Statistical Rethinking
2023

15. Social Networks

What Motivates Sharing?

"Up in our country we are human! And since we are human we help each other. We don't like to hear anybody say thanks for that. What I get today you may get tomorrow. Up here we say that by gifts one makes slaves and by whips one makes dogs."

Quoted in Peter Freuchen's 1961 book about the Inuit

What Motivates Sharing?

data(KosterLeckie)

Year of food transfers among 25 households in Arang Dak

25!/(2!(25-2)!) = 300 dyads

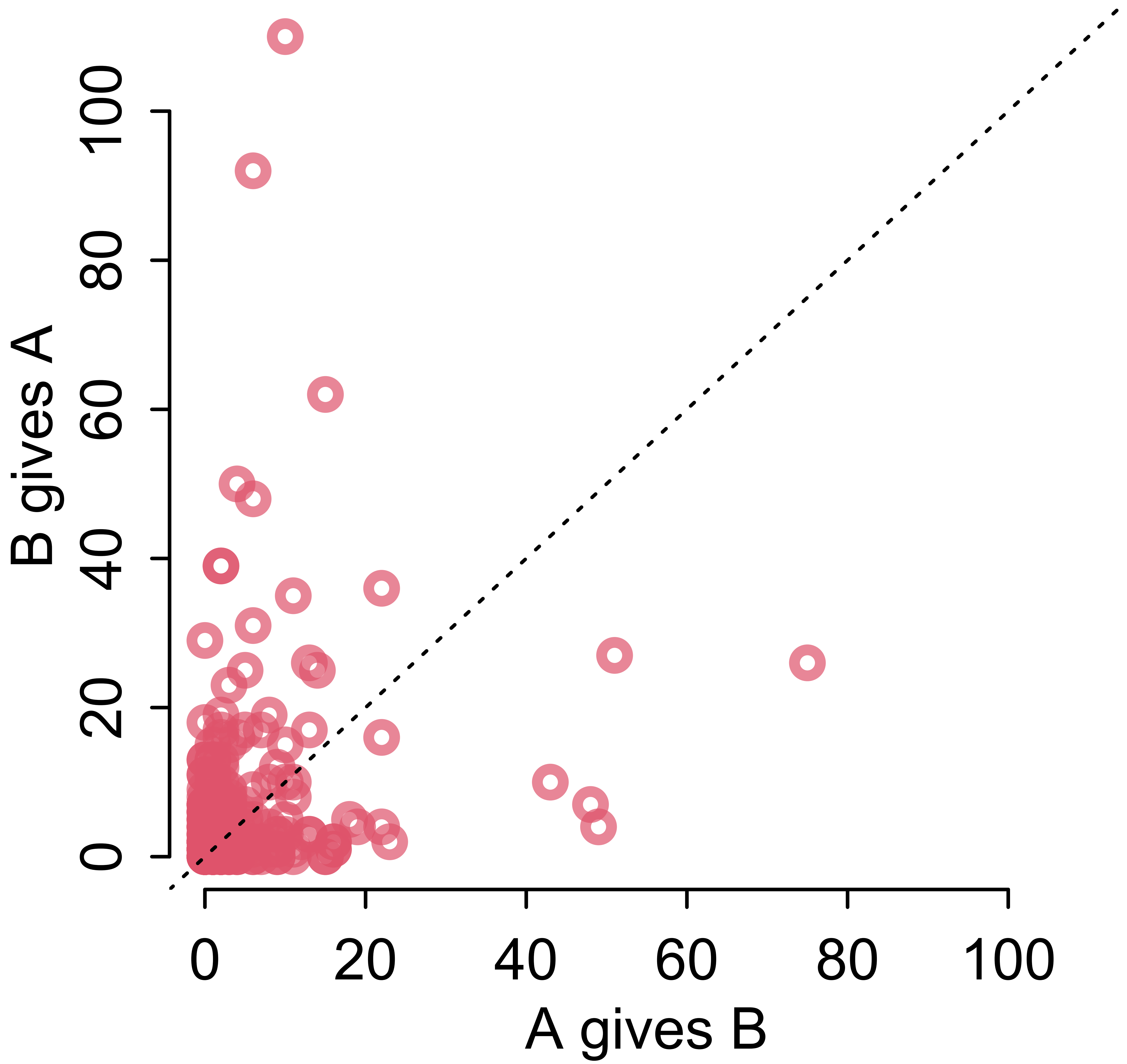
2871 observed transfers between households

How much sharing explained by reciprocity?
How much by generalized giving?

Koster & Leckie 2014

Photo: Dr Karl Frost

What Motivates Sharing?



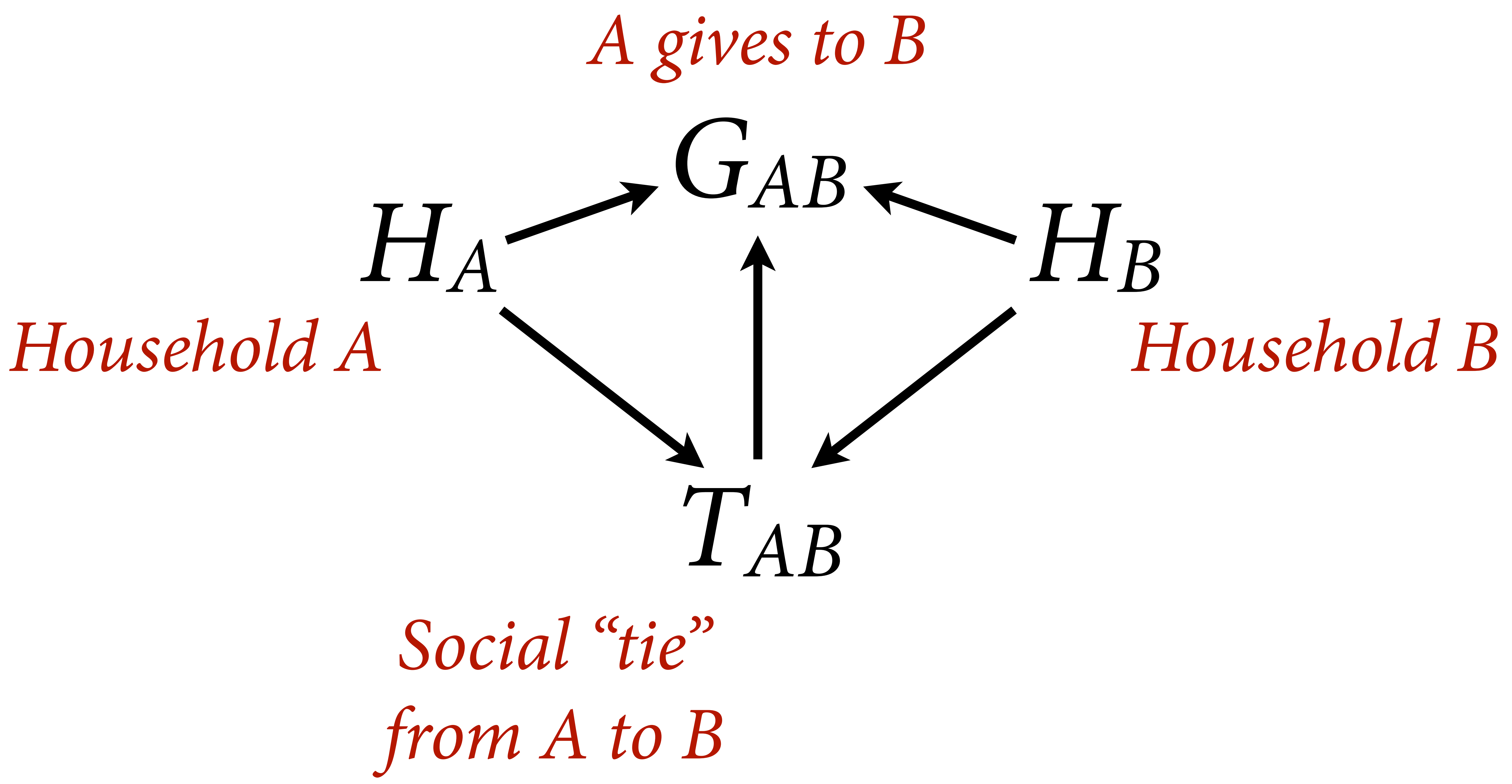
Koster & Leckie 2014

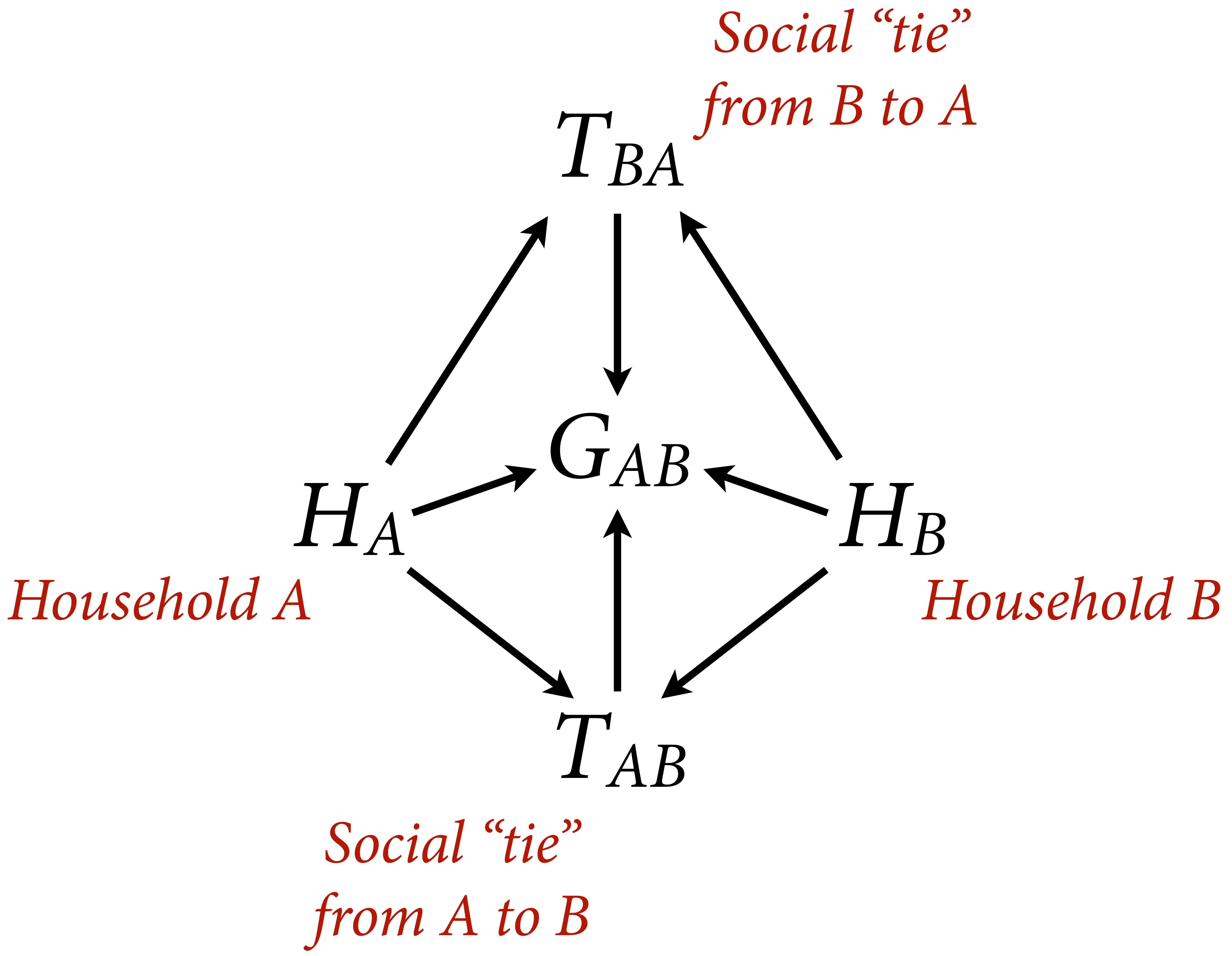
A gives to B

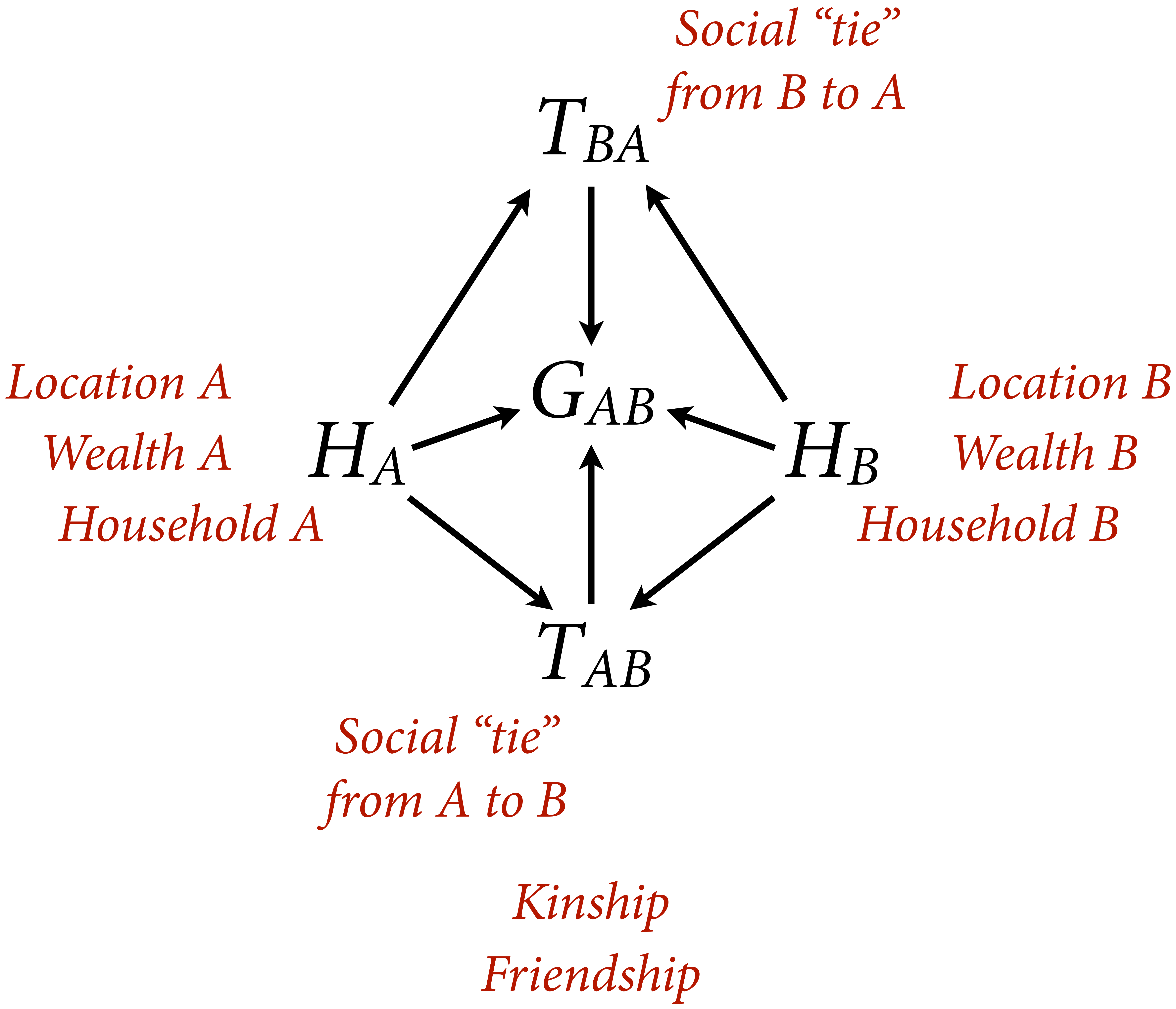
$$\begin{tikzcd} & G_{AB} & \\ H_A \arrow[ur] & & H_B \arrow[ul] \end{tikzcd}$$

Household A

Household B


A gives to B
G_{AB}
H_A
H_B
Household A
Household B
T_{AB}
Social "tie"
from A to B


Social "tie"
from B to A
T_{BA}
G_{AB}
H_A
Household A
H_B
Household B
T_{AB}
Social "tie"
from A to B


Social "tie"
from B to A
T_{BA}
Location A
Wealth A
Household A
H_A
G_{AB}
H_B
Location B
Wealth B
Household B
T_{AB}
Social "tie"
from A to B
Kinship
Friendship

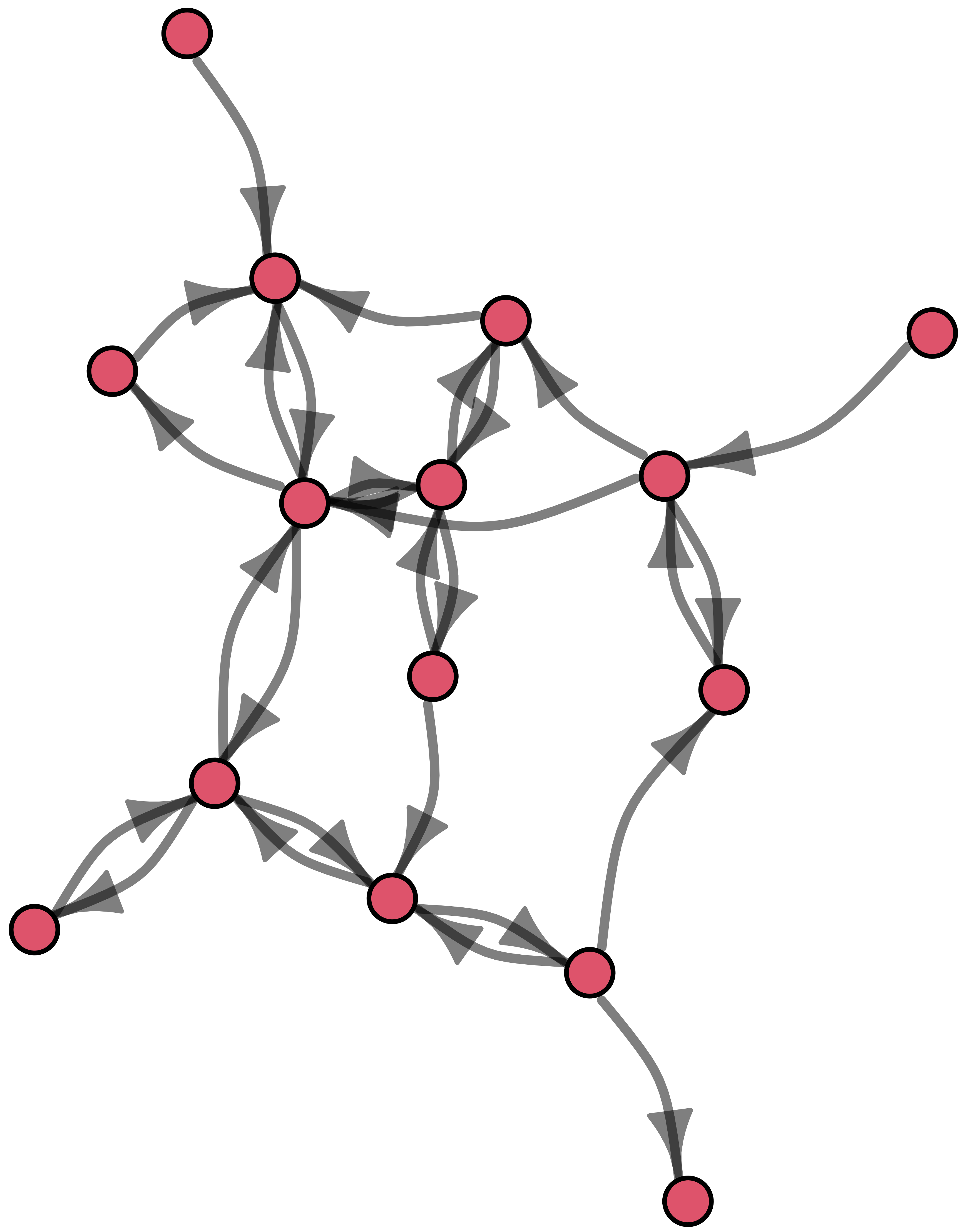
What Motivates Sharing?

