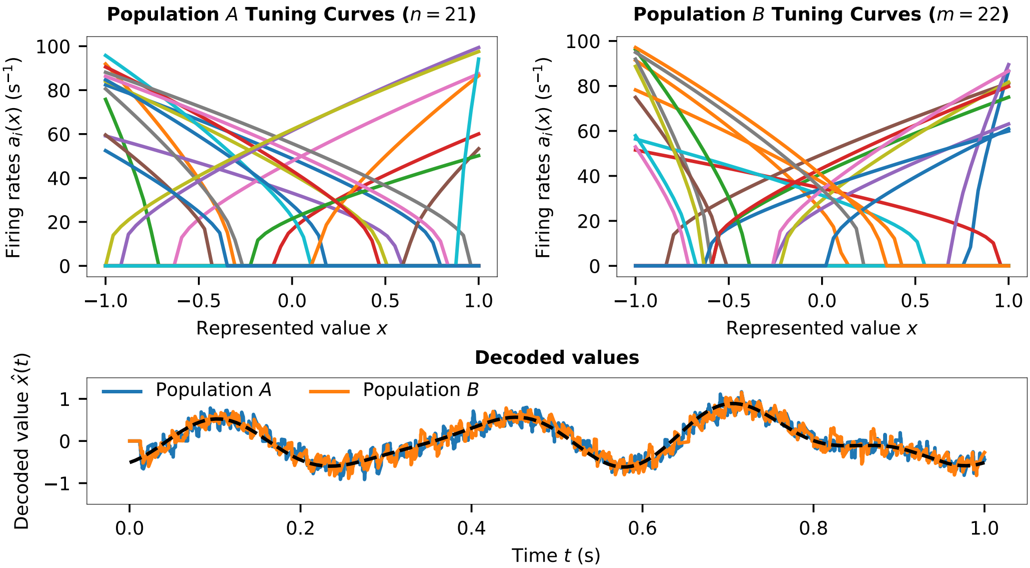
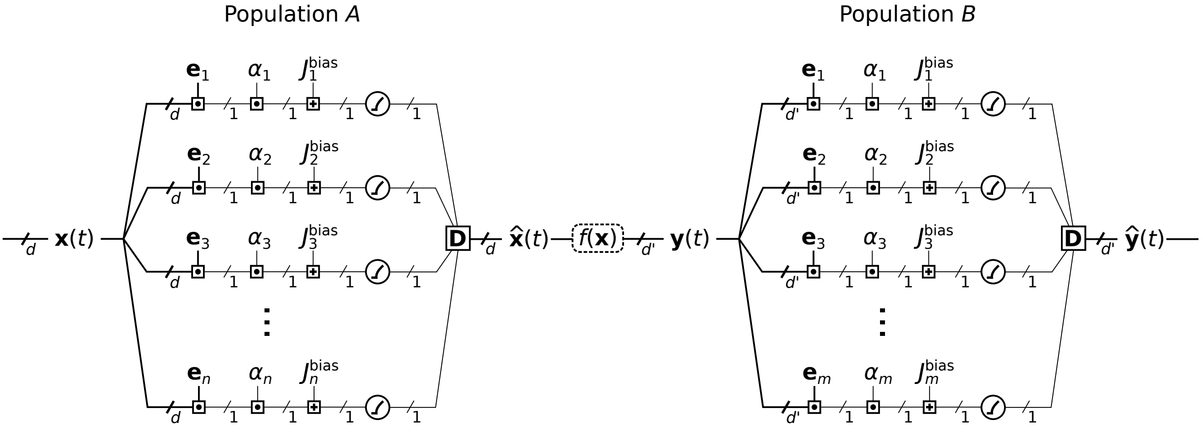


