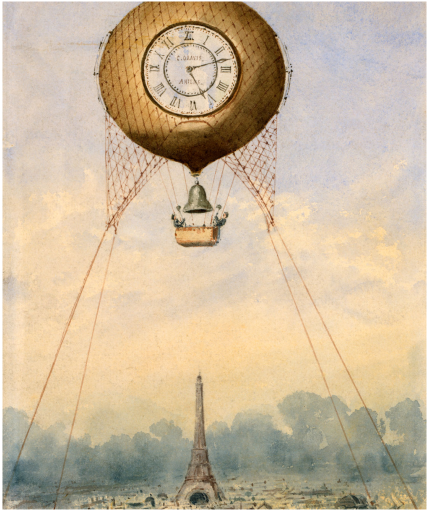
# SYDE 556/750

## Simulating Neurobiological Systems
## Lecture 4: Temporal Representations

Andreas Stöckel

January 22 & 28, 2020



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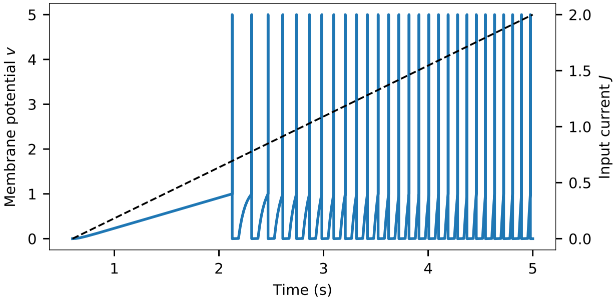
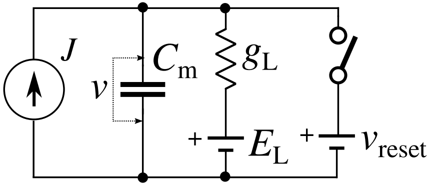
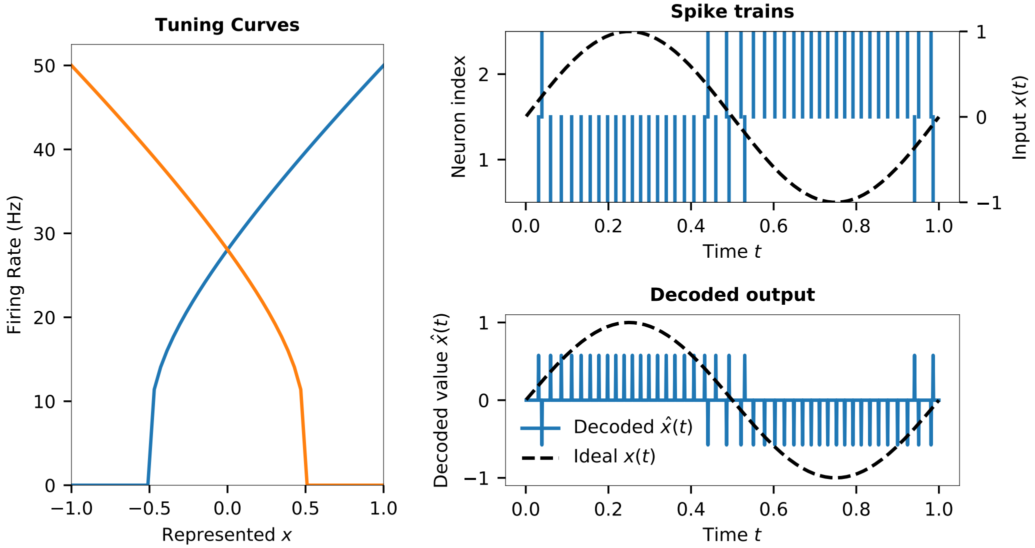
# Reminder: The LIF Neuron