T_{AB} and T_{BA} are not observable

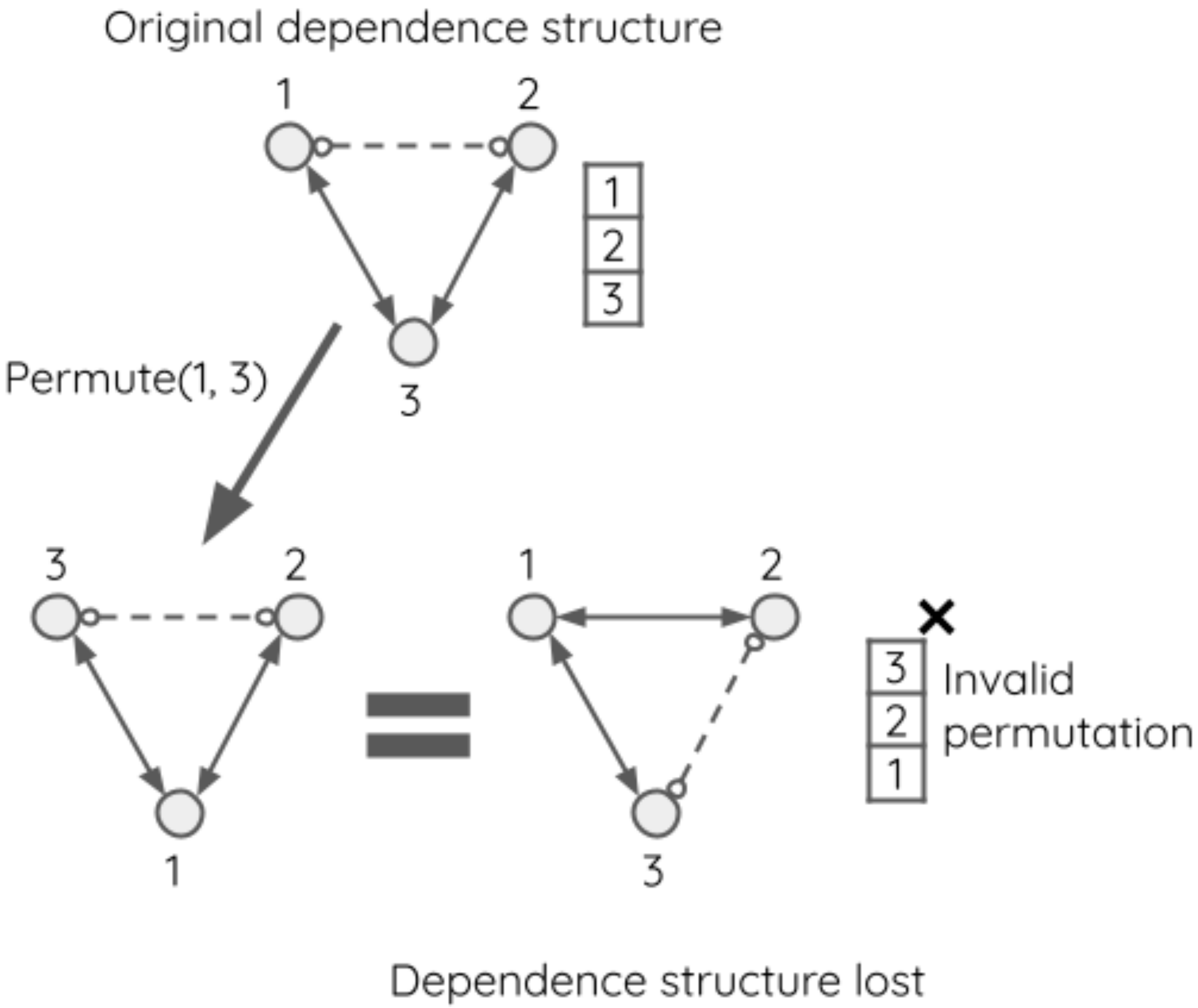
Social network: Pattern of directed exchange

Social networks are abstractions, are not data

What is a principled approach?



Resist Adhockery



Hart et al 2021 Common Permutation Methods in Animal Social Network Analysis Do Not Control for Non-independence

Drawing the Social Owl

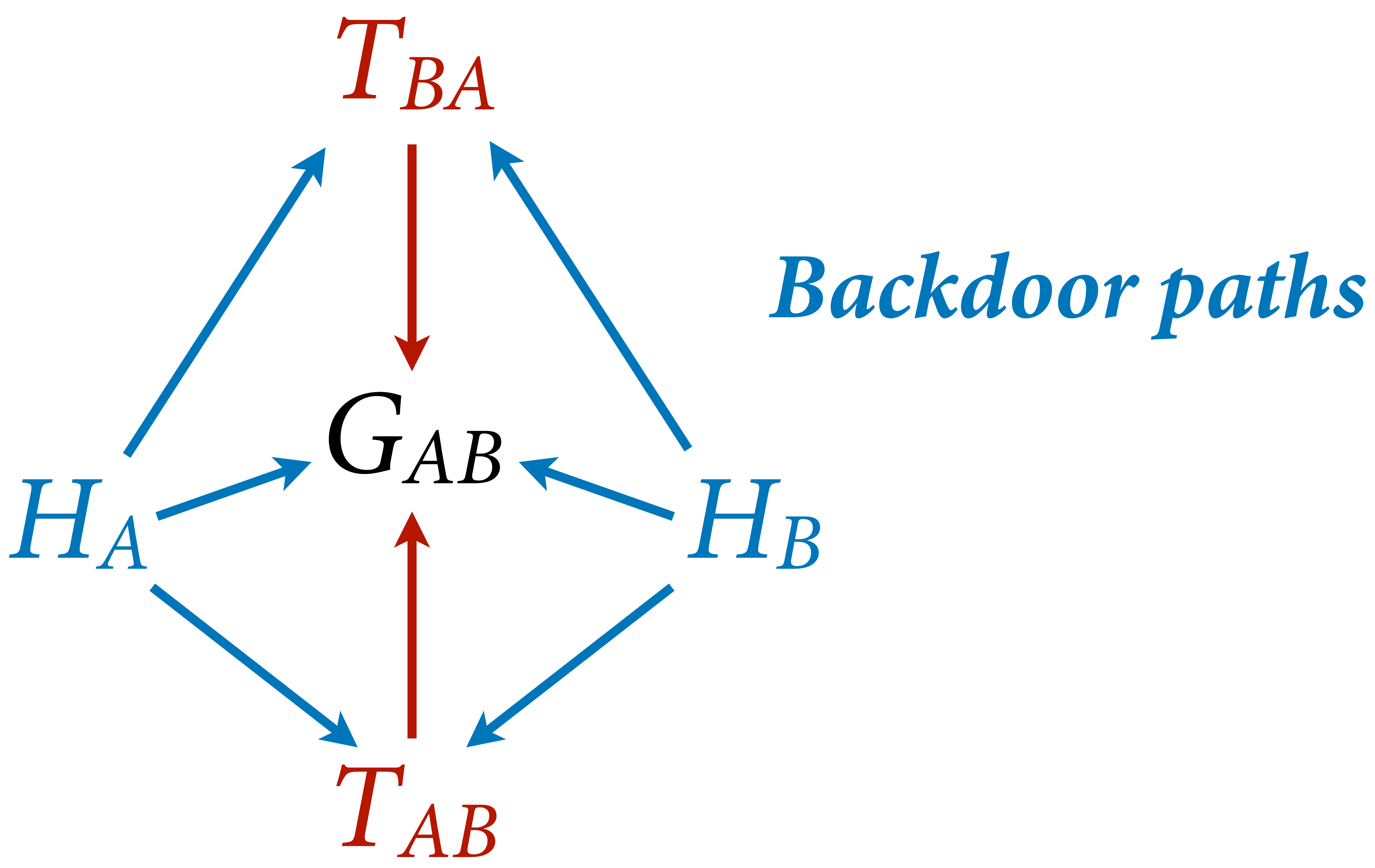
(1) Estimand: Reciprocity & what explains it

(2) Generative model

(3) Statistical model

(4) Analyze sample

$$
\begin{tikzcd}
 & T_{BA} \arrow[d] & \\
H_A \arrow[ur] \arrow[r] \arrow[dr] & G_{AB} & H_B \arrow[ul] \arrow[l] \arrow[dl] \\
 & T_{AB} \arrow[u] &
\end{tikzcd}
$$



T_{BA}
G_{AB}
H_A
H_B
T_{AB}
Backdoor paths

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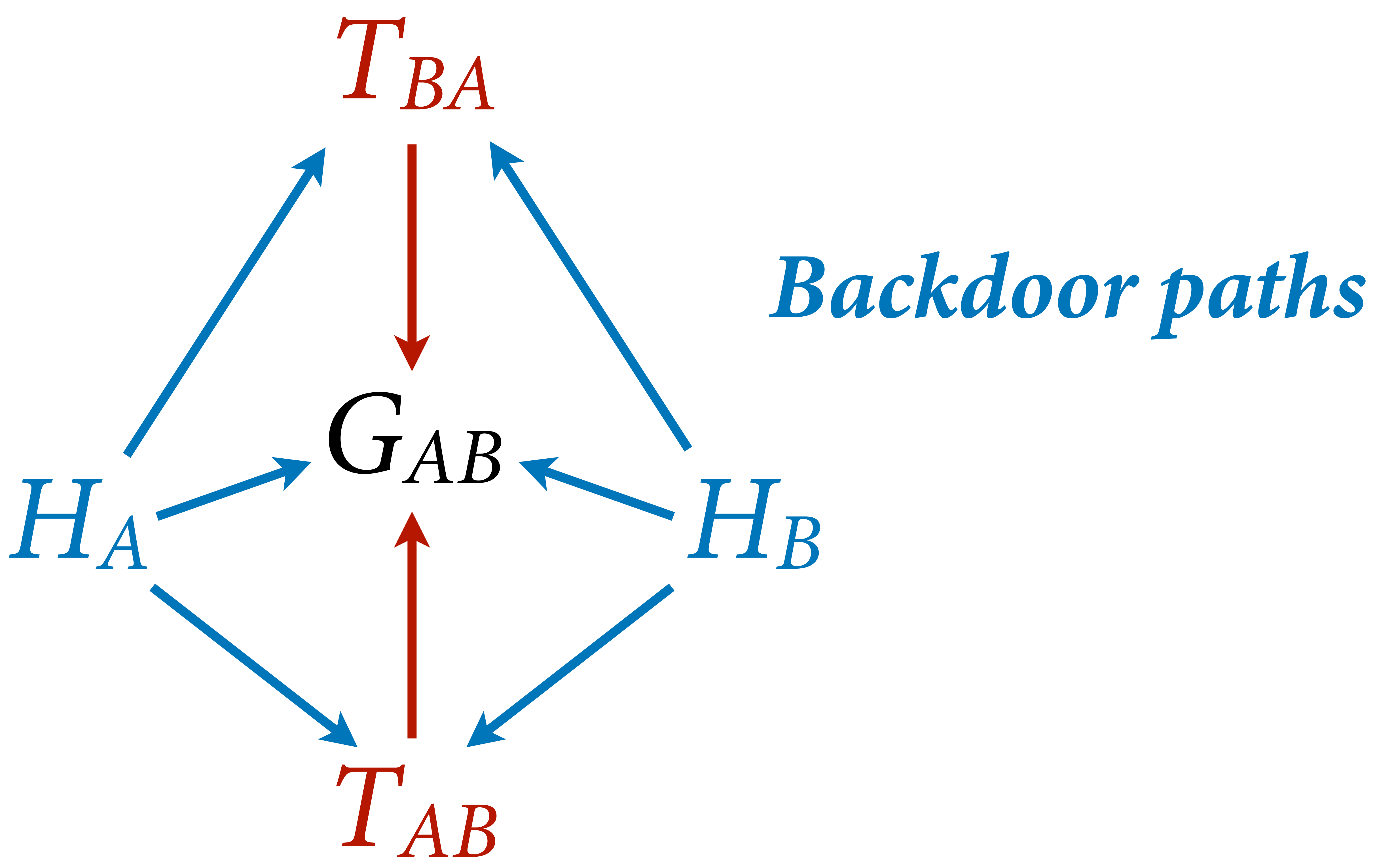
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H_A \arrow[ur] \arrow[dr] & G_{AB} & H_B \arrow[ul] \arrow[dl] \\
 & T_{AB} \arrow[u] &
\end{tikzcd}$$

Begin simple, model tie formation

```r
# N households
N <- 25
dyads <- t(combn(N,2))
N_dyads <- nrow(dyads)
```

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```

```
> dyads
      [,1] [,2]
 [1,]    1    2
 [2,]    1    3
 [3,]    1    4
 [4,]    1    5
 [5,]    1    6
 [6,]    1    7
 [7,]    1    8
 [8,]    1    9
 [9,]    1   10
[10,]    1   11
[11,]    1   12
[12,]    1   13
[13,]    1   14
[14,]    1   15
[15,]    1   16
[16,]    1   17
[17,]    1   18
[18,]    1   19
[19,]    1   20
[20,]    1   21
[21,]    1   22
[22,]    1   23
[23,]    1   24
[24,]    1   25
[25,]    2    3
[26,]    2    4
[27,]    2    5
[28,]    2    6
[29,]    2    7
[30,]    2    8
[31,]    2    9
[32,]    2   10
[33,]    2   11
[34,]    2   12
[35,]    2   13
[36,]    2   14
[37,]    2   15
[38,]    2   16
[39,]    2   17
[40,]    2   18
[41,]    2   19
[42,]    2   20
[43,]    2   21
[44,]    2   22
[45,]    2   23
[46,]    2   24
[47,]    2   25
[48,]    3    4
[49,]    3    5
[50,]    3    6
[51,]    3    7
[52,]    3    8
[53,]    3    9
[54,]    3   10
[55,]    3   11
[56,]    3   12
[57,]    3   13
[58,]    3   14
[59,]    3   15
[60,]    3   16
[61,]    3   17
[62,]    3   18
[63,]    3   19
[64,]    3   20
[65,]    3   21
[66,]    3   22
[67,]    3   23
[68,]    3   24
[69,]    3   25
[70,]    4    5
[71,]    4    6
[72,]    4    7
[73,]    4    8
[74,]    4    9
[75,]    4   10
[76,]    4   11
[77,]    4   12
[78,]    4   13
[79,]    4   14
[80,]    4   15
[81,]    4   16
[82,]    4   17
[83,]    4   18
[84,]    4   19
[85,]    4   20
[86,]    4   21
[87,]    4   22
[88,]    4   23
[89,]    4   24
[90,]    4   25
[91,]    5    6
```

```r
# N households
N <- 25
dyads <- t(combn(N,2))
N_dyads <- nrow(dyads)

# simulate "friendships" in which ties are shared
f <- rbern(N_dyads,0.1) # 10% of dyads are friends
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# there can be ties that are not reciprocal
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y <- matrix(NA,N,N) # matrix of ties
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for ( i in 1:N ) for ( j in 1:N ) {
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        # directed tie from i to j
        ids <- sort( c(i,j) )
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        p_tie <- f[the_dyad] + (1-f[the_dyad])*inv_logit( alpha )
        y[i,j] <- rbern( 1 , p_tie )
    }
}#ij
```

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```

friends share ties

```
# now simulate gifts
giftsAB <- rep(NA,N_dyads)
giftsBA <- rep(NA,N_dyads)
lambda <- log(c(0.5,2)) # rates of giving for y=0,y=1
for ( i in 1:N_dyads ) {
    A <- dyads[i,1]
    B <- dyads[i,2]
    giftsAB[i] <- rpois( 1 , exp( lambda[1+y[A,B]] ) )
    giftsBA[i] <- rpois( 1 , exp( lambda[1+y[B,A]] ) )
}
```