# Communication Channel Experiment: Same input signal



# A Tale of Two Populations (II)



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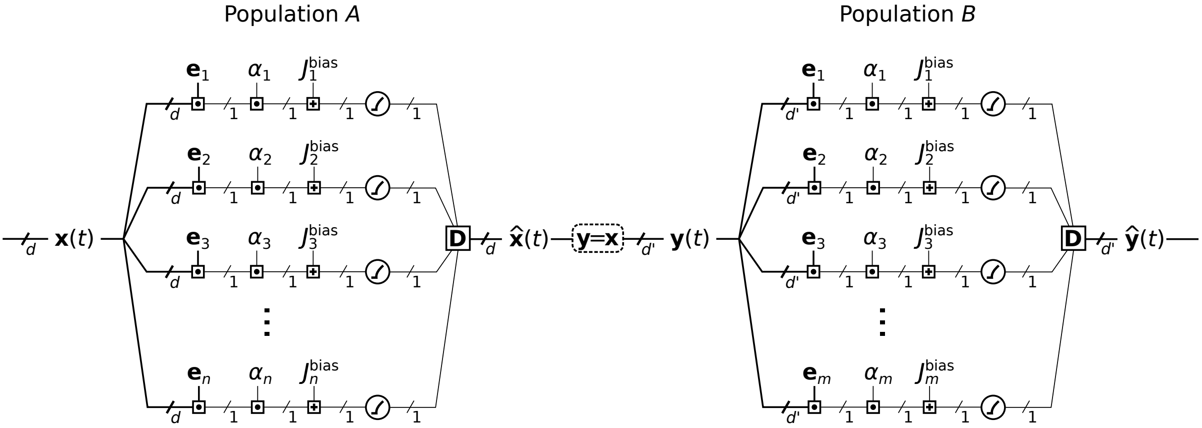
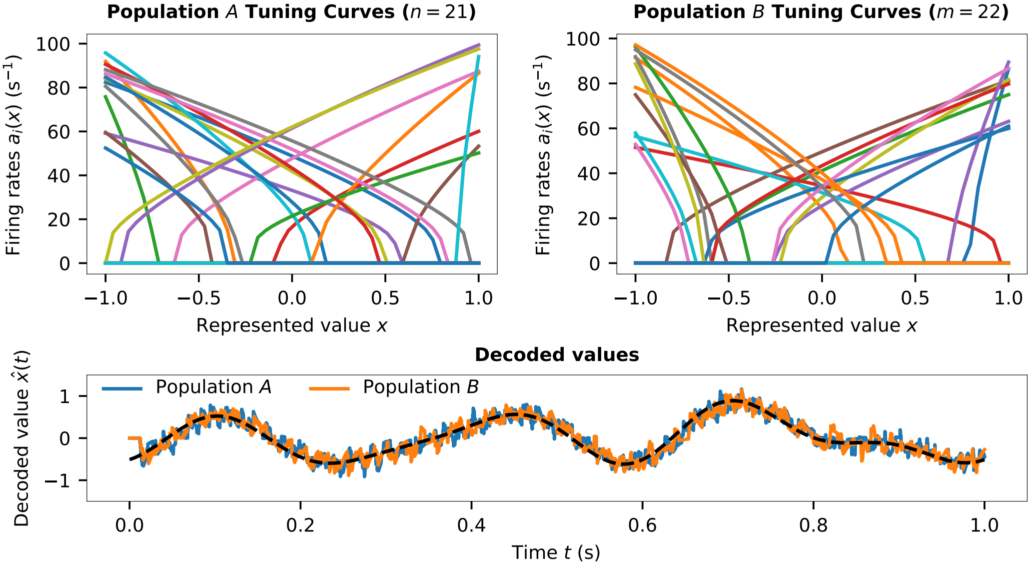
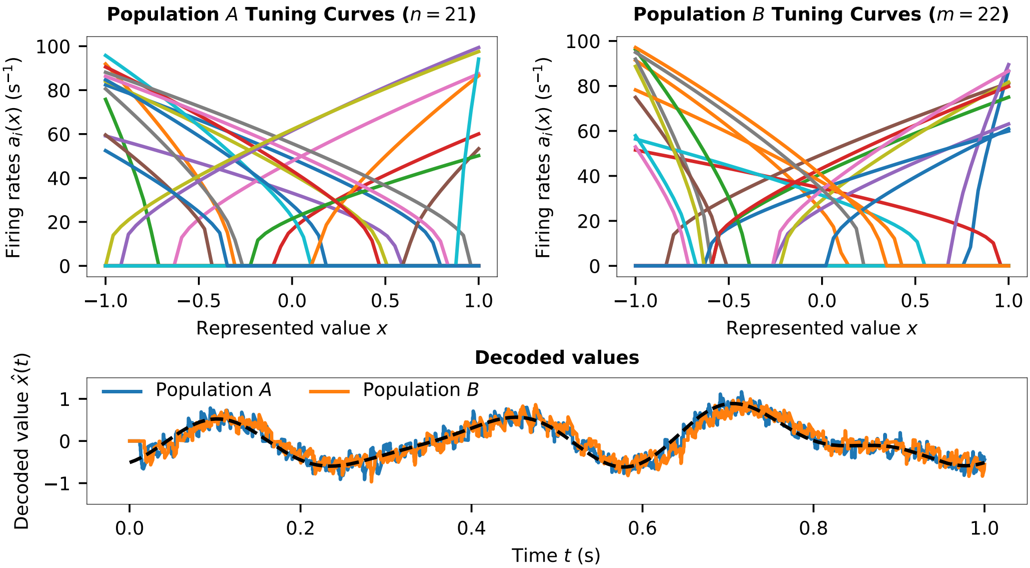