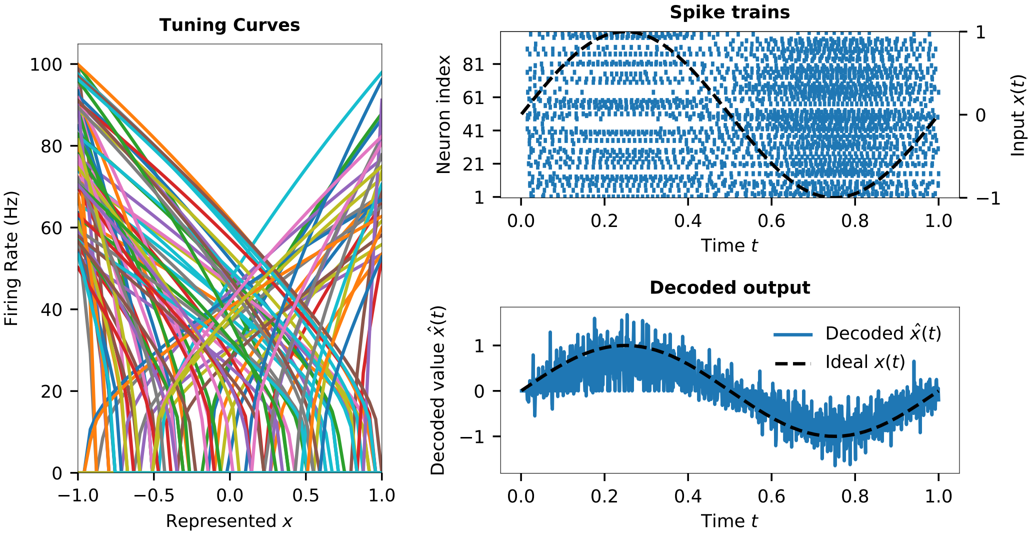




$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} v(t) &= -\frac{1}{\tau_\mathrm{RC}}\big(v(t) - J\big)\,, && \text{if } v(t) < 1\,, \\
v(t) &\leftarrow \delta(t - t_\mathrm{th})\,, && \text{if } t = t_\mathrm{th}\,, \\
v(t) &\leftarrow 0\,, && \text{if } t > t_\mathrm{th} \text{ and } t \geq t_\mathrm{th} + \tau_\mathrm{ref}\,,
\end{aligned}
$$

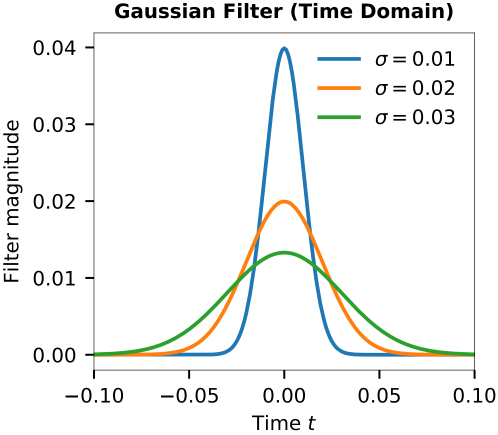
# Temporal Decoding of Two Neurons



# Temporal Decoding of One Hundred Neurons



# Filtering by Convolution




## Gaussian Filter

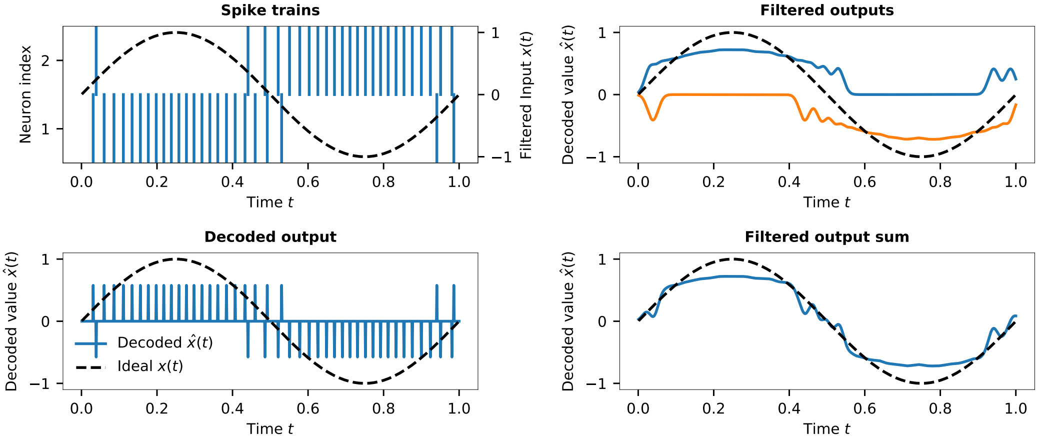
$$h(t) = c\exp\left(\frac{-t^2}{\sigma^2}\right)$$

where $c$ chosen s.t. $\int_{-\infty}^{\infty} h(t)\,\mathrm{d}t = 1$

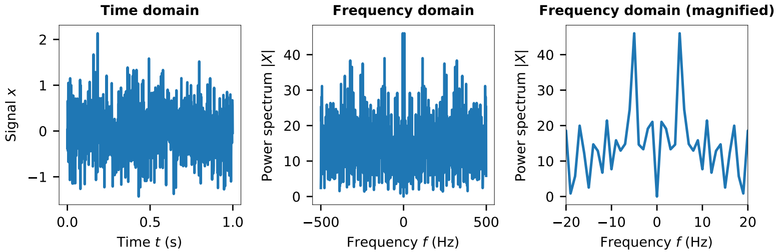
## Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,\mathrm{d}\tau$$