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    giftsAB[i] <- rpois( 1 , exp( lambda[1+y[A,B]] ) )
    giftsBA[i] <- rpois( 1 , exp( lambda[1+y[B,A]] ) )
}
```
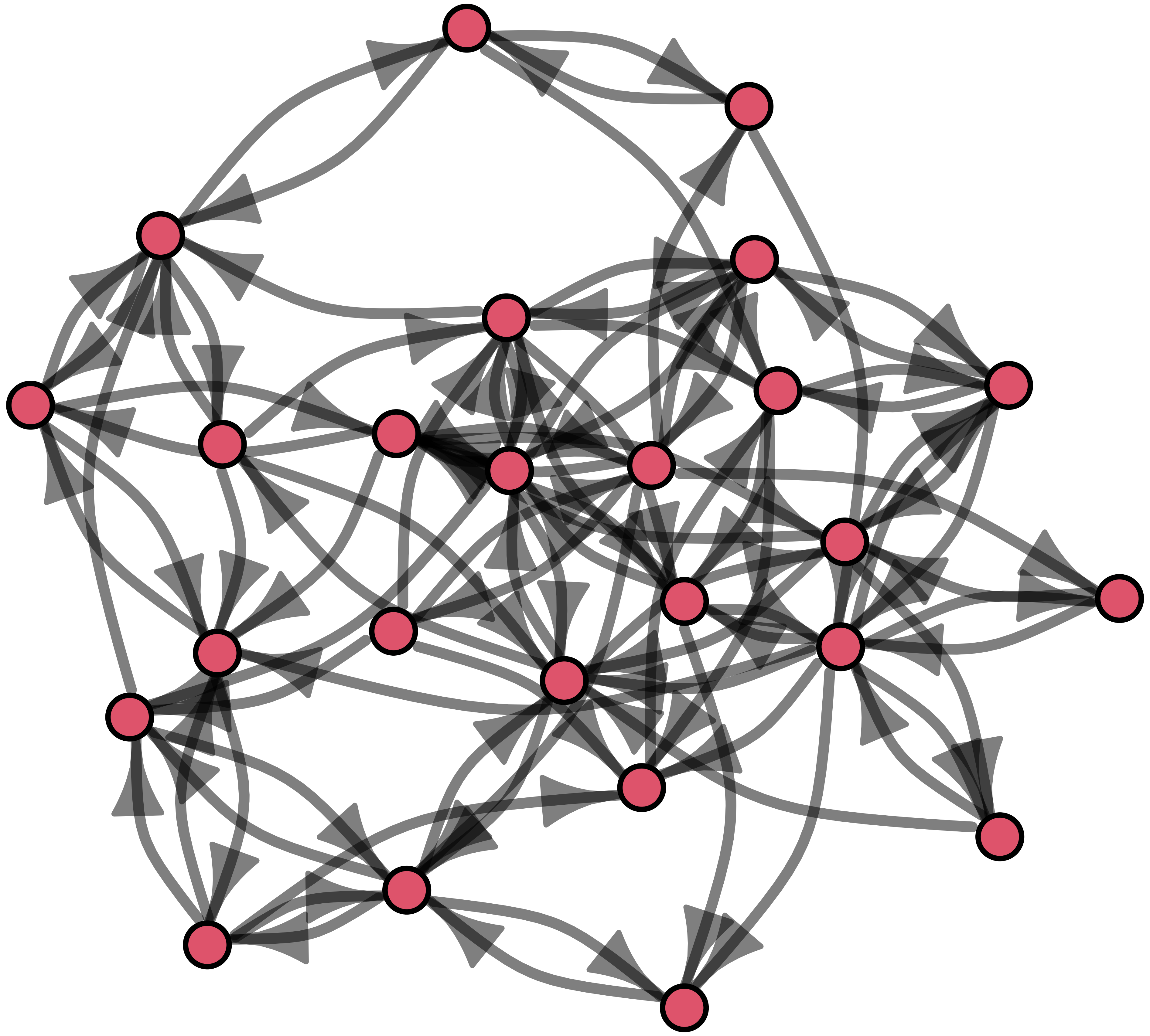

```r
# draw network
library(igraph)
sng <- graph_from_adjacency_matrix(y)
lx <- layout_nicely(sng)
vcol <- "#DE536B"
plot(sng , layout=lx , vertex.size=8 ,
edge.arrow.size=0.75 , edge.width=2 ,
edge.curved=0.35 , vertex.color=vcol ,
edge.color=grau() , asp=0.9 , margin = -0.05 ,
vertex.label=NA )
```



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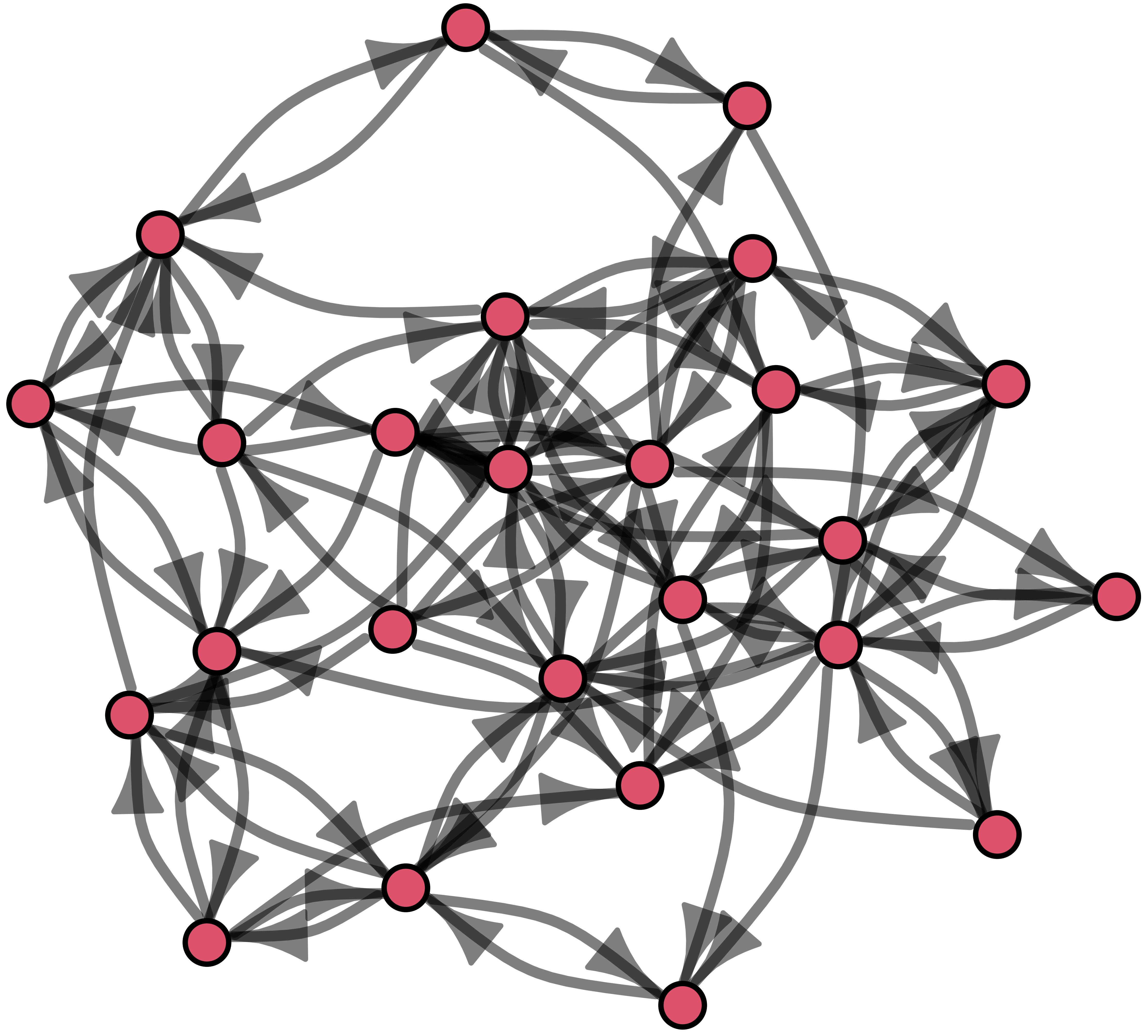

Gifts A to B

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB}$$

average

Tie A to B

Gifts A to B

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB}$$

average

Tie A to B

Gifts B to A

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA}$$

Tie B to A

$$G_{AB} \sim \text{Poisson}(\lambda_{AB}) \qquad G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} \qquad \log(\lambda_{BA}) = \alpha + T_{BA}$$

The AB dyad $$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \underbrace{\rho\sigma^2}_{\text{covariance within dyads}} & \underbrace{\sigma^2}_{\text{variance among ties}} \end{bmatrix} \right)$$

$$G_{AB} \sim \text{Poisson}(\lambda_{AB}) \qquad G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} \qquad \log(\lambda_{BA}) = \alpha + T_{BA}$$

partial pooling for network ties

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

$$\rho \sim \text{LKJCorr}(2)$$

$$\sigma \sim \text{Exponential}(1)$$

$$\alpha \sim \text{Normal}(0,1)$$

```r
# dyad model
f_dyad <- alist(
    GAB ~ poisson( lambdaAB ),
    GBA ~ poisson( lambdaBA ),
    log(lambdaAB) <- a + T[D,1] ,
    log(lambdaBA) <- a + T[D,2] ,
    a ~ normal(0,1),

    ## dyad effects
    transpars> matrix[N_dyads,2]:T <-
            compose_noncentered( rep_vector(sigma_T,2) , L_Rho_T , Z ),
    matrix[2,N_dyads]:Z ~ normal( 0 , 1 ),
    cholesky_factor_corr[2]:L_Rho_T ~ lkj_corr_cholesky( 2 ),
    sigma_T ~ exponential(1),

    ## compute correlation matrix for dyads
    gq> matrix[2,2]:Rho_T <<- Chol_to_Corr( L_Rho_T )
)

mGD <- ulam( f_dyad , data=sim_data , chains=4 , cores=4 , iter=2000 )
```

```r
# dyad model
f_dyad <- alist(
    GAB ~ poisson( lambdaAB ),
    GBA ~ poisson( lambdaBA ),
    log(lambdaAB) <- a + T[D,1] ,
    log(lambdaBA) <- a + T[D,2] ,
    a ~ normal(0,1),

    ## dyad effects
    transpars> matrix[N_dyads,2]:T <-
            compose_noncentered( rep_vector(sigma_T,2) , L_Rho_T , Z ),
    matrix[2,N_dyads]:Z ~ normal( 0 , 1 ),
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```
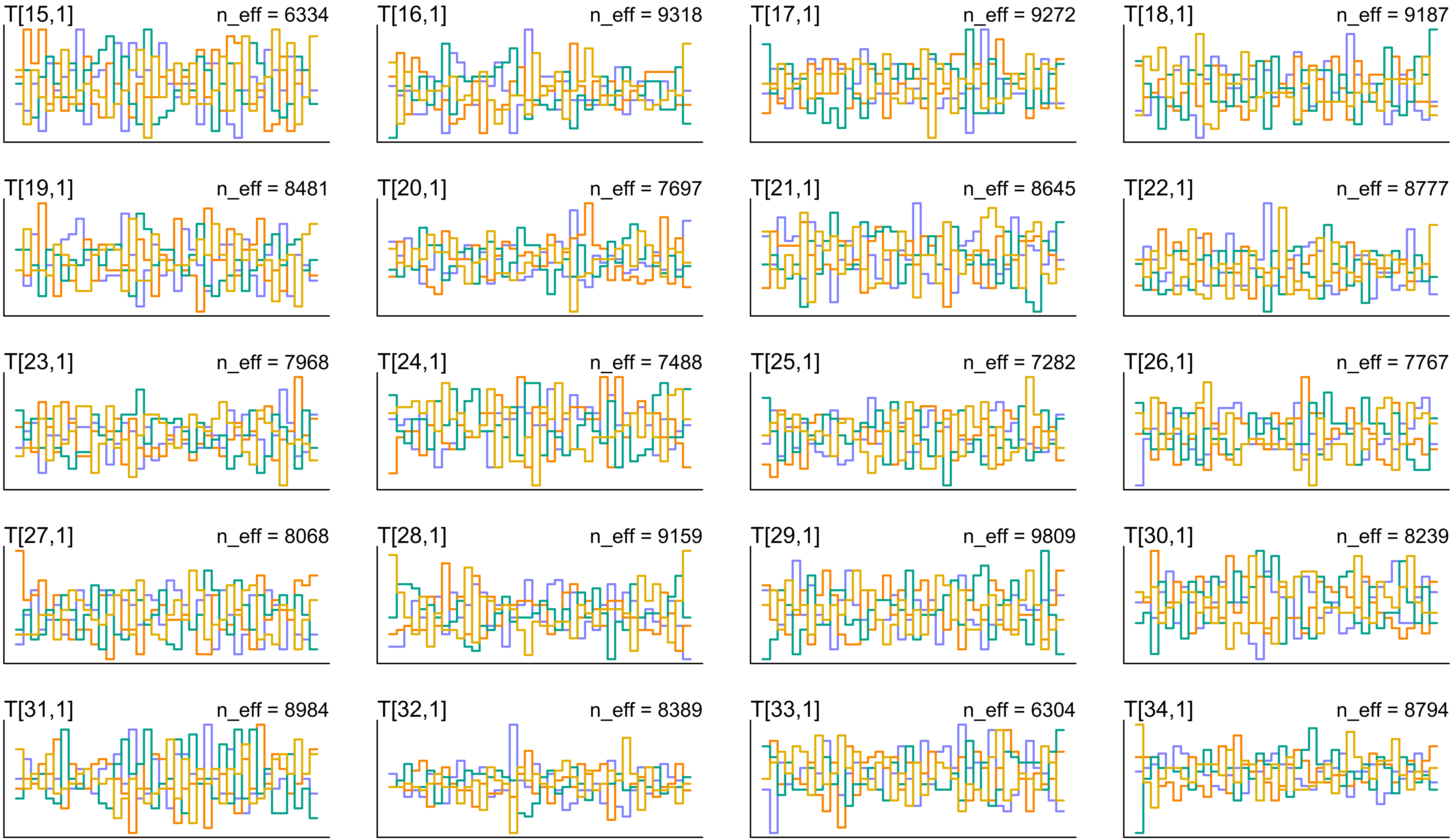
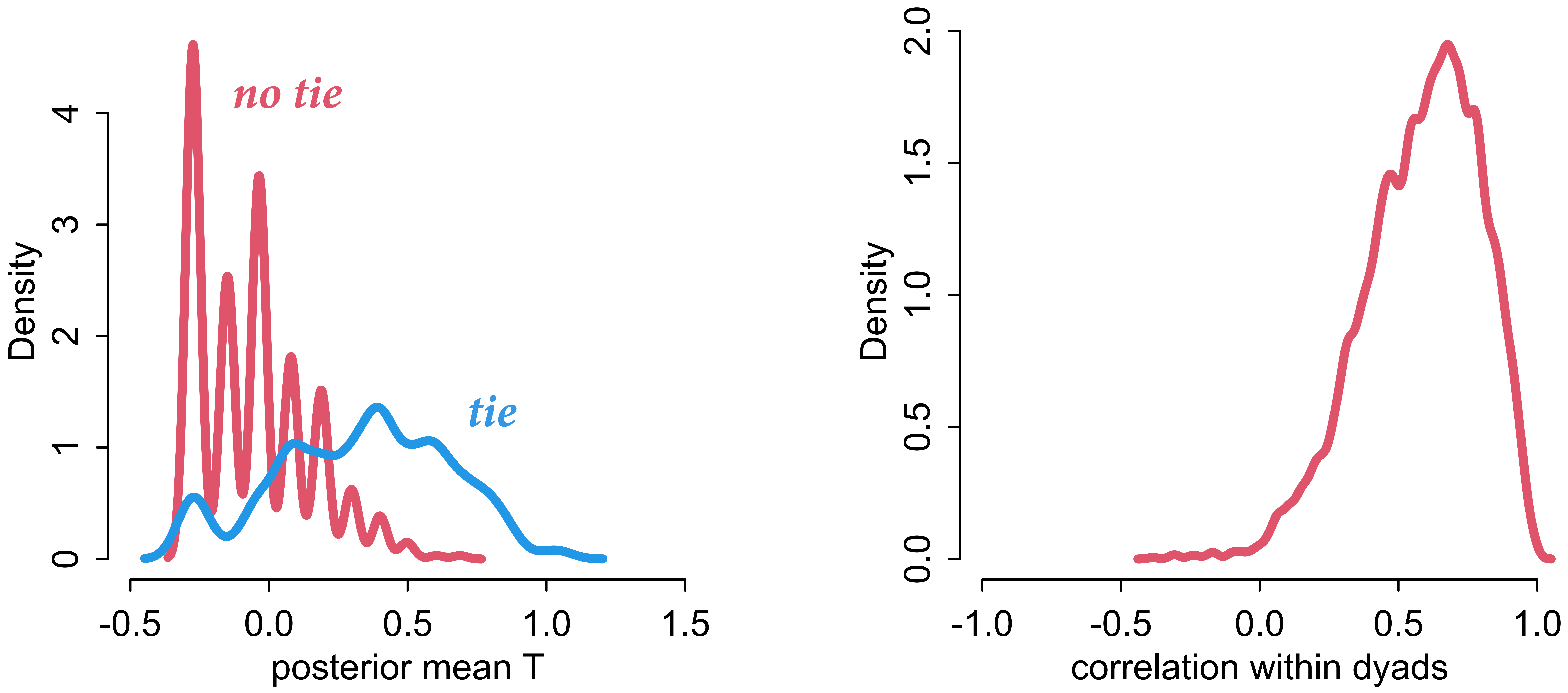
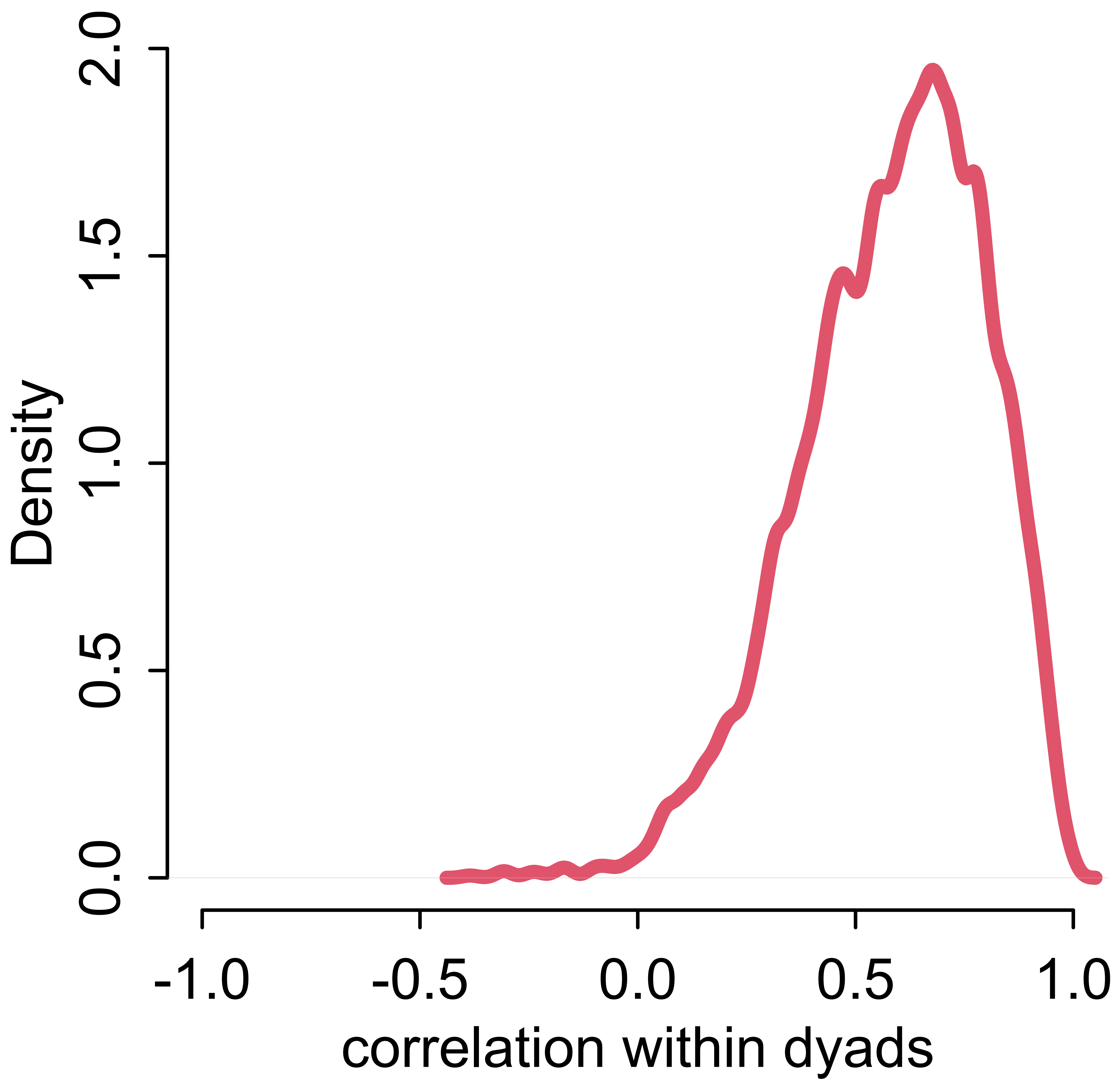