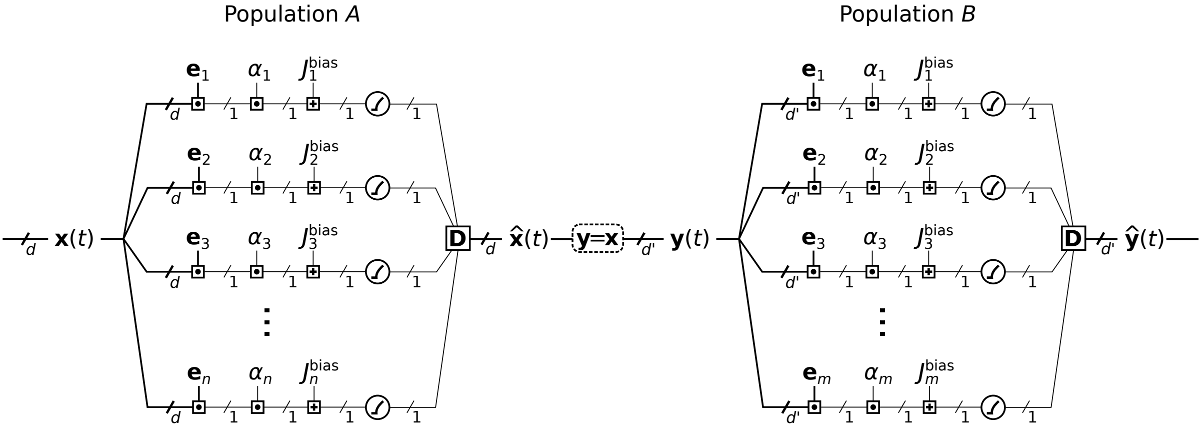
# Communication Channel Experiment: Populations in series



# Communication Channel Experiment: Populations in series



# Computing Synaptic Weights: Step 1 – Encoding Matrix



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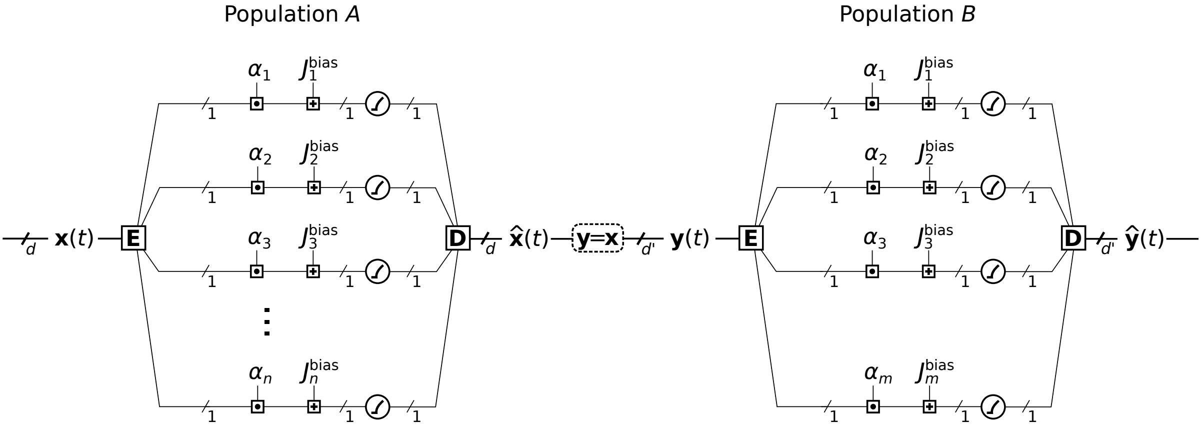
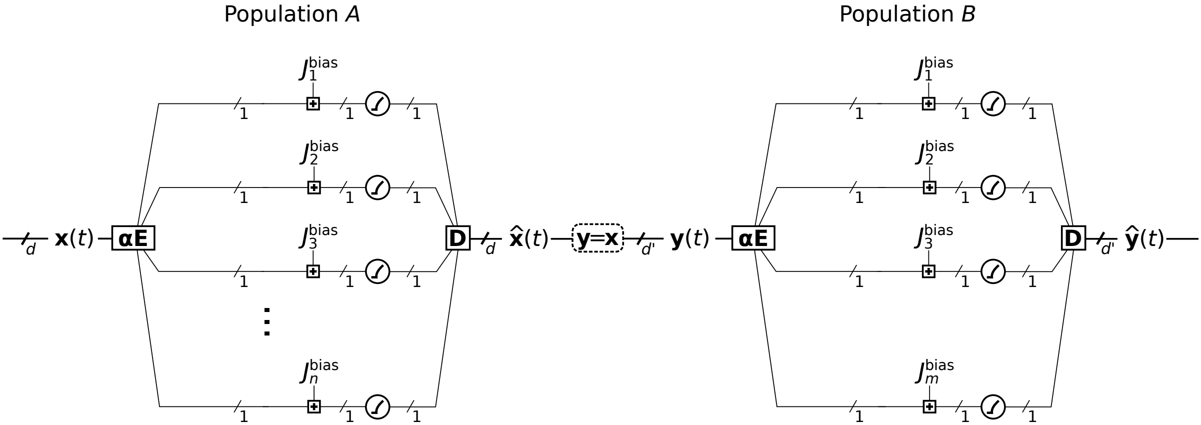
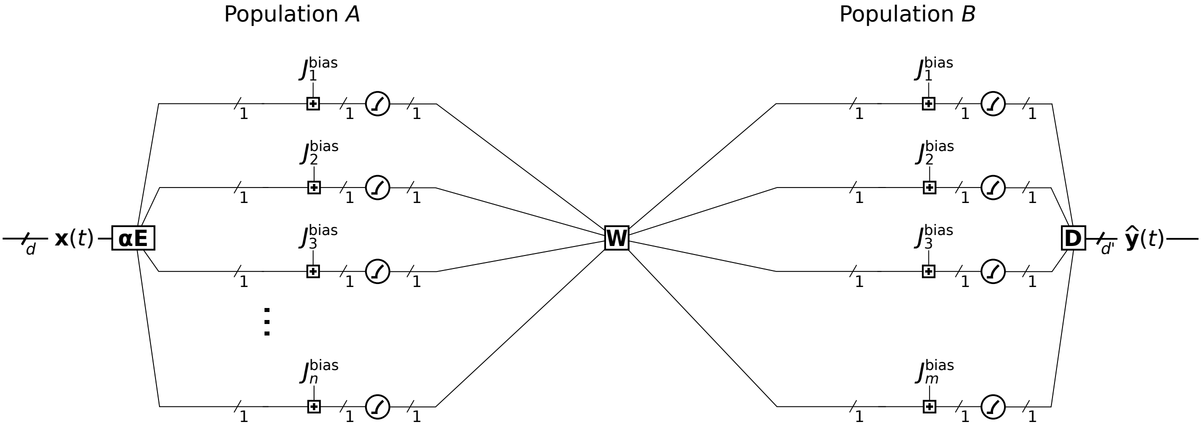