# Filtering a Spike Train

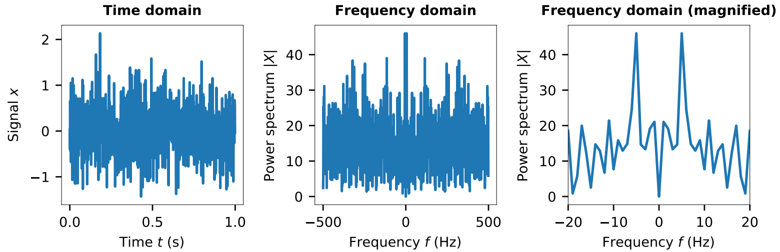


# Random Signals

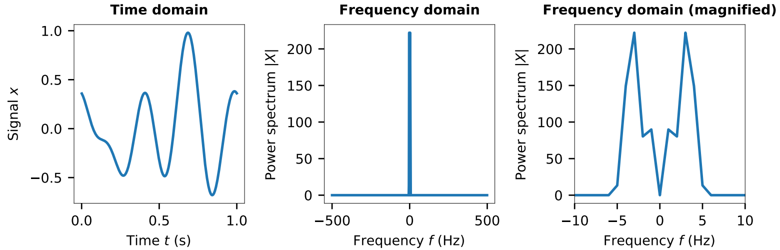



**White Noise**
(zero mean)

# Random Signals

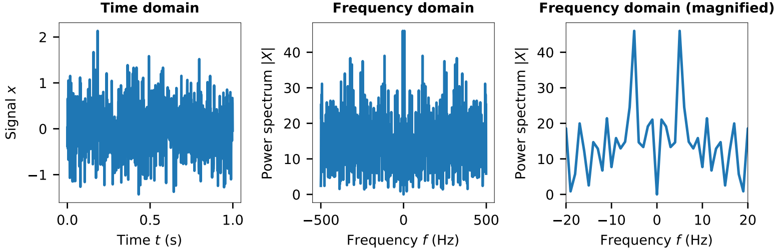



**White Noise**
(zero mean)

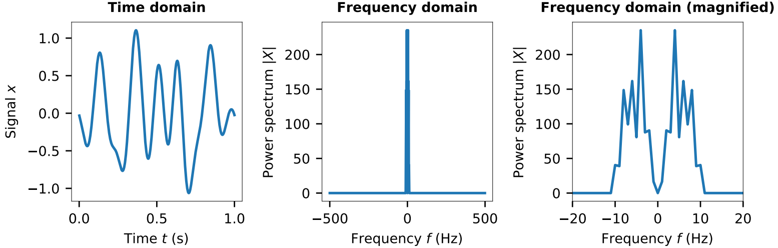



**Bandlimited**
White Noise
(zero mean,
5 Hz bandwidth)

# Random Signals

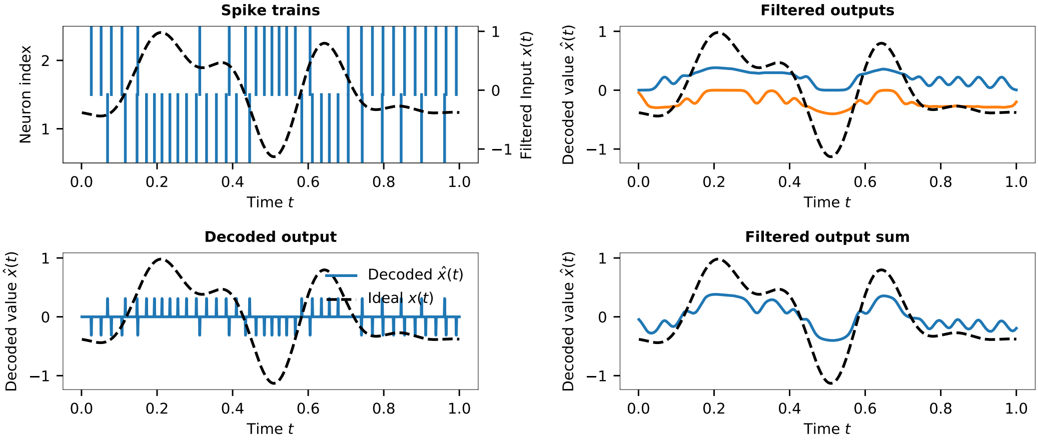



**White Noise**
(zero mean)

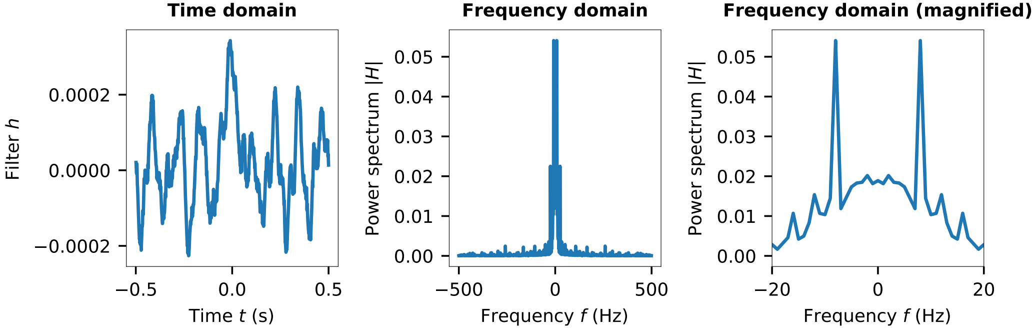