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

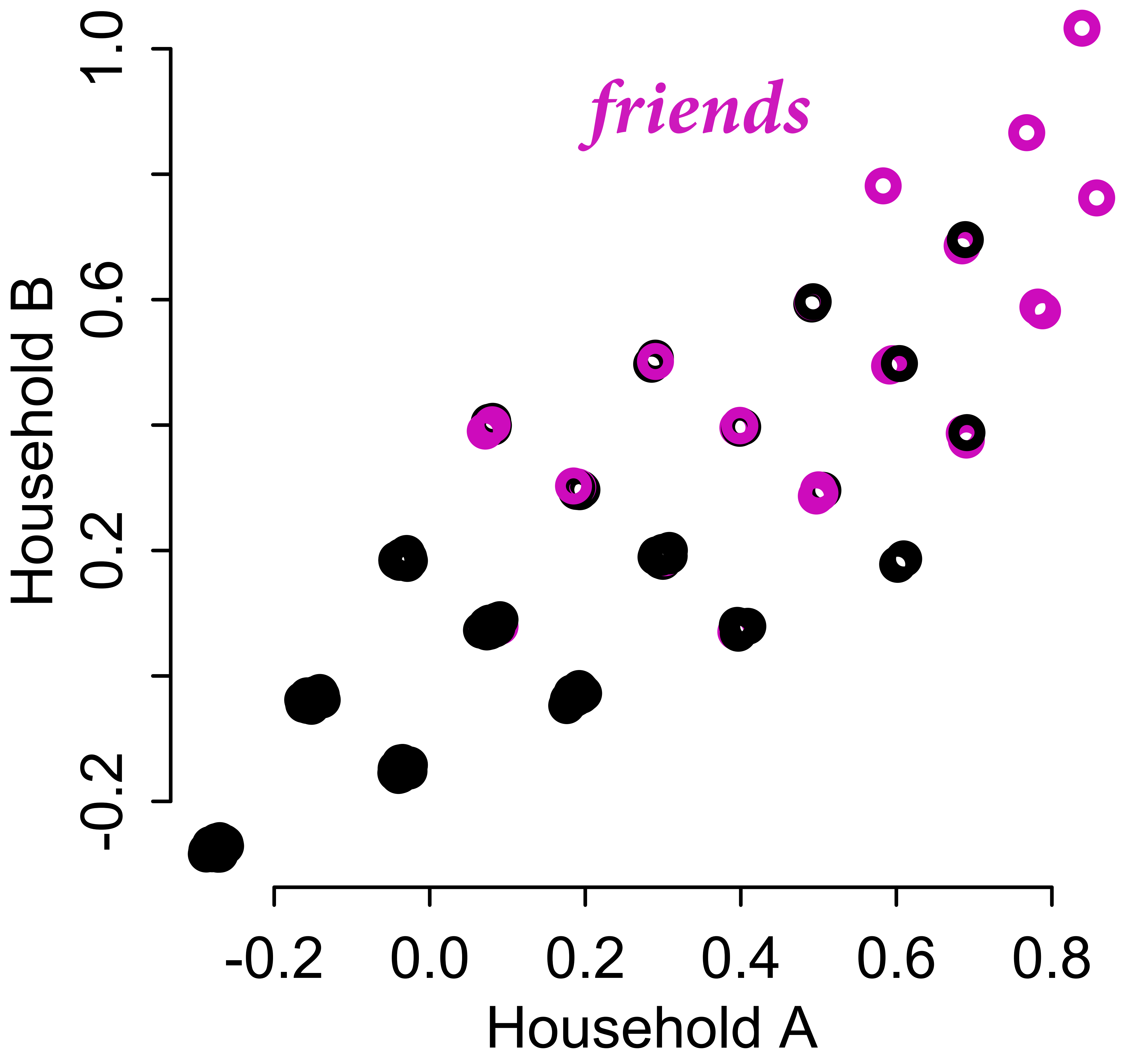

T[15,1]
n_eff = 6334
T[16,1]
n_eff = 9318
T[17,1]
n_eff = 9272
T[18,1]
n_eff = 9187
T[19,1]
n_eff = 8481
T[20,1]
n_eff = 7697
T[21,1]
n_eff = 8645
T[22,1]
n_eff = 8777
T[23,1]
n_eff = 7968
T[24,1]
n_eff = 7488
T[25,1]
n_eff = 7282
T[26,1]
n_eff = 7767
T[27,1]
n_eff = 8068
T[28,1]
n_eff = 9159
T[29,1]
n_eff = 9809
T[30,1]
n_eff = 8239
T[31,1]
n_eff = 8984
T[32,1]
n_eff = 8389
T[33,1]
n_eff = 6304
T[34,1]
n_eff = 8794

Posterior ties



Posterior ties





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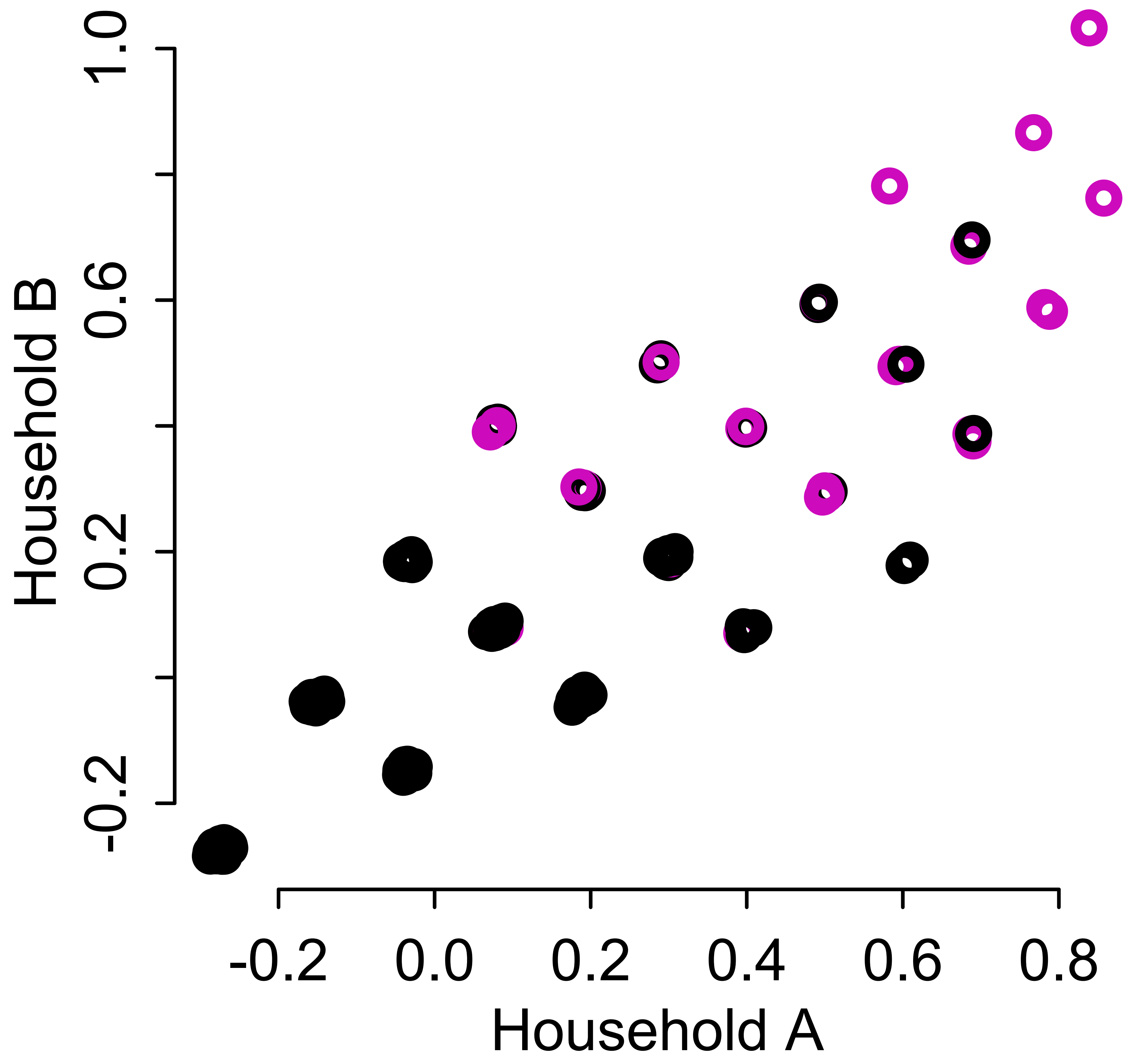
(4) Analyze sample



```r
# analyze sample
kl_data <- list(
    N_dyads = nrow(kl_dyads),
    N_households = max(kl_dyads$hidB),
    D = 1:nrow(kl_dyads),
    HA = kl_dyads$hidA,
    HB = kl_dyads$hidB,
    GAB = kl_dyads$giftsAB,
    GBA = kl_dyads$giftsBA )

mGDkl <- ulam( f_dyad , data=kl_data , chains=4 , cores=4 , iter=2000 )

precis( mGDkl , depth=3 , pars=c("a","Rho_T","sigma_T") )
```

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kl_data <- list(
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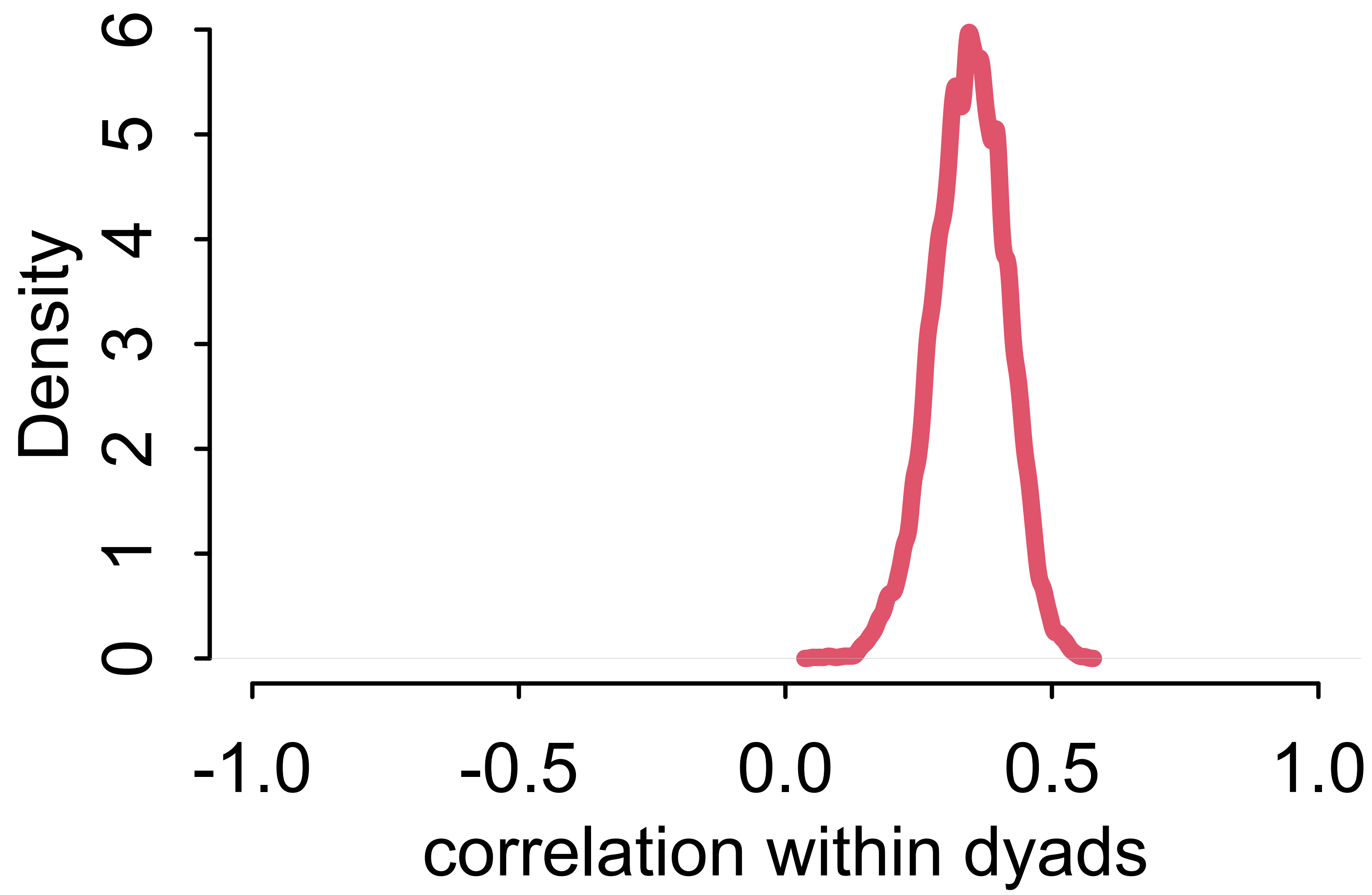
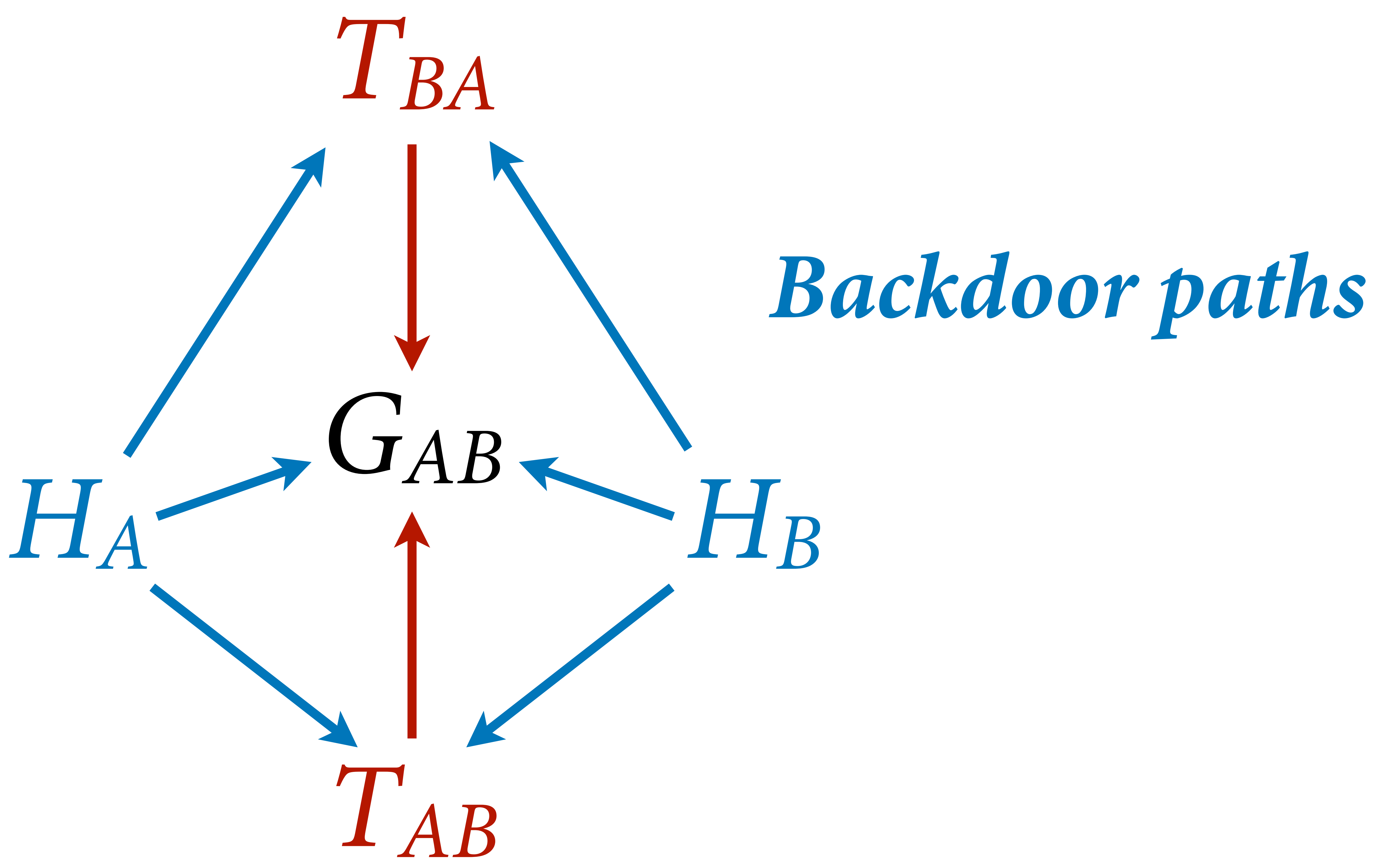
```

	mean	sd	5.5%	94.5%	n_eff	Rhat4
a	0.55	0.08	0.42	0.68	2246	1.00
Rho_T[1,1]	1.00	0.00	1.00	1.00	NaN	NaN
Rho_T[1,2]	0.35	0.07	0.24	0.45	1351	1.00
Rho_T[2,1]	0.35	0.07	0.24	0.45	1351	1.00
Rho_T[2,2]	1.00	0.00	1.00	1.00	NaN	NaN
sigma_T	1.44	0.06	1.35	1.55	1249	1.01




T_{BA}
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PAUSE

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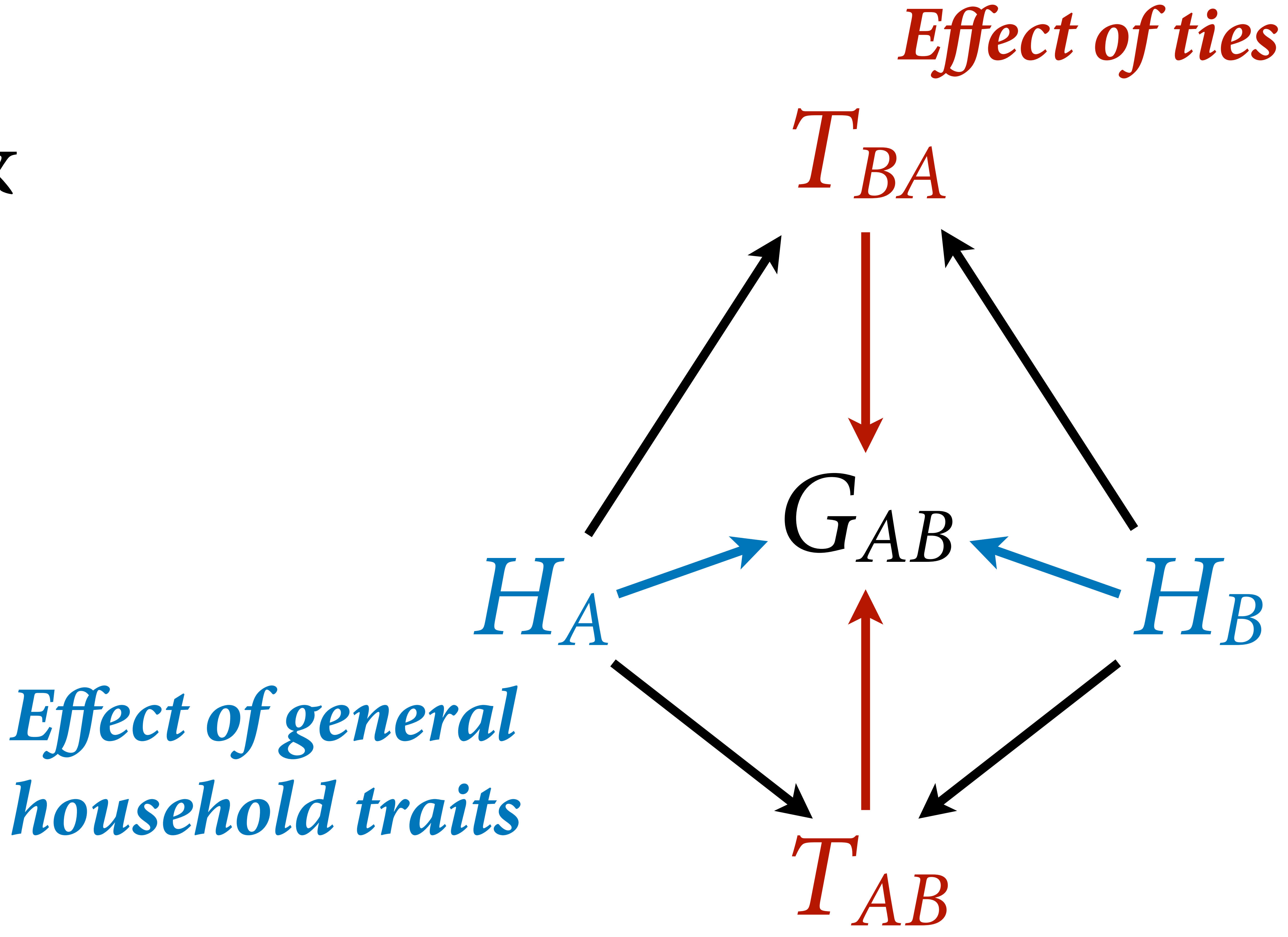

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dyads <- t(combn(N,2))
N_dyads <- nrow(dyads)

# simulate "friendships" in which ties are shared
f <- rbern(N_dyads,0.1) # 10% of dyads are friends

# now simulate directed ties for all individuals
# there can be ties that are not reciprocal
alpha <- (-3) # base rate of ties; -3 ~= 0.05
y <- matrix(NA,N,N) # edge list
for ( i in 1:N ) for ( j in 1:N ) {
    if ( i != j ) {
        # directed tie from i to j
        ids <- sort( c(i,j) )
        the_dyad <- which( dyads[,1]==ids[1] & dyads[,2]==ids[2] )
        p_tie <- f[the_dyad] + (1-f[the_dyad])*inv_logit( alpha )
        y[i,j] <- rbern( 1 , p_tie )
    }
}#ij
```

```r
# simulate wealth
W <- rnorm(N) # standardized relative wealth in community
bWG <- 0.5 # effect of wealth on giving - rich give more
bWR <- (-1) # effect of wealth on receiving - rich get less / poor get more

# now simulate gifts
giftsAB <- rep(NA,N_dyads)
giftsBA <- rep(NA,N_dyads)
lambda <- log(c(0.5,2)) # rates of giving for y=0,y=1
for ( i in 1:N_dyads ) {
    A <- dyads[i,1]
    B <- dyads[i,2]
    giftsAB[i] <- rpois( 1 , exp( lambda[1+y[A,B]] + bWG*W[A] + bWR*W[B] ) )
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}
```