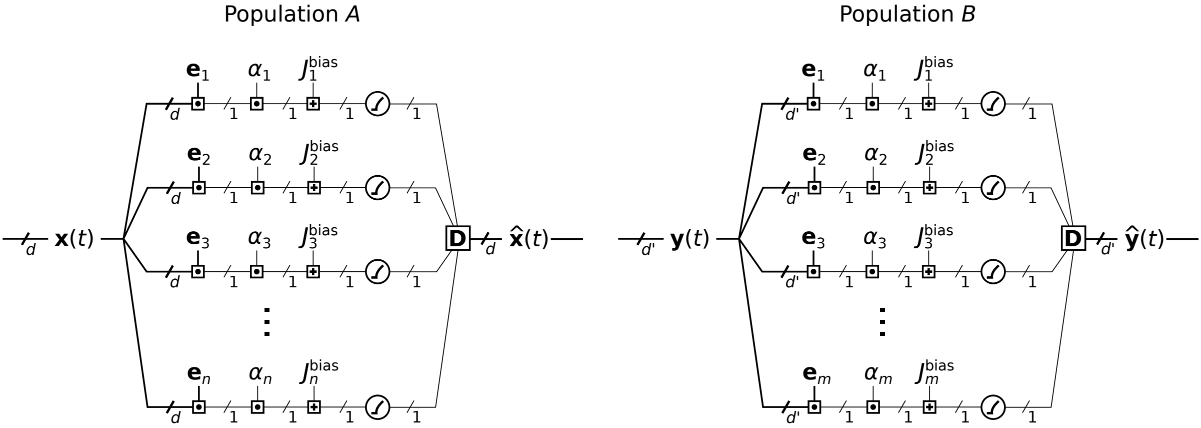
# Computing Synaptic Weights: Step 2 – Scaled Encoding Matrix