**Bandlimited**
White Noise
(zero mean,
10 Hz bandwidth)

# Filtering a Spike Train for a Random Signal

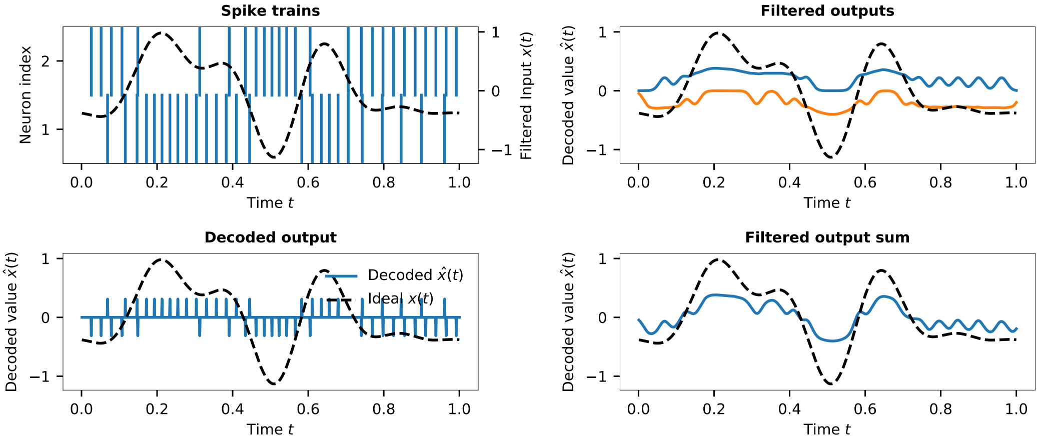


# Optimal Filter

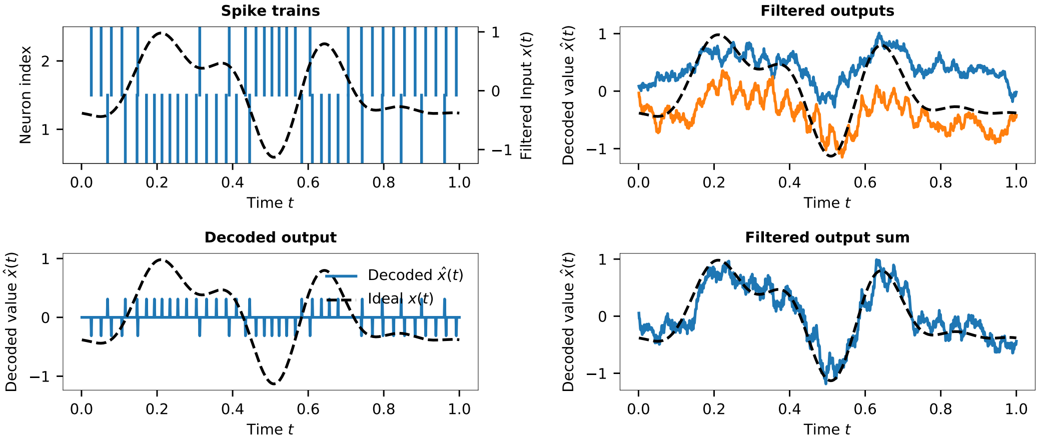



$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

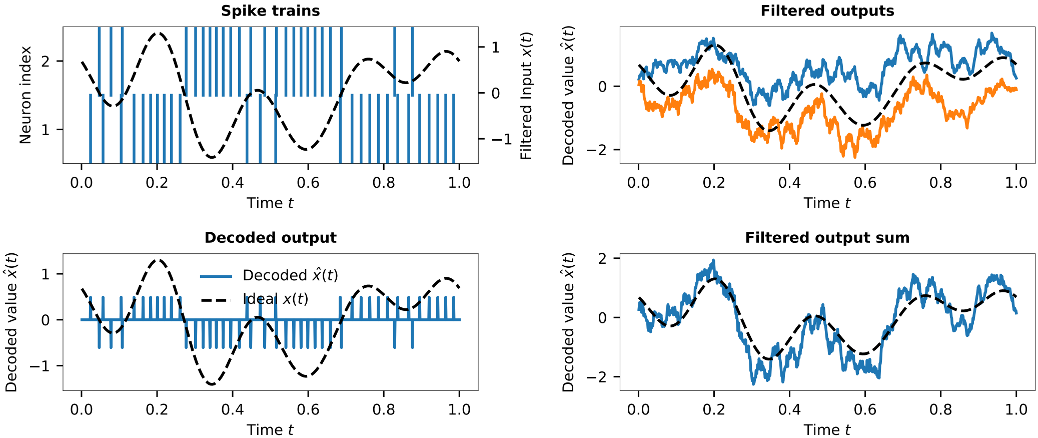
# Filtering a Spike Train for a Random Signal (Optimal Filter)



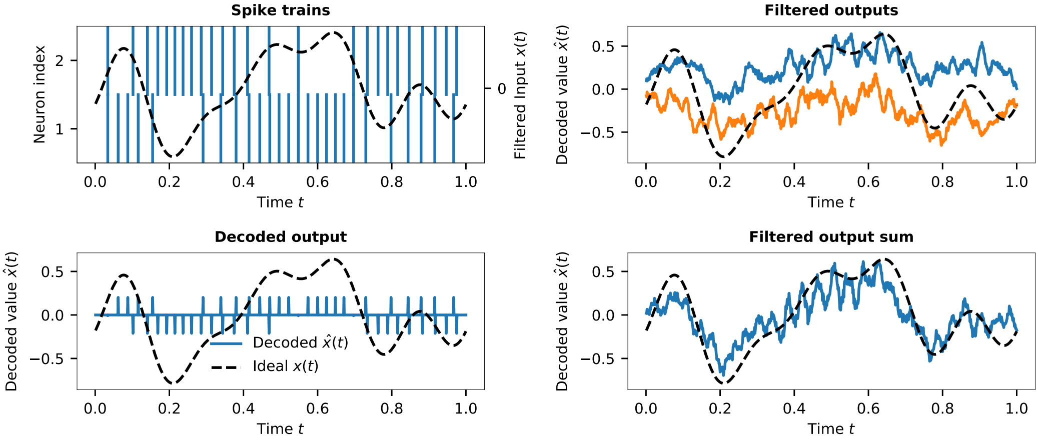
# Filtering a Spike Train for a Random Signal (Optimal Filter)