$$G_{AB} \sim \text{Poisson}(\lambda_{AB}) \qquad G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} \qquad \log(\lambda_{BA}) = \alpha + T_{BA}$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$\sigma \sim \text{Exponential}(1)$$

$$\alpha \sim \text{Normal}(0,1)$$

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$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} + G_A + R_B$$

A's generalized giving

B's generalized receiving

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA}$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} G_A \\ R_A \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_G^2 & r\sigma_G\sigma_R \\ r\sigma_G\sigma_R & \sigma_R^2 \end{bmatrix} \right)$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$\rho \sim \text{LKJCorr}(2)$$

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$$\alpha \sim \text{Normal}(0,1)$$

$$\begin{pmatrix} G_A \\ R_A \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} \sigma_G^2 & r\sigma_G\sigma_R \\ r\sigma_G\sigma_R & \sigma_R^2 \end{bmatrix}}_{\text{Covariance matrix of household giving \& receiving}} \right)$$

A's giving & receiving

Covariance matrix of household giving & receiving

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} + G_A + R_B$$

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

$$\rho \sim \text{LKJCorr}(2)$$

$$\sigma \sim \text{Exponential}(1)$$

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$$\begin{pmatrix} G_A \\ R_A \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{R}_{GR}, \mathbf{S}_{GR} \right)$$

A's giving & receiving

Correlation matrix

Standard deviations

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} + G_A + R_B$$

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$\mathbf{S}_{GR} \sim \text{Exponential}(1)$$

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$$\mathbf{R}_{GR} \sim \text{LKJCorr}(2)$$

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25 households

300 dyads

600 counts / 2871 transfers

602 social network parameters

53 household parameters

```r
# general model
f_general <- alist(
    GAB ~ poisson( lambdaAB ),
    GBA ~ poisson( lambdaBA ),
    log(lambdaAB) <- a + T[D,1] + gr[HA,1] + gr[HB,2],
    log(lambdaBA) <- a + T[D,2] + gr[HB,1] + gr[HA,2],
    a ~ normal(0,1),

    ## dyad effects - non-centered
    transpars> matrix[N_dyads,2]:T <-
            compose_noncentered(rep_vector(sigma_T,2),L_Rho_T,Z),
    matrix[2,N_dyads]:Z ~ normal( 0 , 1 ),
    cholesky_factor_corr[2]:L_Rho_T ~ lkj_corr_cholesky( 2 ),
    sigma_T ~ exponential(1),

   ## gr matrix of varying effects
    transpars> matrix[N_households,2]:gr <-
            compose_noncentered( sigma_gr , L_Rho_gr , Zgr ),
    matrix[2,N_households]:Zgr ~ normal( 0 , 1 ),
    cholesky_factor_corr[2]:L_Rho_gr ~ lkj_corr_cholesky( 2 ),
    vector[2]:sigma_gr ~ exponential(1),

   ## compute correlation matrixes
    gq> matrix[2,2]:Rho_T <<- Chol_to_Corr( L_Rho_T ),
    gq> matrix[2,2]:Rho_gr <<- Chol_to_Corr( L_Rho_gr )
)
mGDGR <- ulam(f_general,data=sim_data,chains=4,cores=4,iter=2000)
```

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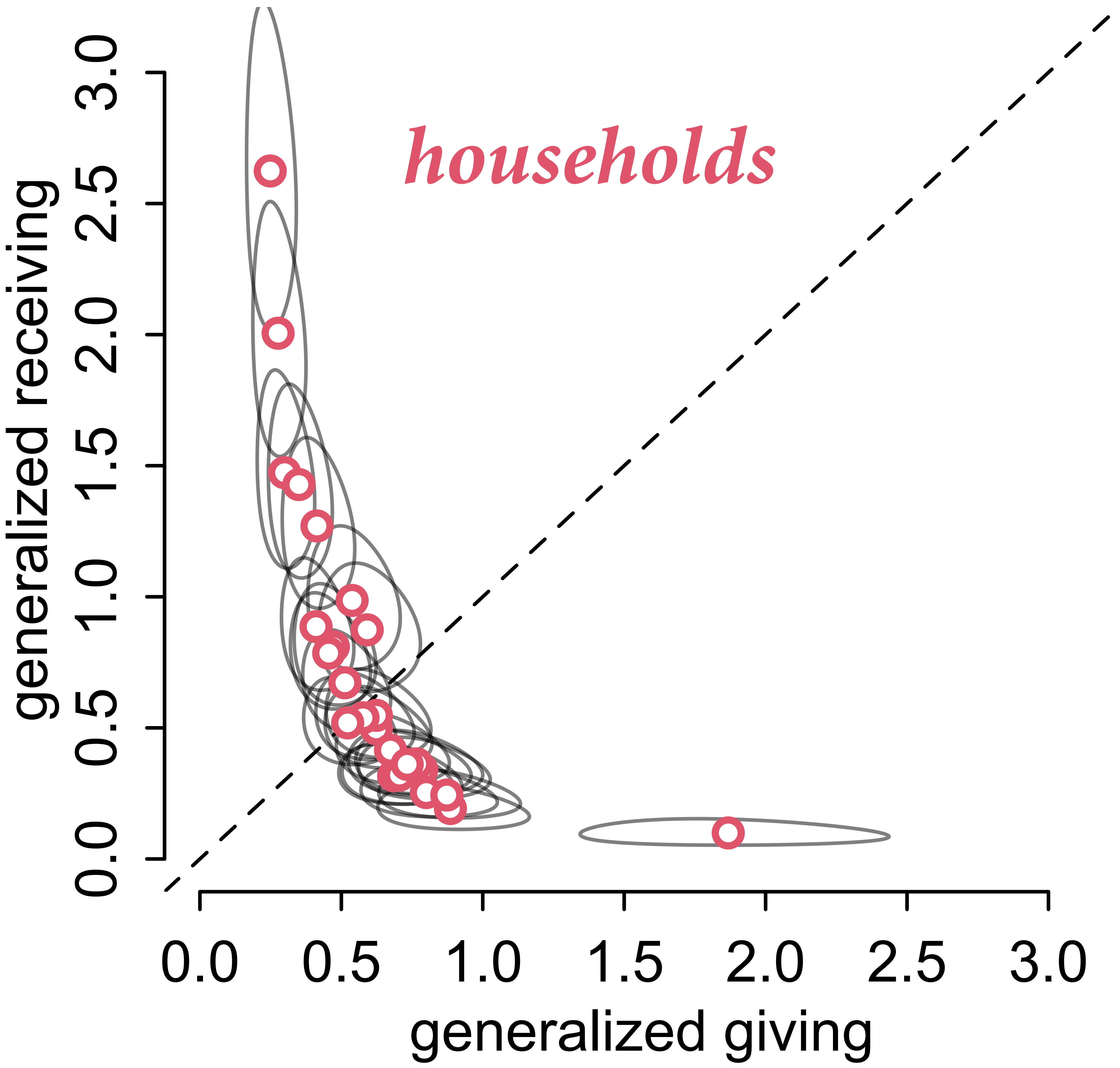
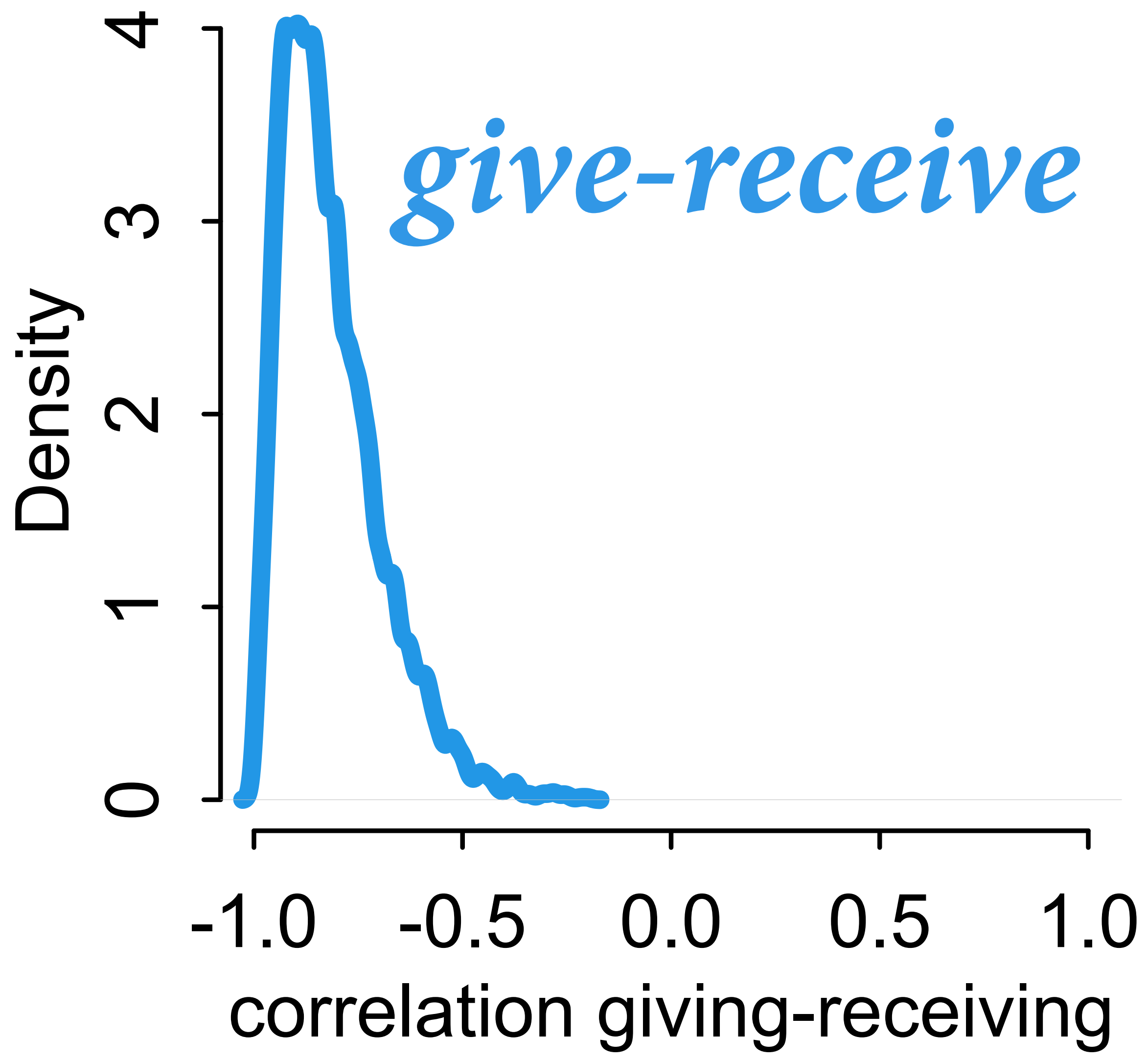
$$\mathbf{R}_{GR} \sim \text{LKJCorr}(2)$$

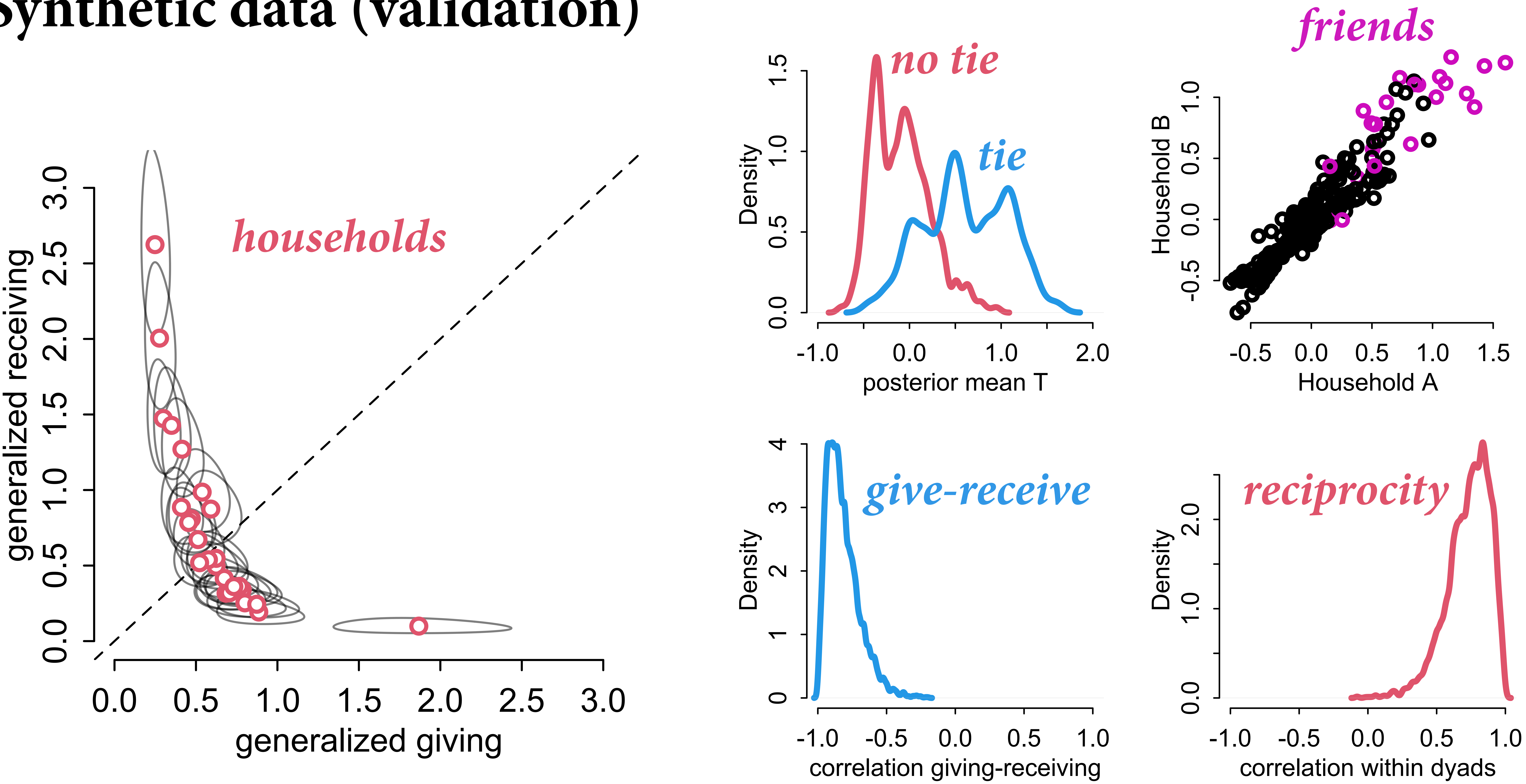
$$\mathbf{S}_{GR} \sim \text{Exponential}(1)$$