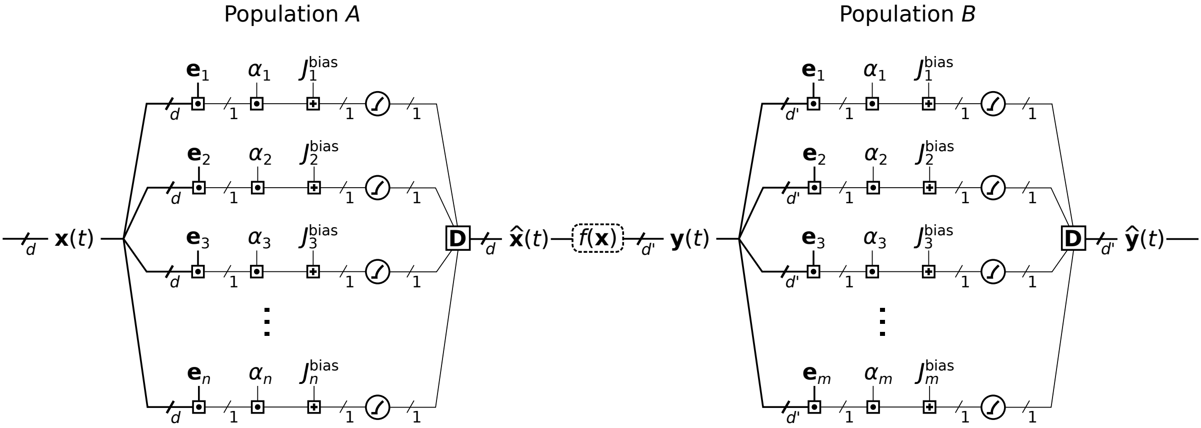
# Computing Synaptic Weights: Step 3 – $\mathbf{W} = \mathbf{ED}$