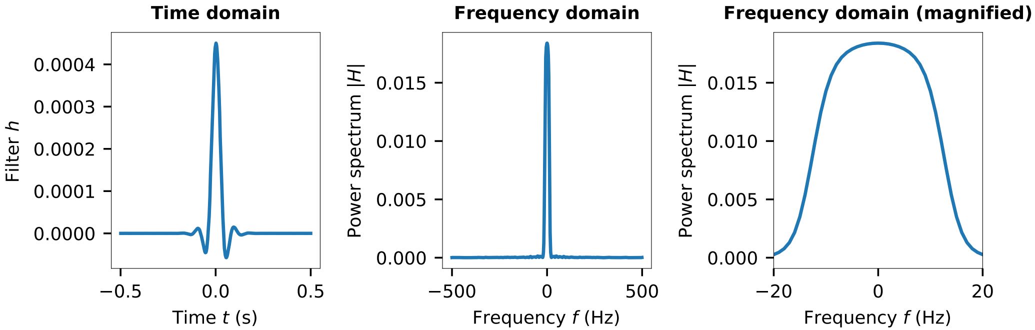
# Filtering a Spike Train for a Random Signal (Optimal Filter)



# Filtering a Spike Train for a Random Signal (Optimal Filter)

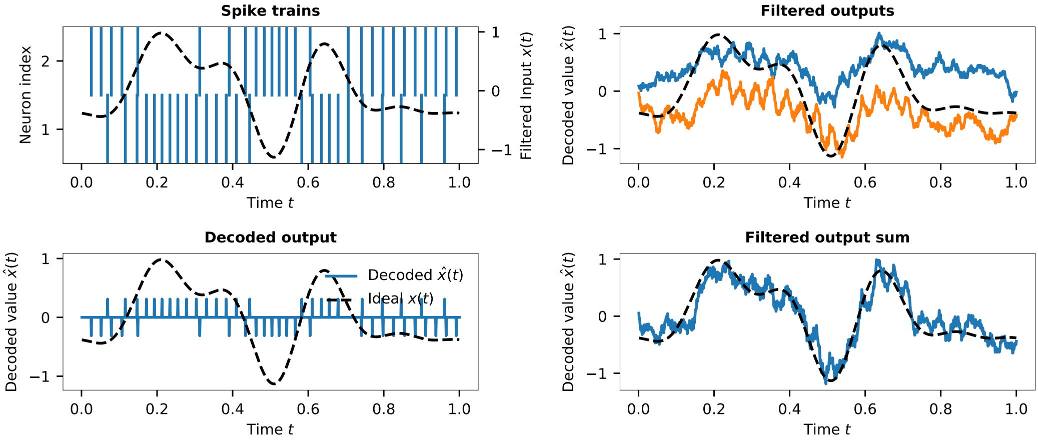


# Optimal Filter (Improved)

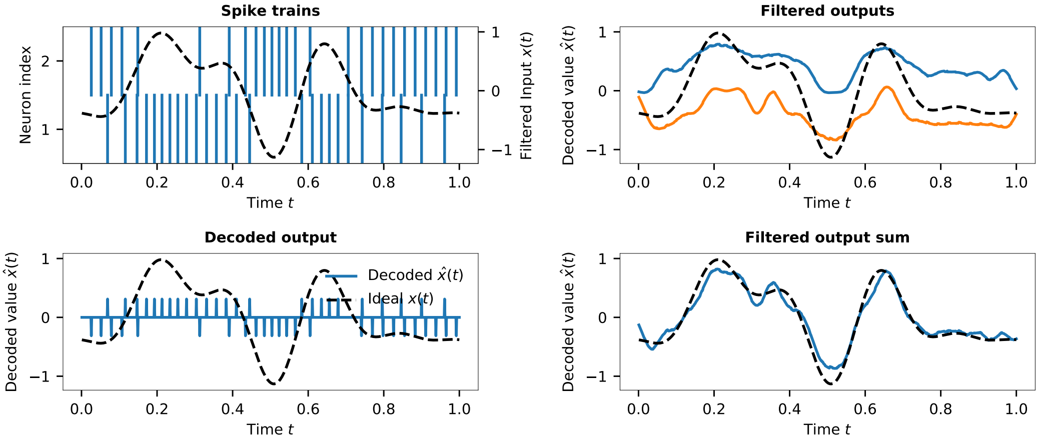



$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$

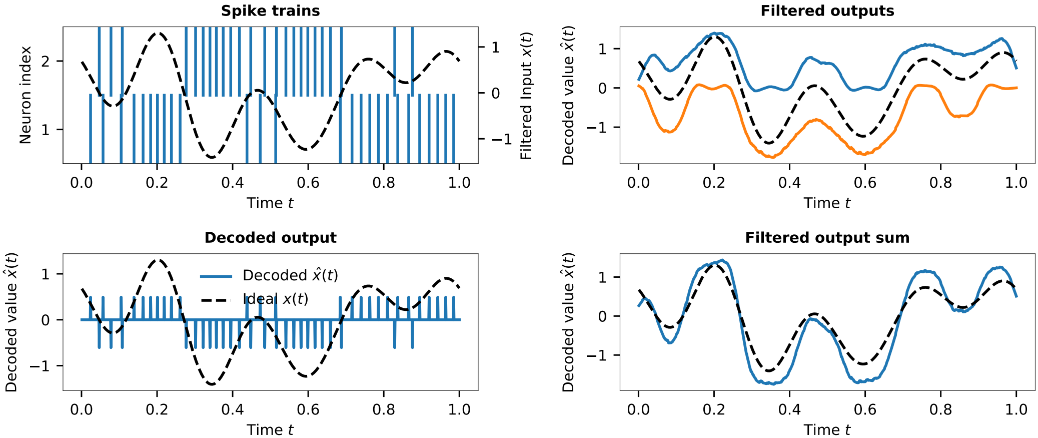
# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



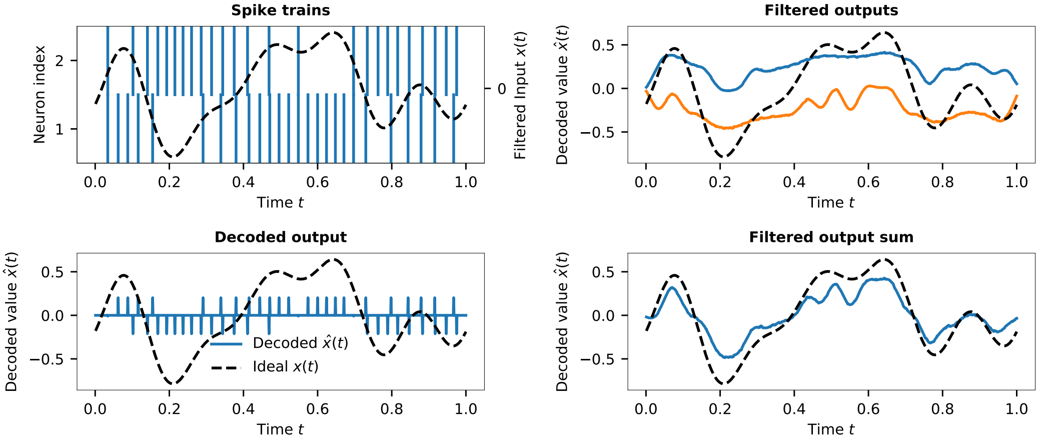
# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



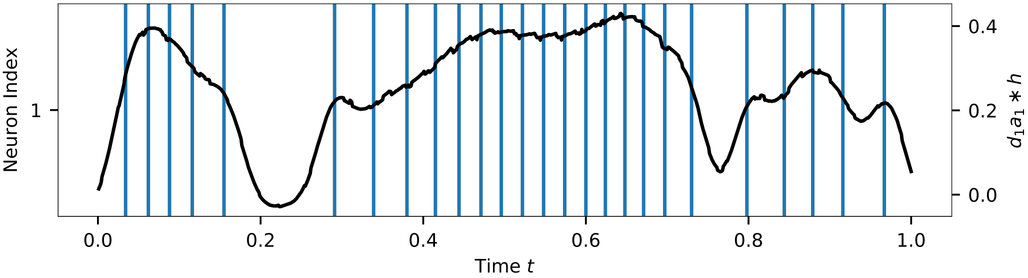
# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



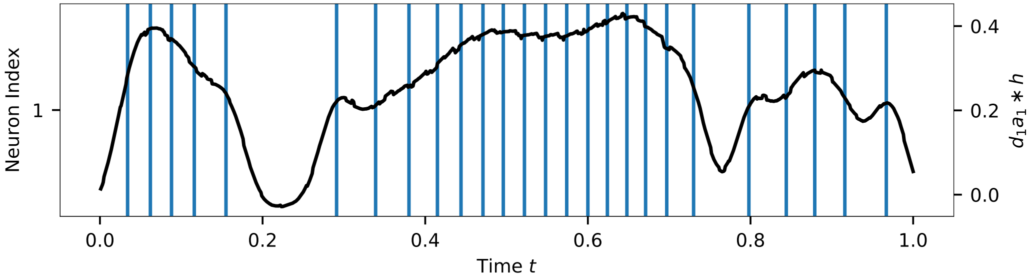
# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