Synthetic data (validation)

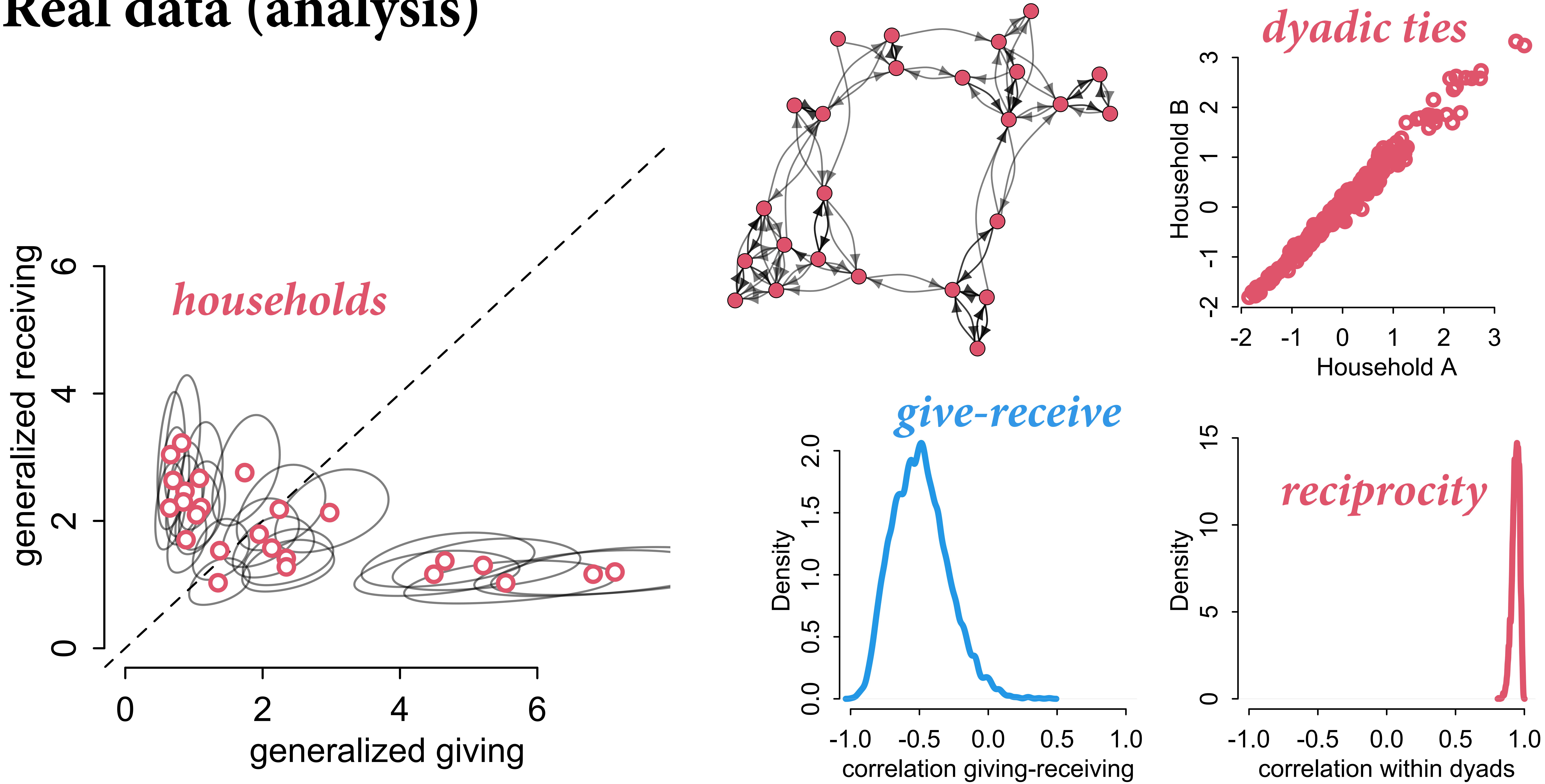




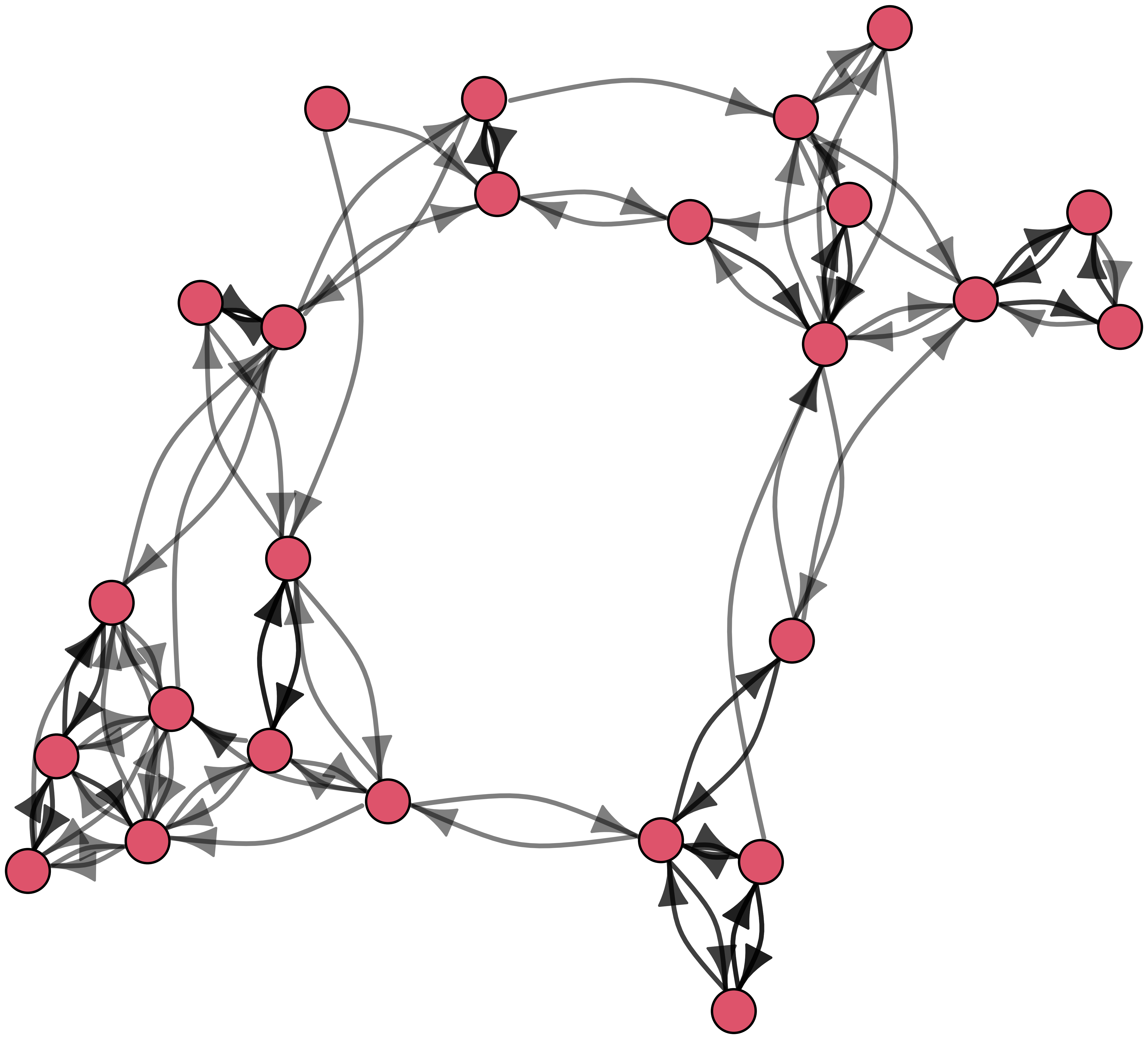
Synthetic data (validation)



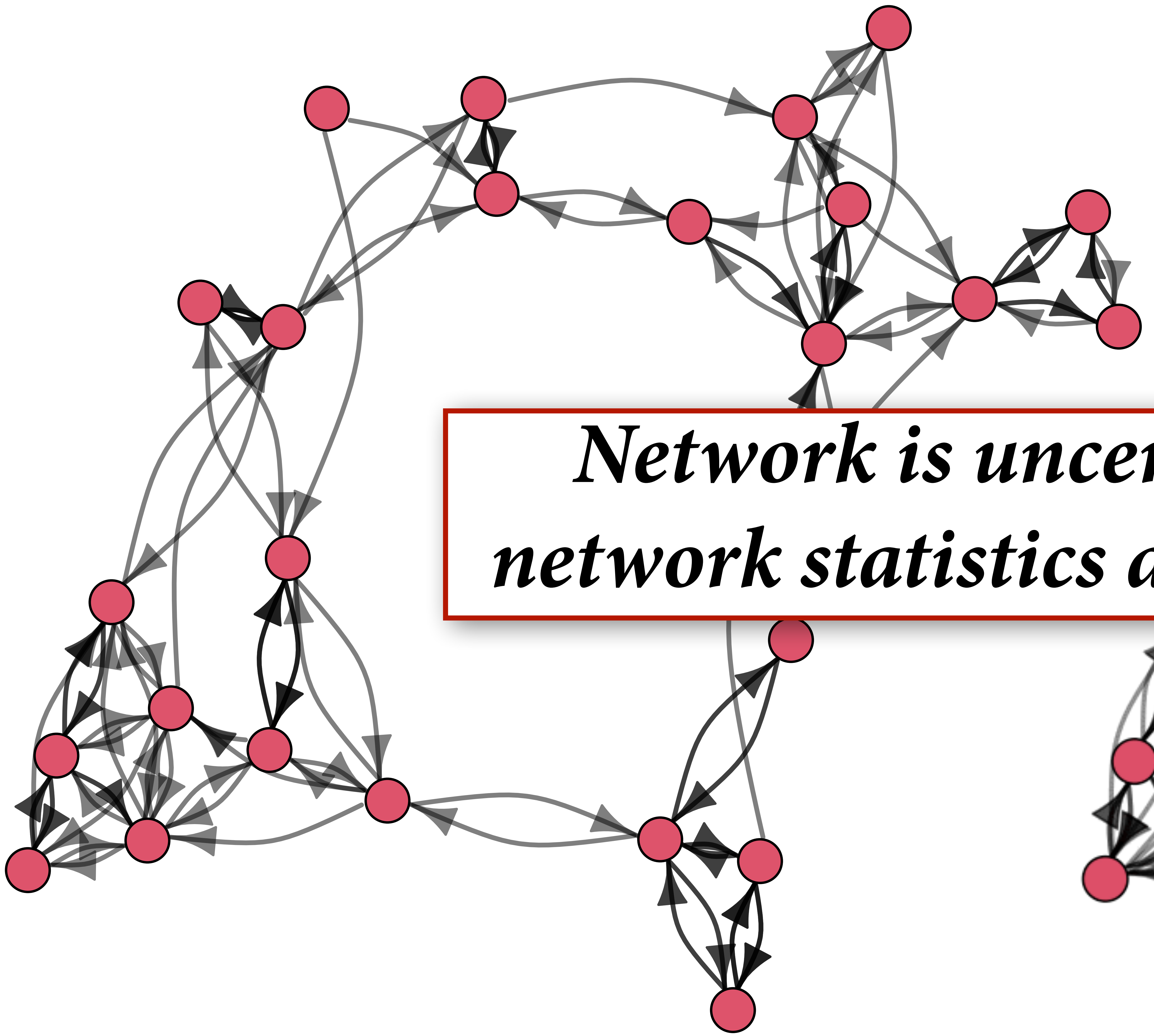
Real data (analysis)

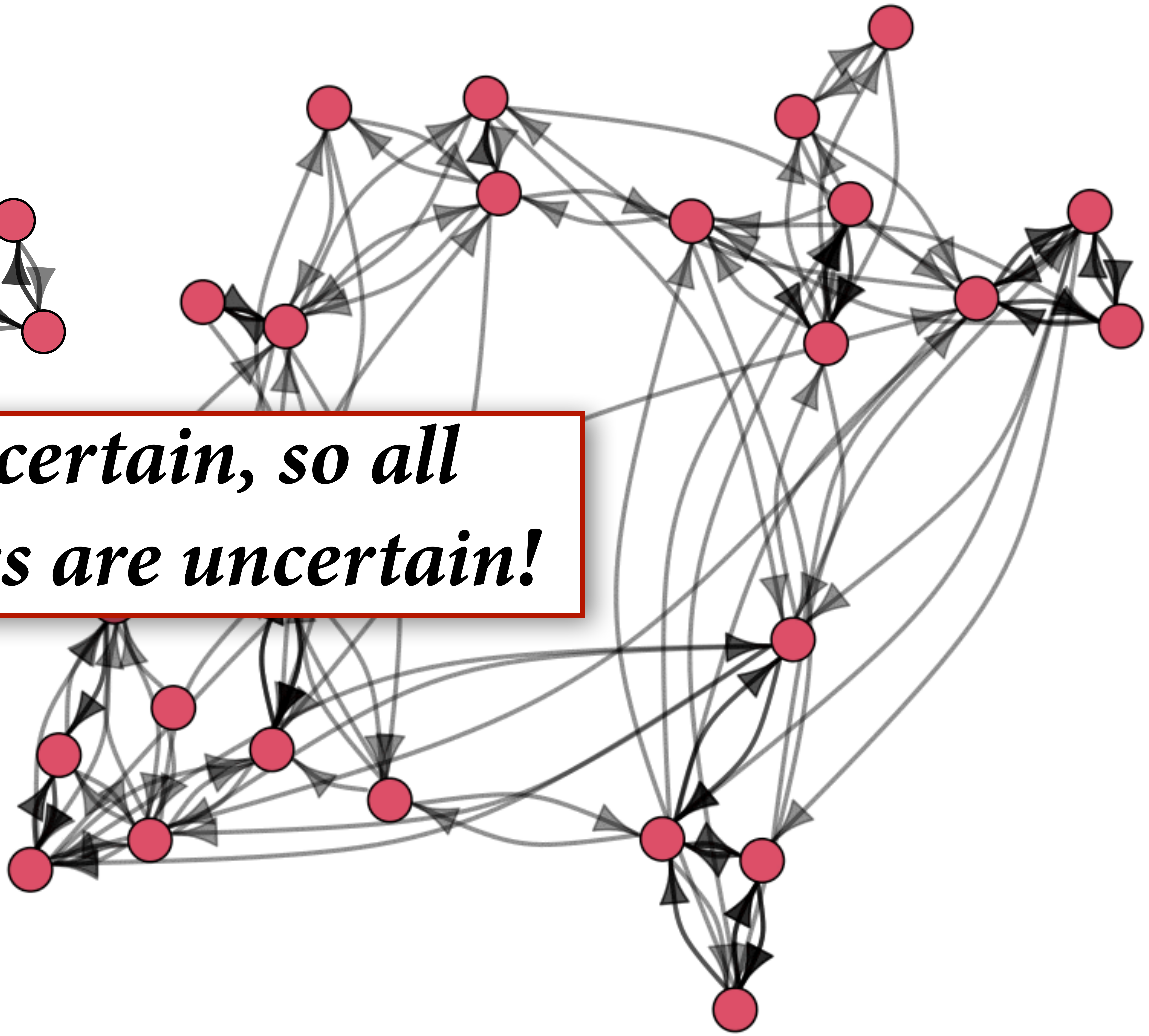


Posterior mean network



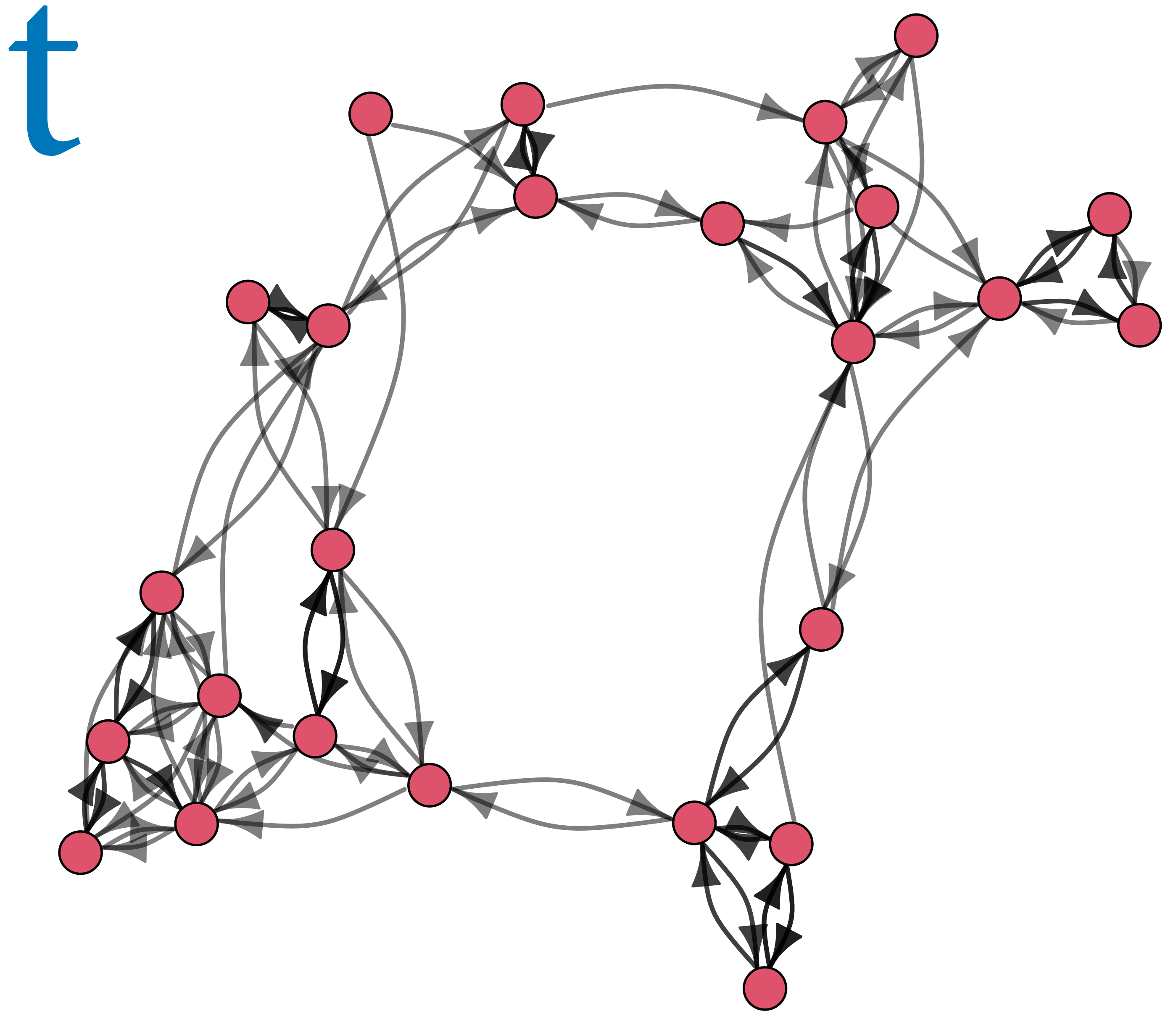
Posterior mean network | **Samples from posterior**





Network is uncertain, so all network statistics are uncertain!