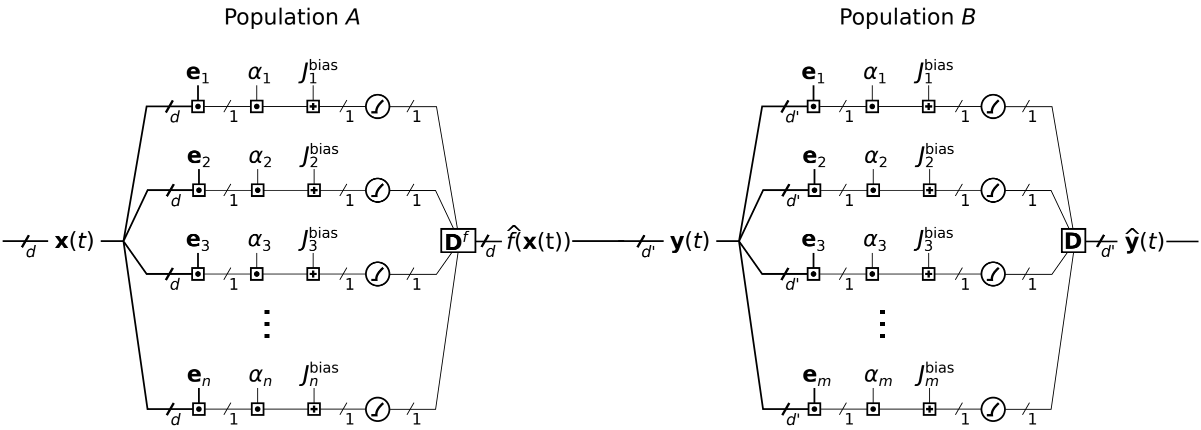
# Computing Functions



# Computing Functions

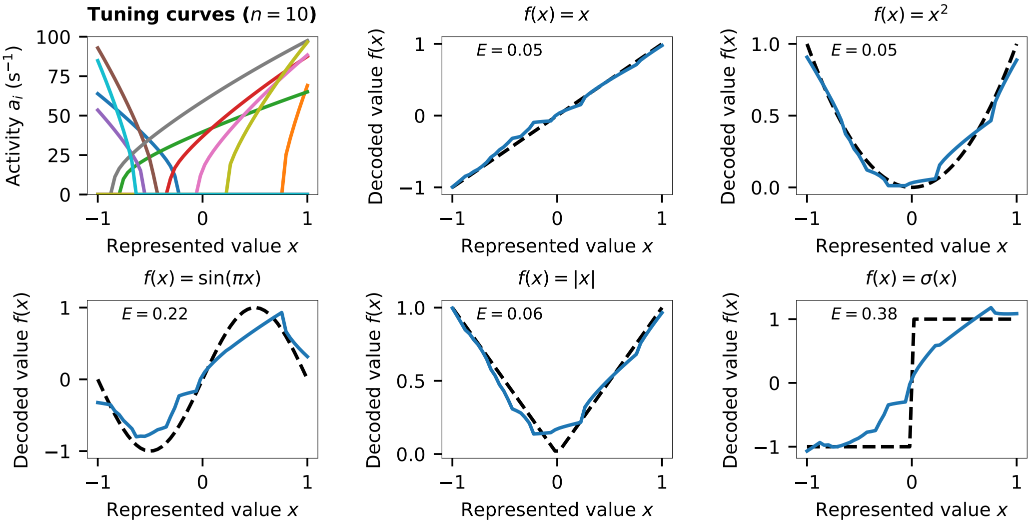


# Computing Functions

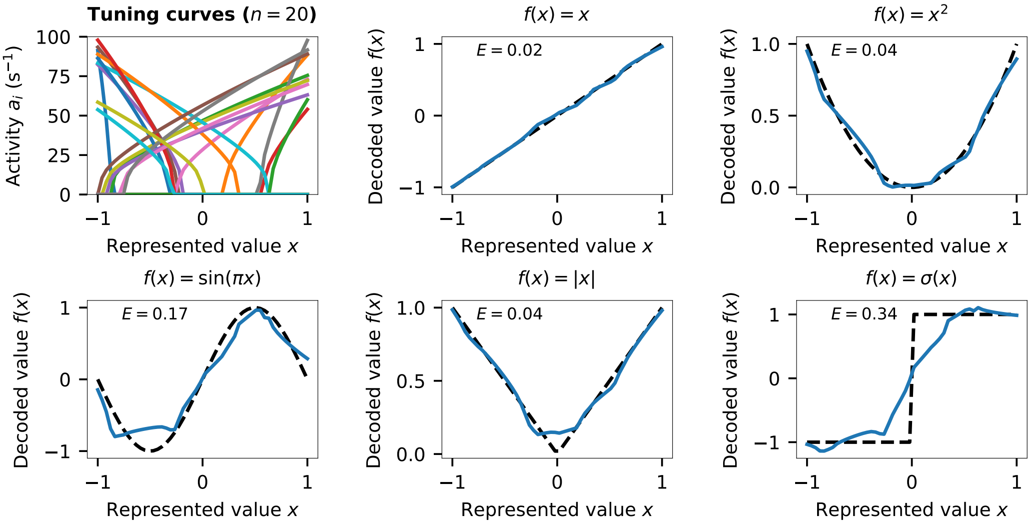



**Function Decoder** $\mathbf{D}^f = \left( (\mathbf{A}\mathbf{A}^\mathsf{T} + N\sigma^2\mathbf{I})^{-1}\mathbf{A}\mathbf{Y}^\mathsf{T} \right)^\mathsf{T}\,, \quad \text{where } (\mathbf{Y})_{ik} = \left(f(\mathbf{x}_k)\right)_i$

# Decoding Functions – Using a Few Neurons



# Decoding Functions – Using More Neurons



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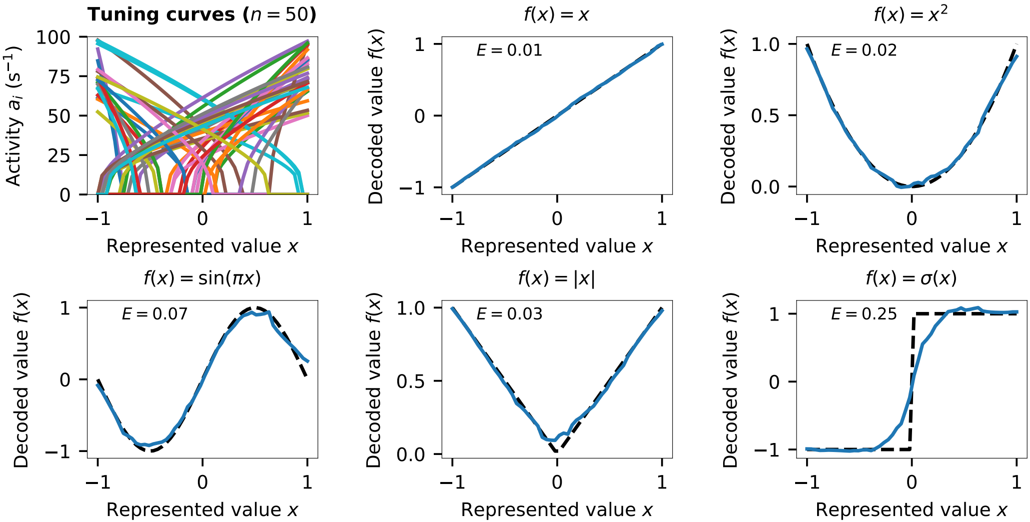