# Pros and Cons of the Optimal Filter



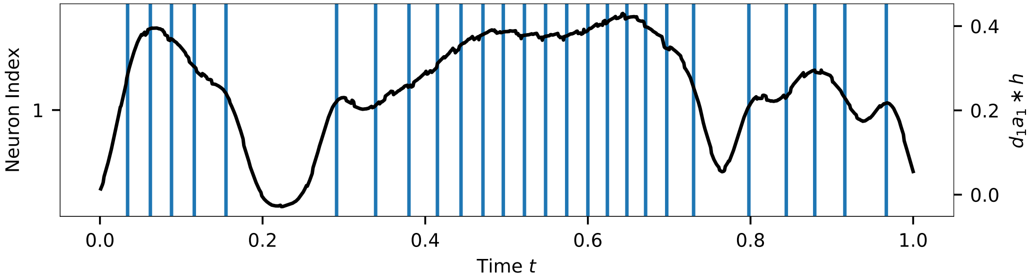
# Pros and Cons of the Optimal Filter




**Precise**

Good for analysing data after the fact

# Pros and Cons of the Optimal Filter



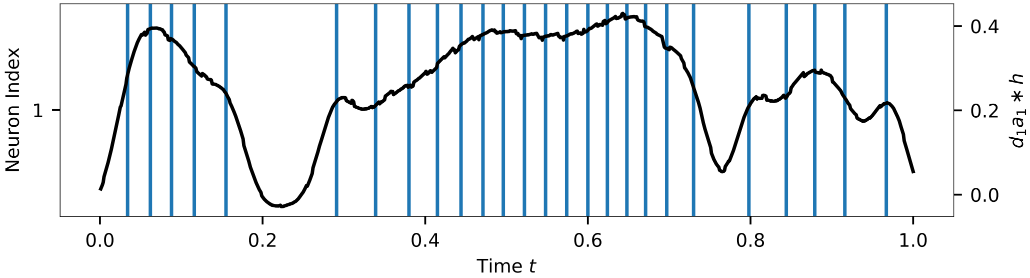

**Precise**

Good for analysing data after the fact

**Non-causal**

Does not describe a biological process

# Pros and Cons of the Optimal Filter

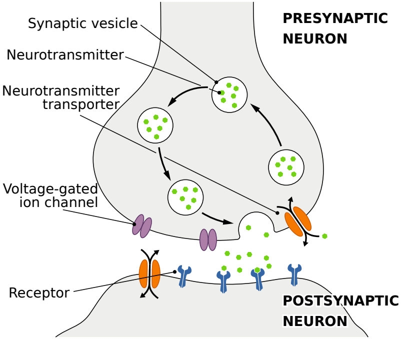


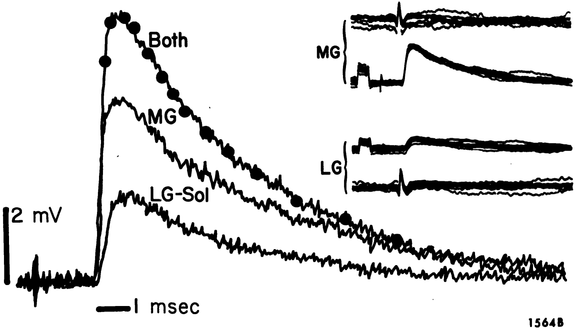

**Precise**
Good for analysing data after the fact

**Non-causal**
Does not describe a biological process

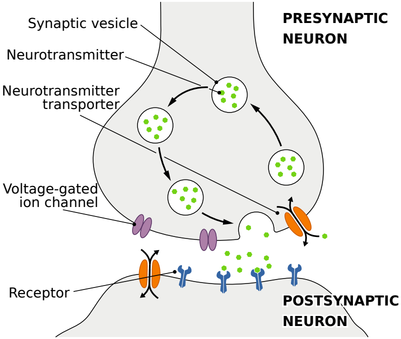
We need to find a mechanism that low-pass filters spikes over time!

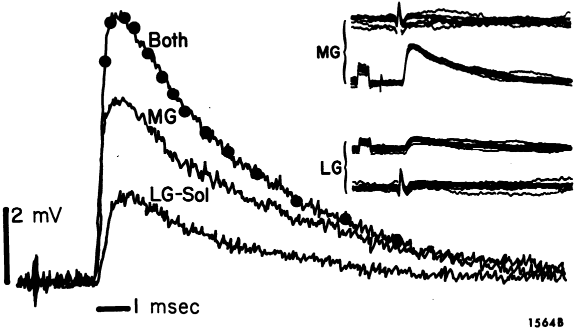
# Synapses as Filters



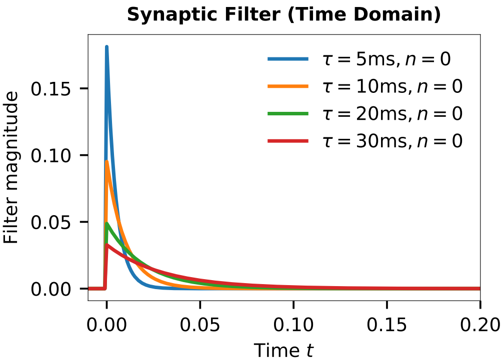
# Synapses as Filters

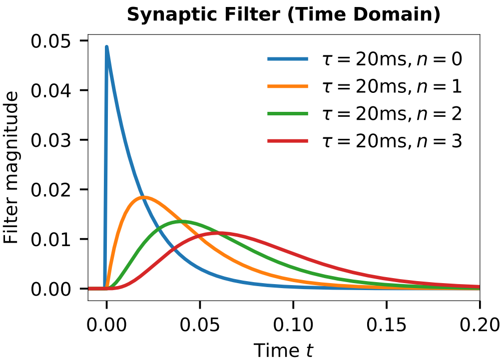



Post-synaptic currents (EPSCs, IPSCs) are low-pass filtered spike trains!