Social Networks Don't Exist



Varying effects are placeholders

Can model the network ties (using dyad features)

Can model the giving/receiving (using household features)

Relationships can cause other relationships

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + T_{AB} + G_A + R_B$$

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$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + \mathcal{T}_{AB} + \mathcal{G}_A + \mathcal{R}_B$$

$$\mathcal{T}_{AB} = T_{AB} + \beta_A A_{AB}$$

linear model for tie strength

varying effect

effect of association between A&B

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + \mathscr{T}_{AB} + \mathscr{G}_{A} + \mathscr{R}_{B}$$

$$\mathscr{T}_{AB} = T_{AB} + \beta_A A_{AB}$$

$$\mathscr{G}_{A} = G_A + \beta_{W,G} W_A$$

linear model for giving

varying effect

effect of A's wealth on giving

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

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$$\mathscr{G}_A = G_A + \beta_{W,G} W_A$$

$$\mathscr{R}_B = R_B + \beta_{W,R} W_B$$

linear model for receiving

varying effect

effect of B's wealth on receiving

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{BA}) = \alpha + T_{BA} + G_B + R_A$$

$$\begin{pmatrix} T_{AB} \\ T_{BA} \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right)$$

$$\rho \sim \text{LKJCorr}(2)$$

$$\sigma \sim \text{Exponential}(1)$$

$$\alpha \sim \text{Normal}(0,1)$$

$$\begin{pmatrix} G_A \\ R_A \end{pmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{R}_{GR}, \mathbf{S}_{GR} \right)$$

$$\mathbf{R}_{GR} \sim \text{LKJCorr}(2)$$

$$\mathbf{S}_{GR} \sim \text{Exponential}(1)$$

$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + \mathscr{T}_{AB} + \mathscr{G}_A + \mathscr{R}_B$$

$$\mathscr{T}_{AB} = T_{AB} + \beta_A A_{AB}$$

$$\mathscr{G}_A = G_A + \beta_{W,G} W_A$$

$$\mathscr{R}_B = R_B + \beta_{W,R} W_B$$

$$G_{BA} \sim \text{Poisson}(\lambda_{BA})$$

$$\log(\lambda_{AB}) = \alpha + \mathscr{T}_{BA} + \mathscr{G}_B + \mathscr{R}_A$$

$$\mathscr{T}_{BA} = T_{BA} + \beta_A A_{AB}$$

$$\mathscr{G}_B = G_B + \beta_{W,G} W_B$$

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$$G_{AB} \sim \text{Poisson}(\lambda_{AB})$$

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$$\mathcal{T}_{BA} = T_{BA} + \beta_A A_{AB}$$

$$\mathcal{G}_B = G_B + \beta_{W,G} W_B$$

$$\mathcal{R}_A = R_A + \beta_{W,R} W_A$$

```
# general model with features
f_houses <- alist(

    GAB ~ poisson( lambdaAB ),
    GBA ~ poisson( lambdaBA ),

    # A to B
    log(lambdaAB) <- a + TAB + GA + RB,
      TAB <- T[D,1] + bA*A,
      GA <- gr[HA,1] + bW[1]*W[HA] ,
      RB <- gr[HB,2] + bW[2]*W[HB] ,

    # B to A
    log(lambdaBA) <- a + TBA + GB + RA,
      TBA <- T[D,2] + bA*A,
      GB <- gr[HB,1] + bW[1]*W[HB] ,
      RA <- gr[HA,2] + bW[2]*W[HA] ,

    # priors
    a ~ normal(0,1),
    vector[2]:bW ~ normal(0,1),
    bA ~ normal(0,1),
```

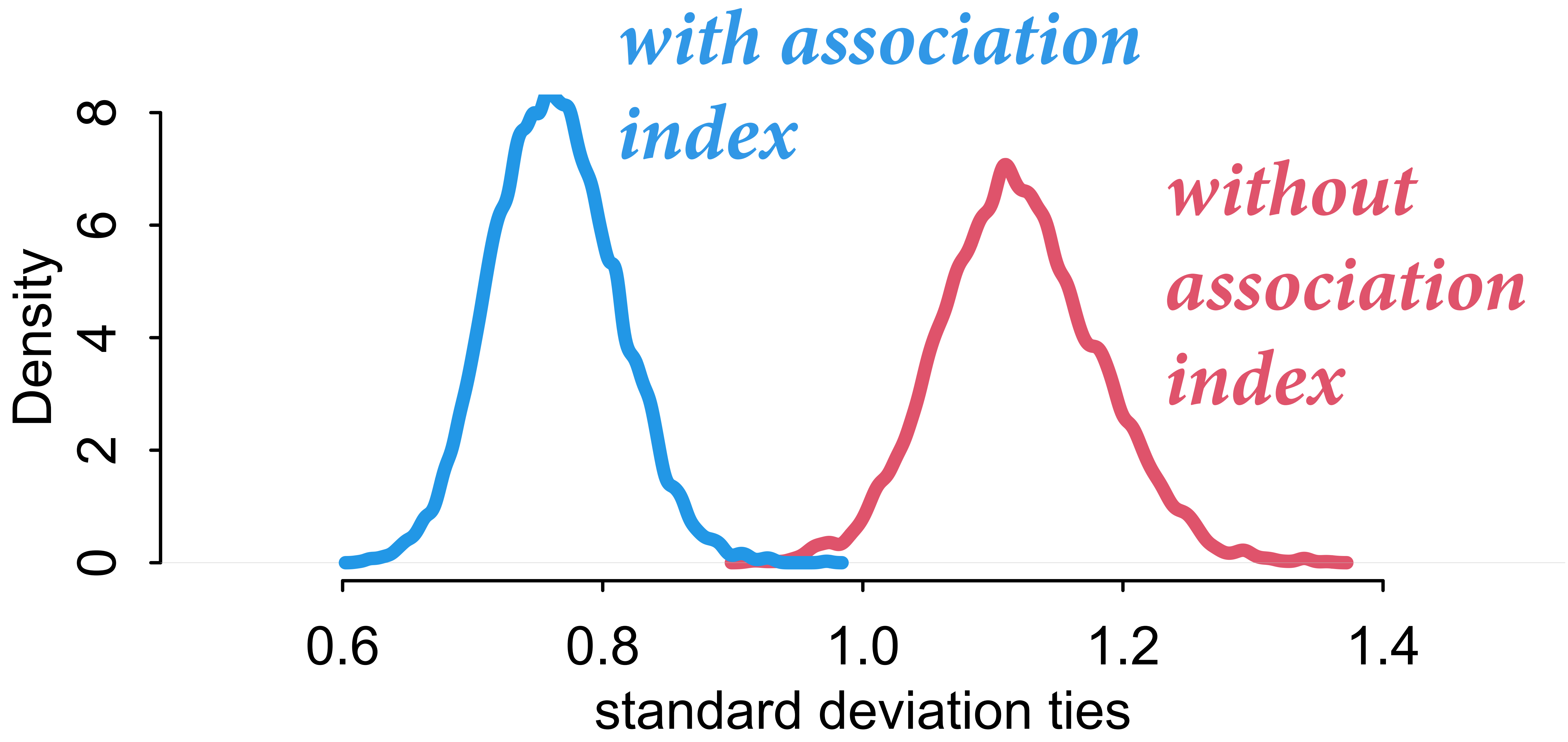
$$G_{AB} \sim \mathrm{Poisson}(\lambda_{AB})$$

$$\log(\lambda_{AB}) = \alpha + \mathscr{T}_{AB} + \mathscr{G}_A + \mathscr{R}_B$$

$$\mathscr{T}_{AB} = T_{AB} + \beta_A A_{AB}$$

$$\mathscr{G}_A = G_A + \beta_{W,G} W_A$$

$$\mathscr{R}_B = R_B + \beta_{W,R} W_B$$



$$G_{BA} \sim \mathrm{Poisson}(\lambda_{BA})$$

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$$\mathscr{T}_{BA} = T_{BA} + \beta_A A_{AB}$$

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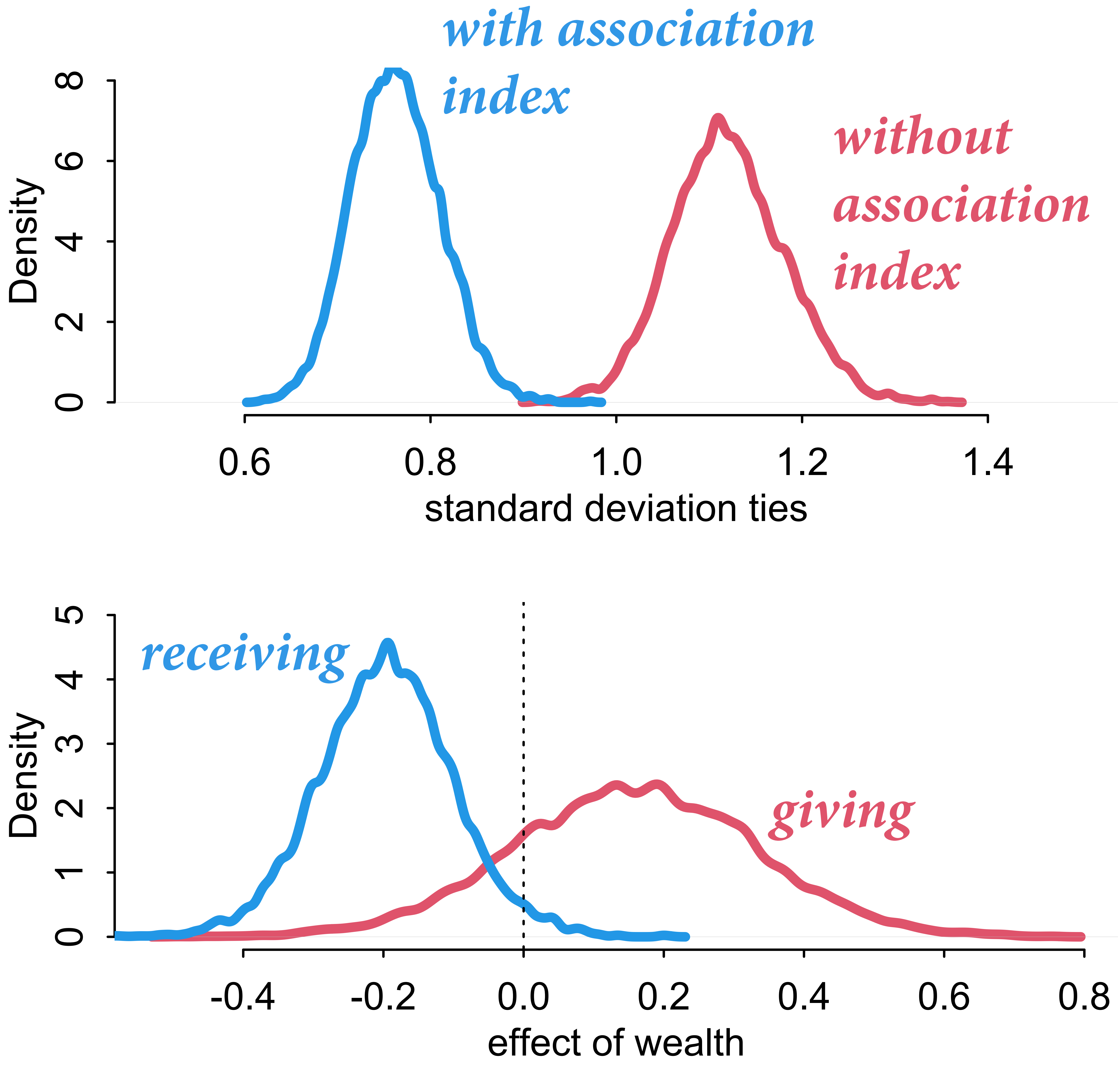
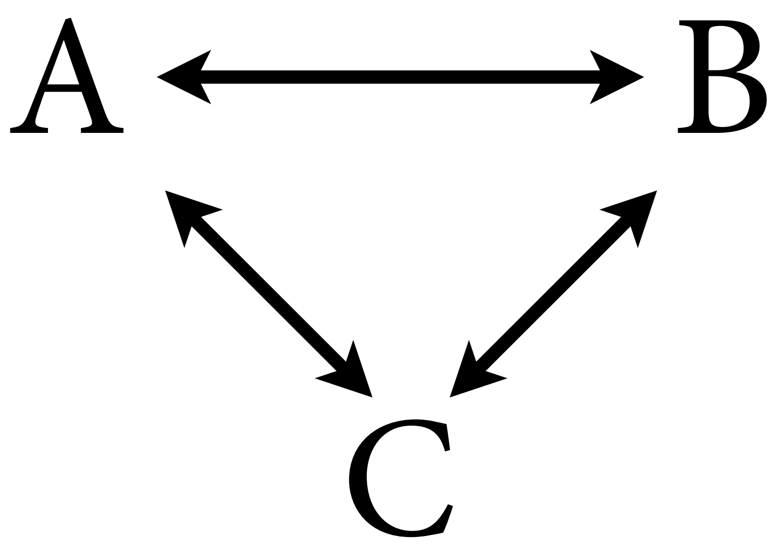