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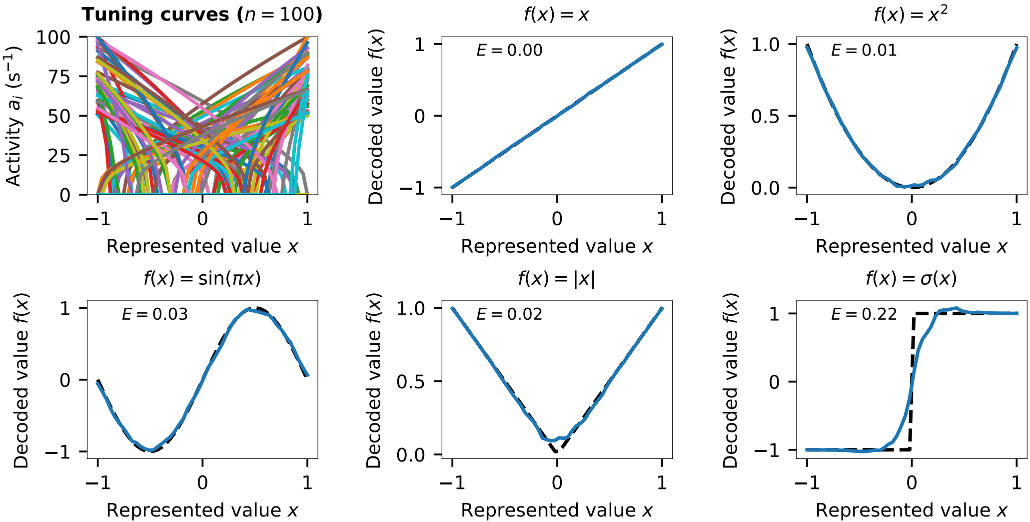
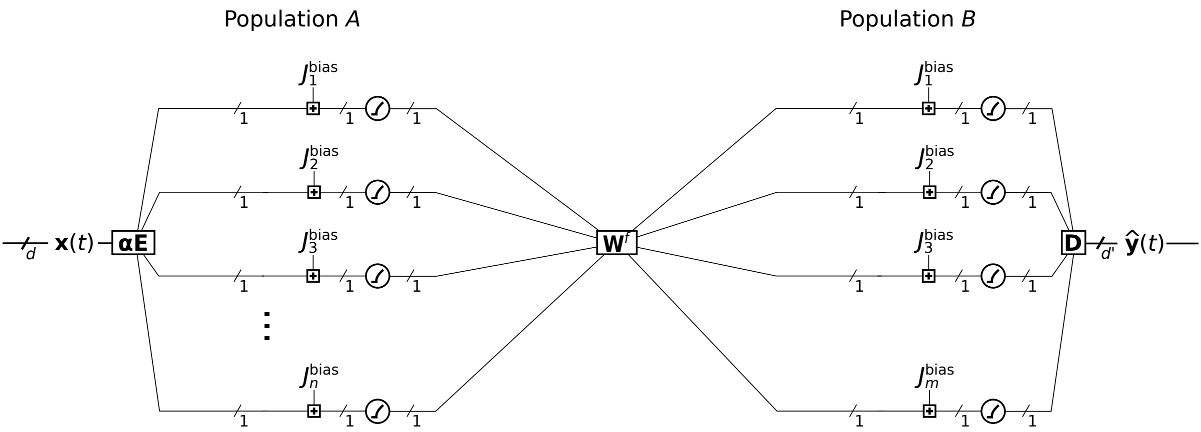


# Computing Functions – Weight Matrix




$$\mathbf{W}^f = \mathbf{E}\mathbf{D}^f$$

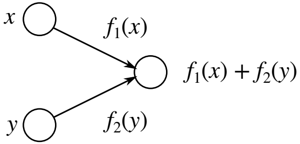
# Computing Multivariate Functions

Homogenous population · Heterogenous population

Linear connection · Inh. connection · Exc. connection

## Linear Superposition

$$W^{f_1}\mathbf{a}_1(\mathbf{x}) + W^{f_2}\mathbf{a}_2(\mathbf{y})$$



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Homogenous population Heterogenous population

Linear connection Inh. connection Exc. connection

## Linear Superposition

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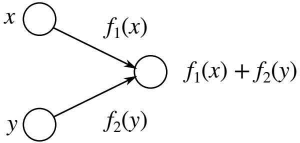
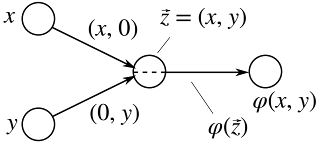



## Nonlinear Functions

Multi-dimensional $\mathbf{z}$



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Homogenous population    Heterogenous population

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## Linear Superposition

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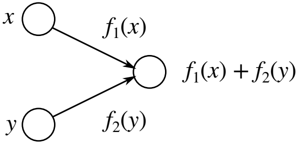
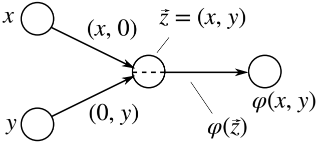