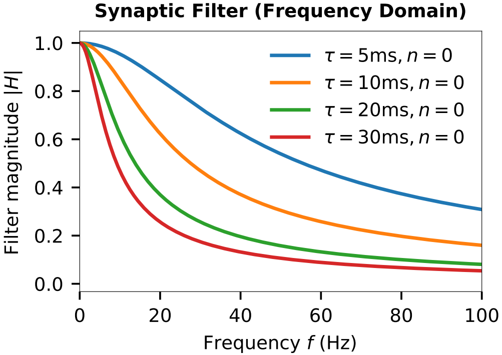
# Exponential Low-Pass Filter (I)

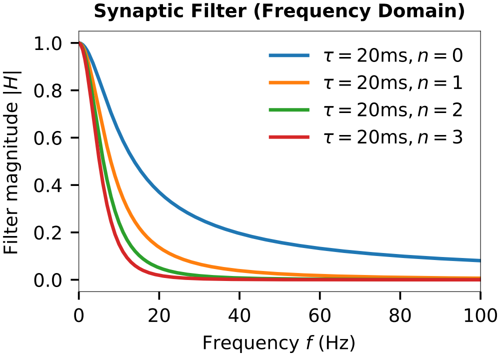



$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0\,, \\ 0 & \text{otherwise}\,, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t\,.$$

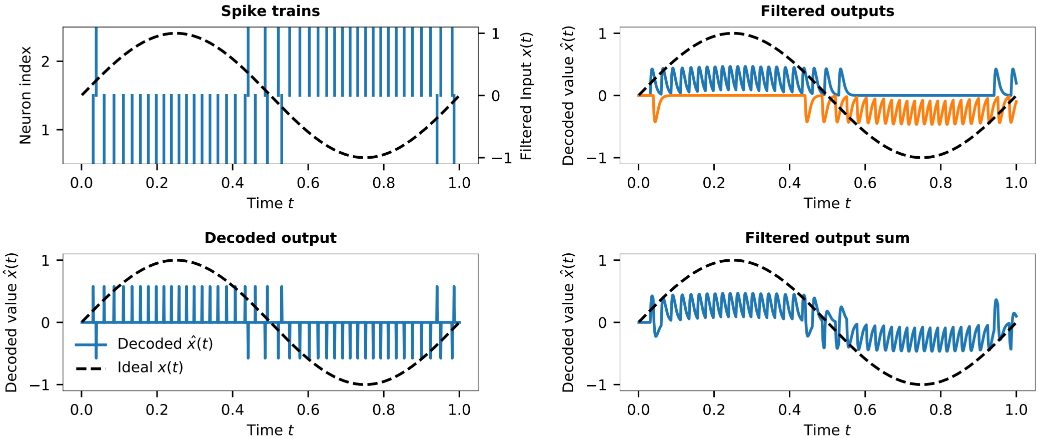
# Exponential Low-Pass Filter (II)



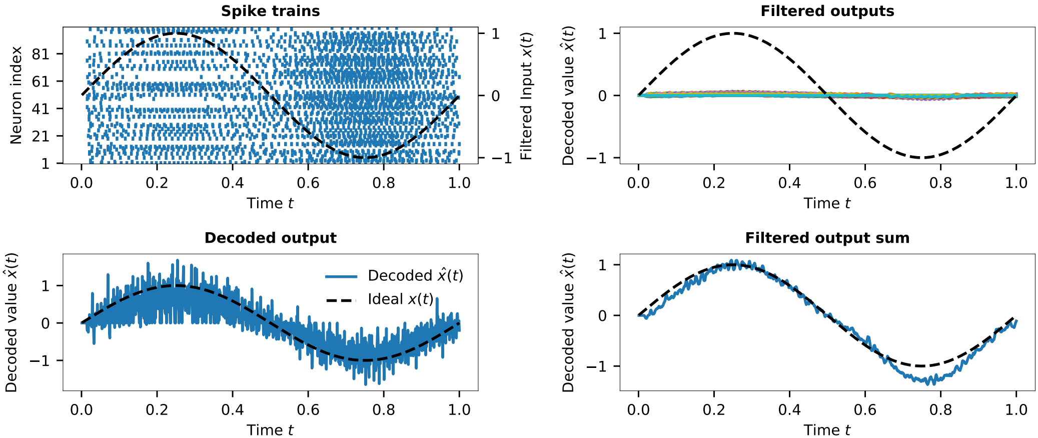

$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0 \,, \\ 0 & \text{otherwise} \,, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t \,.$$

# Example: Synaptic Filter for Two Neurons




$$\tau = 5\,\mathrm{ms},\ n = 1$$

# Example: Synaptic Filter for One Hundred Neurons




$\tau = 5\,\text{ms}, n = 1$

# Image sources

**Title slide**
“Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France.”
Author: Camille Grávis, between 1889 and 1900.
From Wikimedia.