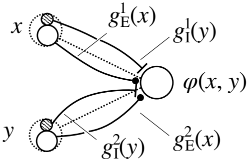

## Nonlinear Functions

Multi-dimensional $\mathbf{z}$



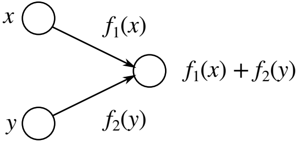

## (Dendritic Computation)

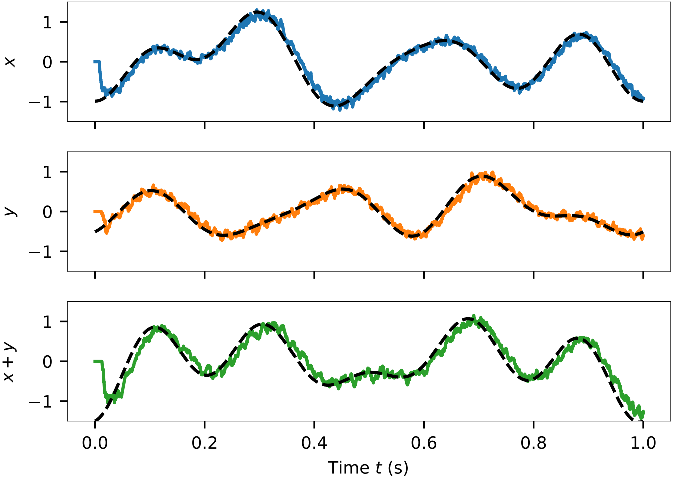
Exploit dendritic nonlinearity



# Computing Multivariate Functions – Linear Superposition

**Linear Superposition**

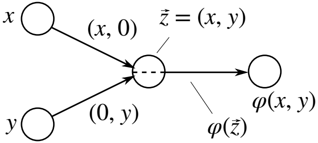


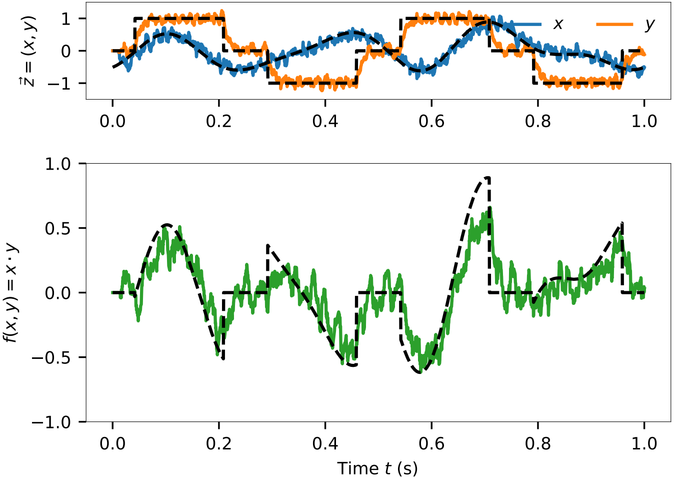



# Computing Multivariate Functions – Multiplication

**Nonlinear Functions**

Multi-dimensional $\mathbf{z}$

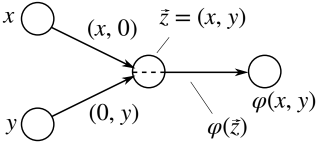


# Computing Multivariate Functions – Multiplication

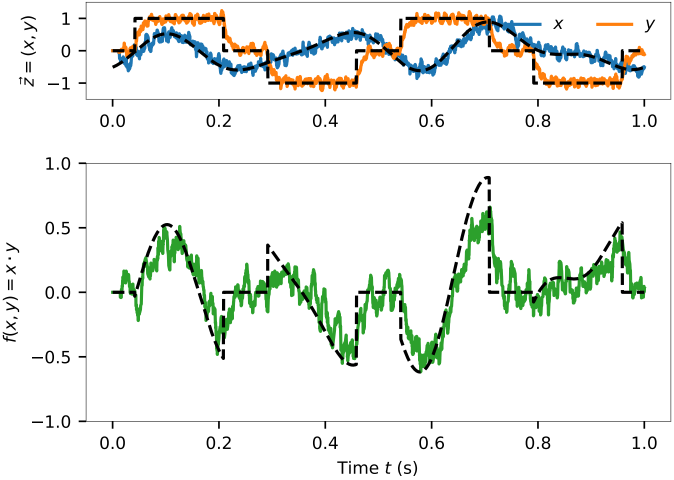
**Nonlinear Functions**

Multi-dimensional $\mathbf{z}$




Multiplication is useful...

- Gating of signals
- Attention effects
- Binding
- Statistical inference



# Image sources

**Title slide**
"Yellow Butterfly"
Author: Albert Bierstadt, circa 1890.
From Wikimedia.