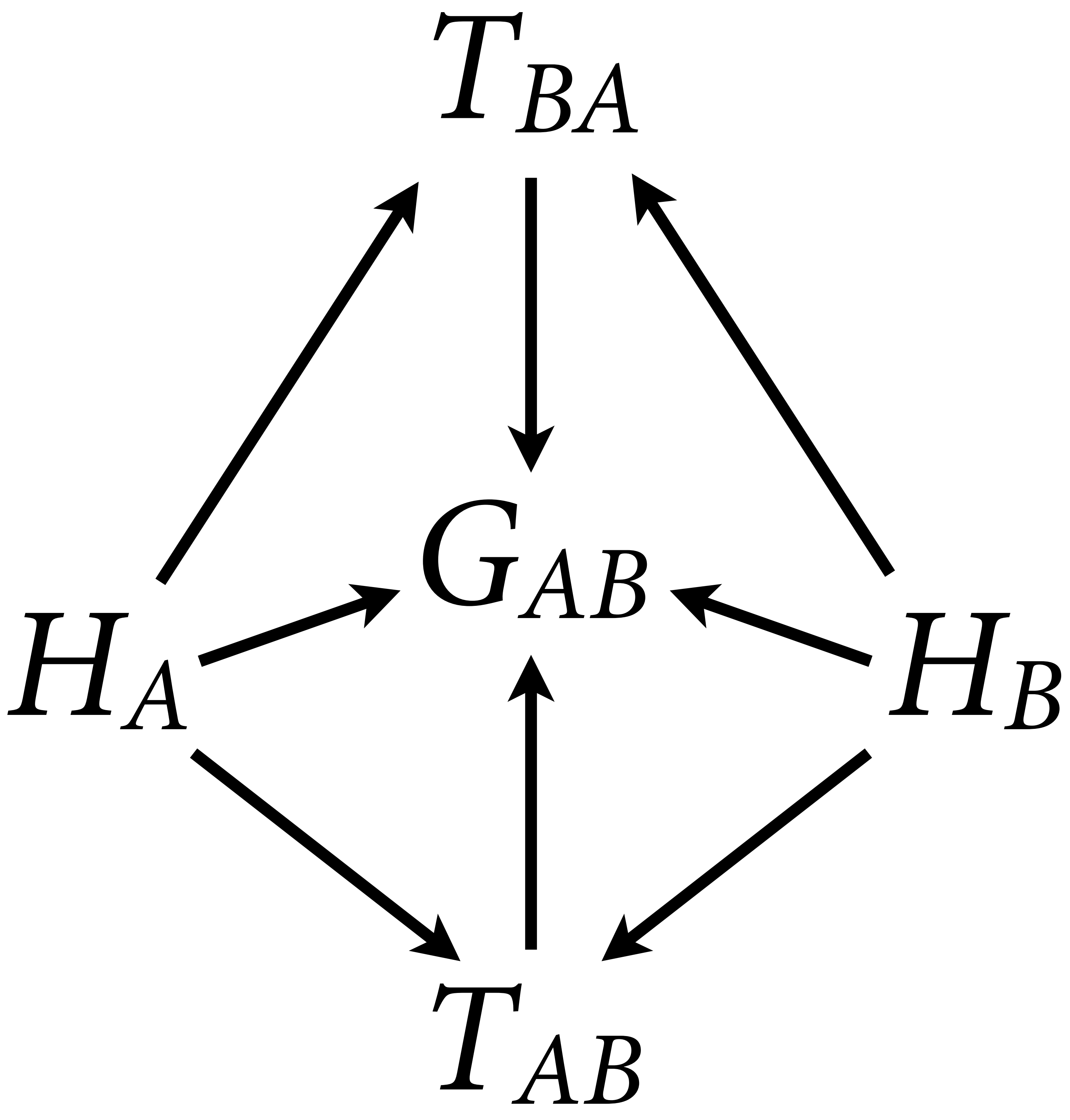
Additional Structure: Triangles

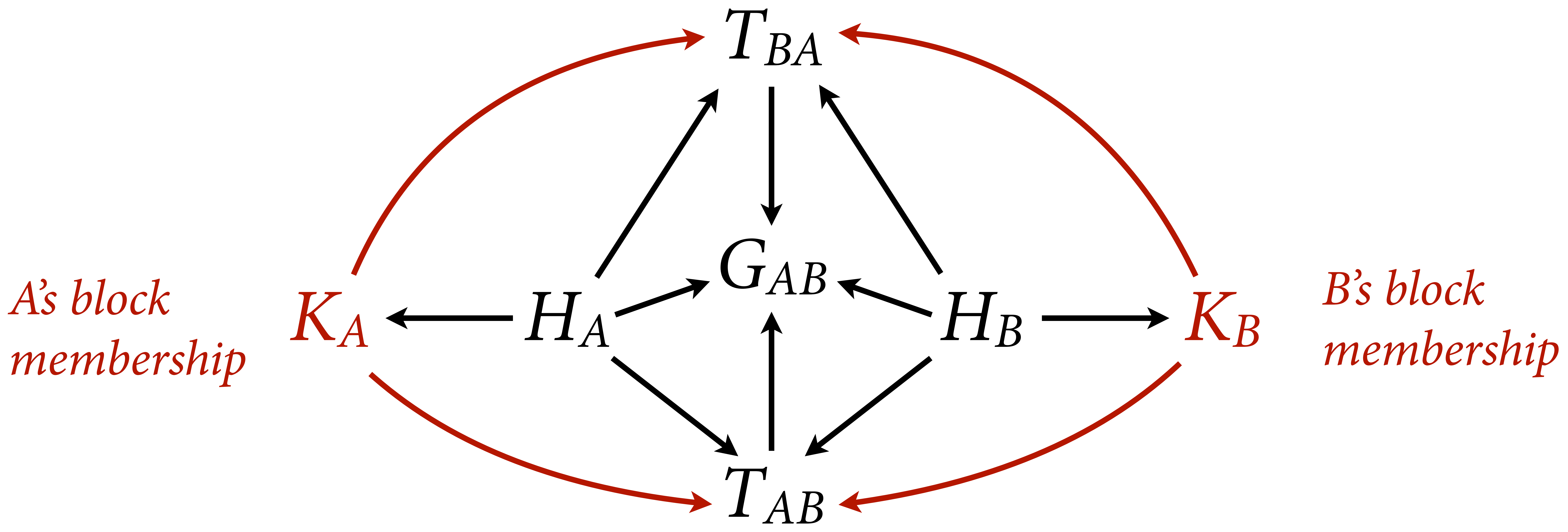
Relationships tend to come in triangles

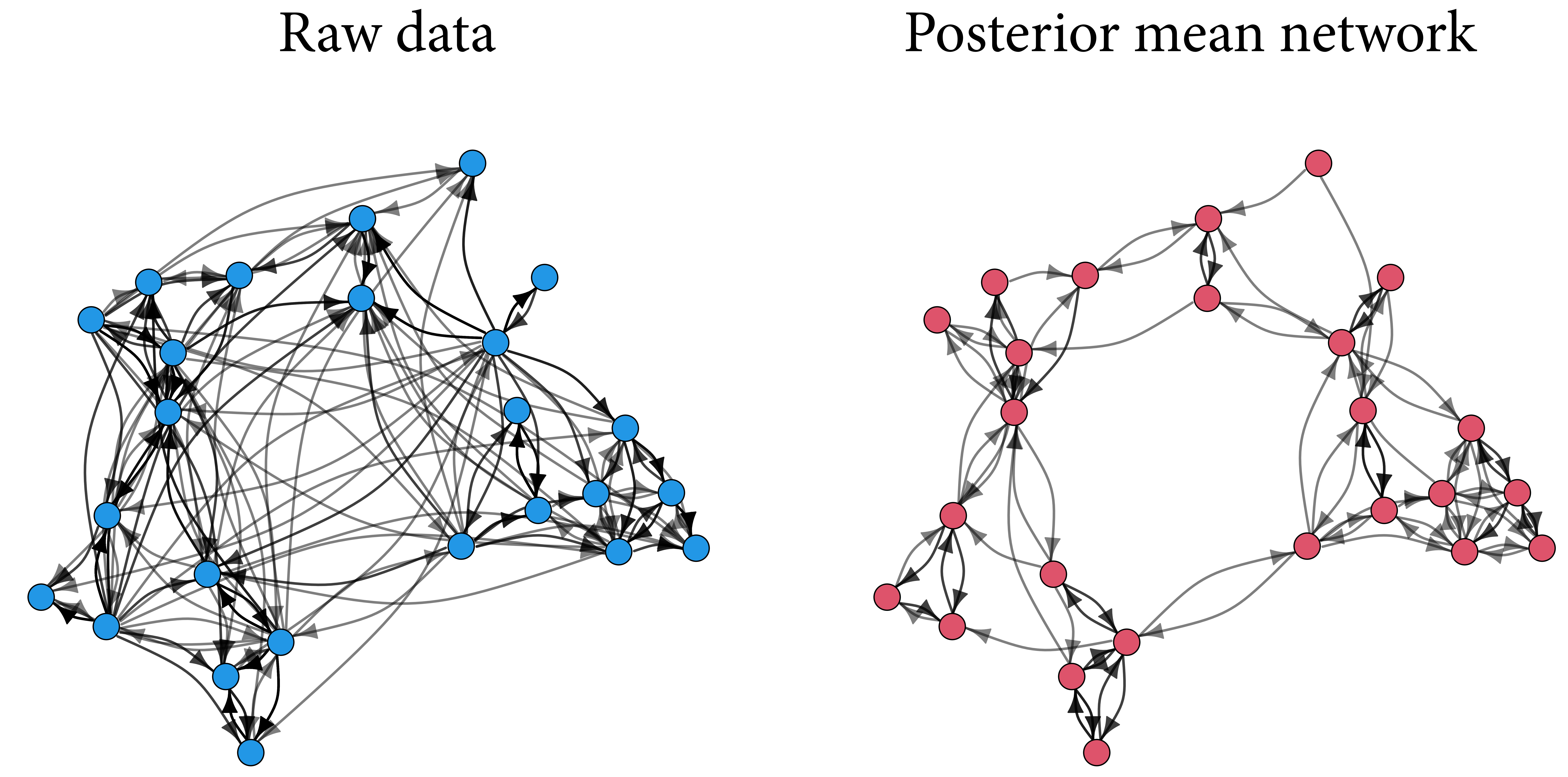
Triangle closure:



Block models: Ties more common within certain groups (family, office, *Stammtisch*)




T_{BA}
G_{AB}
A's block membership
K_A
H_A
H_B
K_B
B's block membership
T_{AB}



Raw data
Posterior mean network

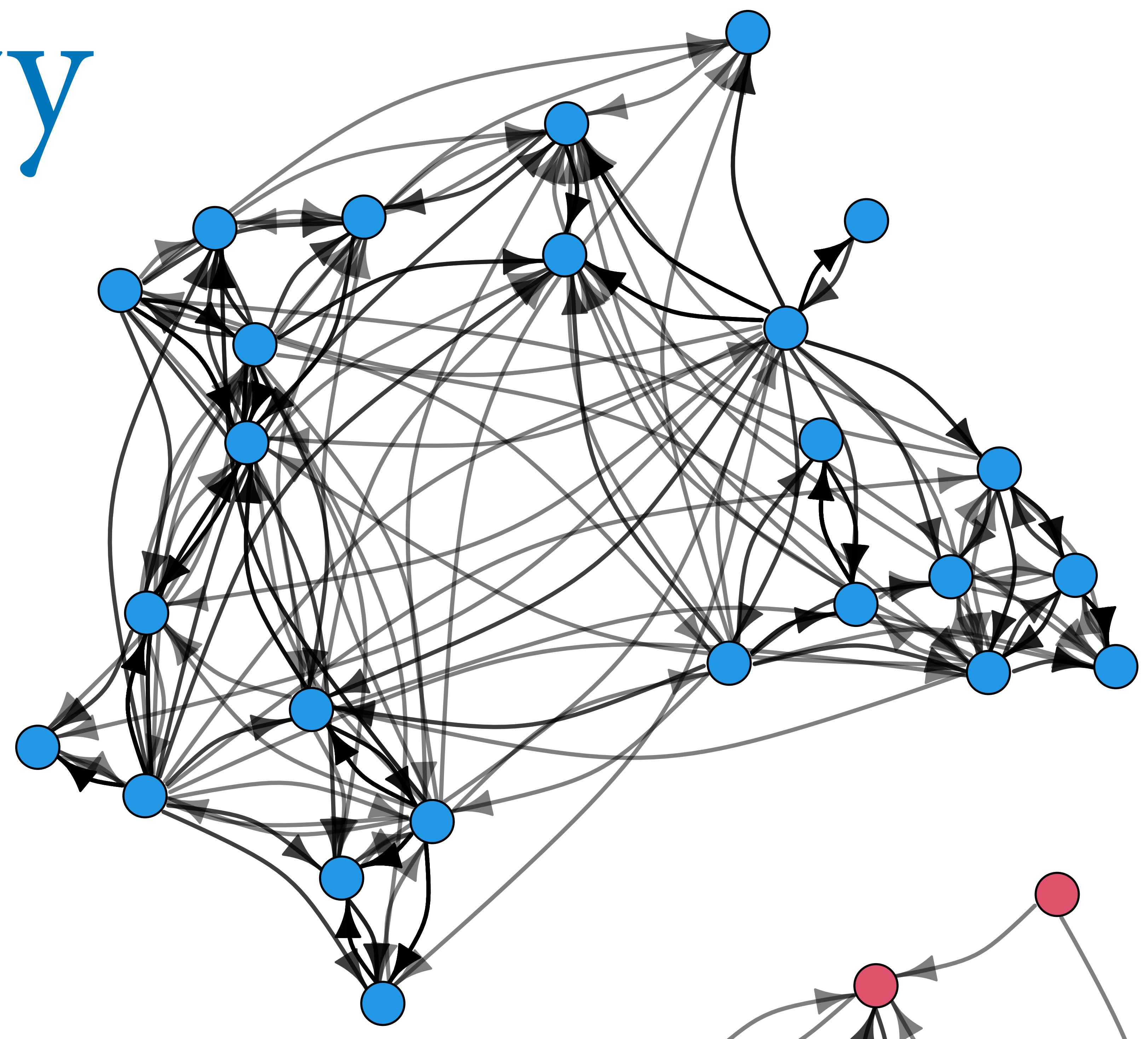
Varying effects as technology

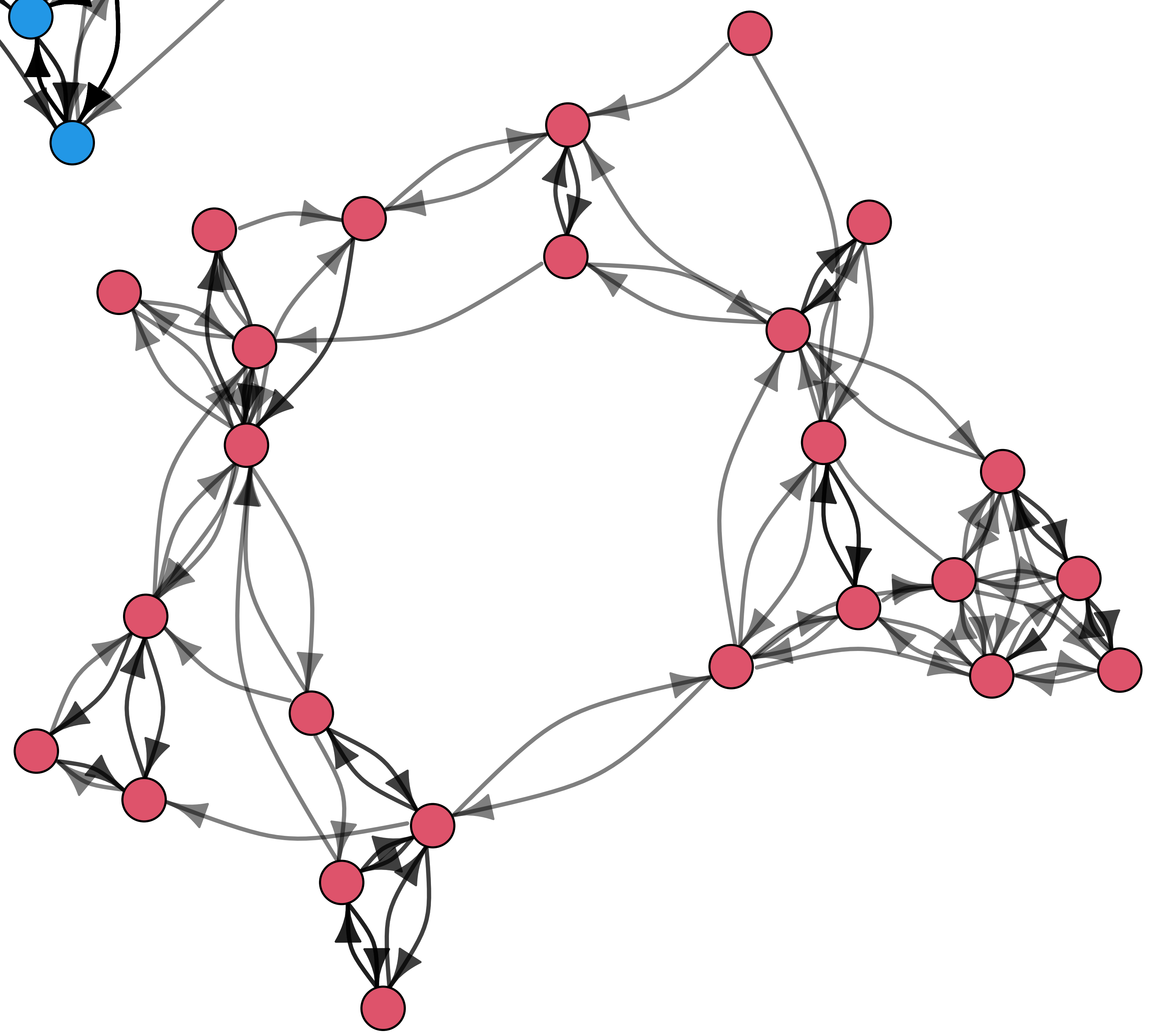
Social networks try to express *regularities* of observations

Inferred social network is *regularized*, a structured varying effect

Analogous problems: phylogeny, space, heritability, knowledge, personality

What happens when the clusters are not discrete but continuous? Age, distance, time, similarity





Course Schedule

Week 1	Bayesian inference	Chapters 1, 2, 3
Week 2	Linear models & Causal Inference	Chapter 4
Week 3	Causes, Confounds & Colliders	Chapters 5 & 6
Week 4	Overfitting / MCMC	Chapters 7, 8, 9
Week 5	Generalized Linear Models	Chapters 10, 11
Week 6	Ordered categories & Multilevel models	Chapters 12 & 13
Week 7	More Multilevel models	Chapters 13 & 14
Week 8	Social Networks & Gaussian Processes	Chapter 14
Week 9	Measurement & Missingness	Chapter 15
Week 10	Generalized Linear Madness	Chapter 16

https://github.com/rmcelreath/stat_rethinking_2023

BONUS

Constructed Variables Are Bad

Folk tradition of building outcome variables as a back-alley form of "control": ratios, differences, transformations

Body Mass Index (BMI) = mass/height^2

rates/ratios: per capita, per unit time

differences: change scores, difference from reference

All of these are usually bad

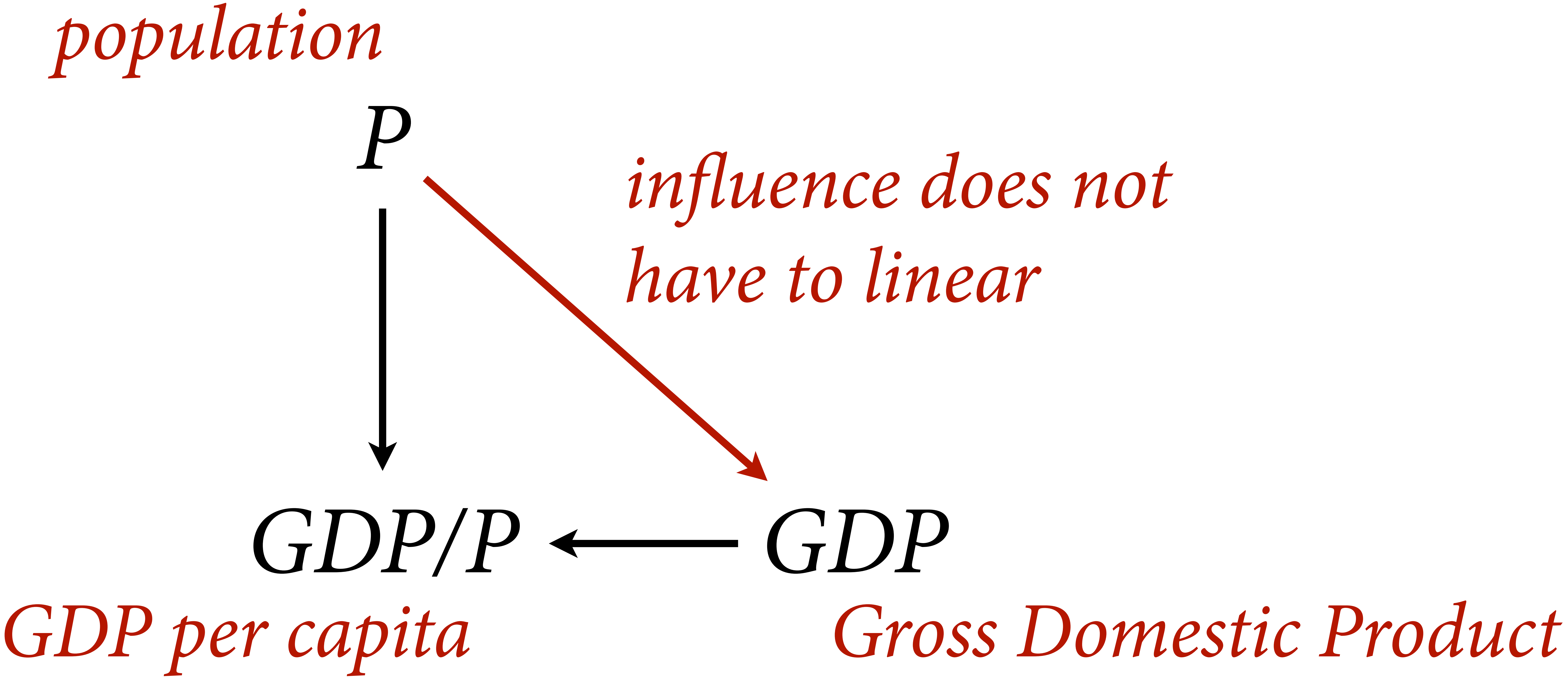
Per Capitated

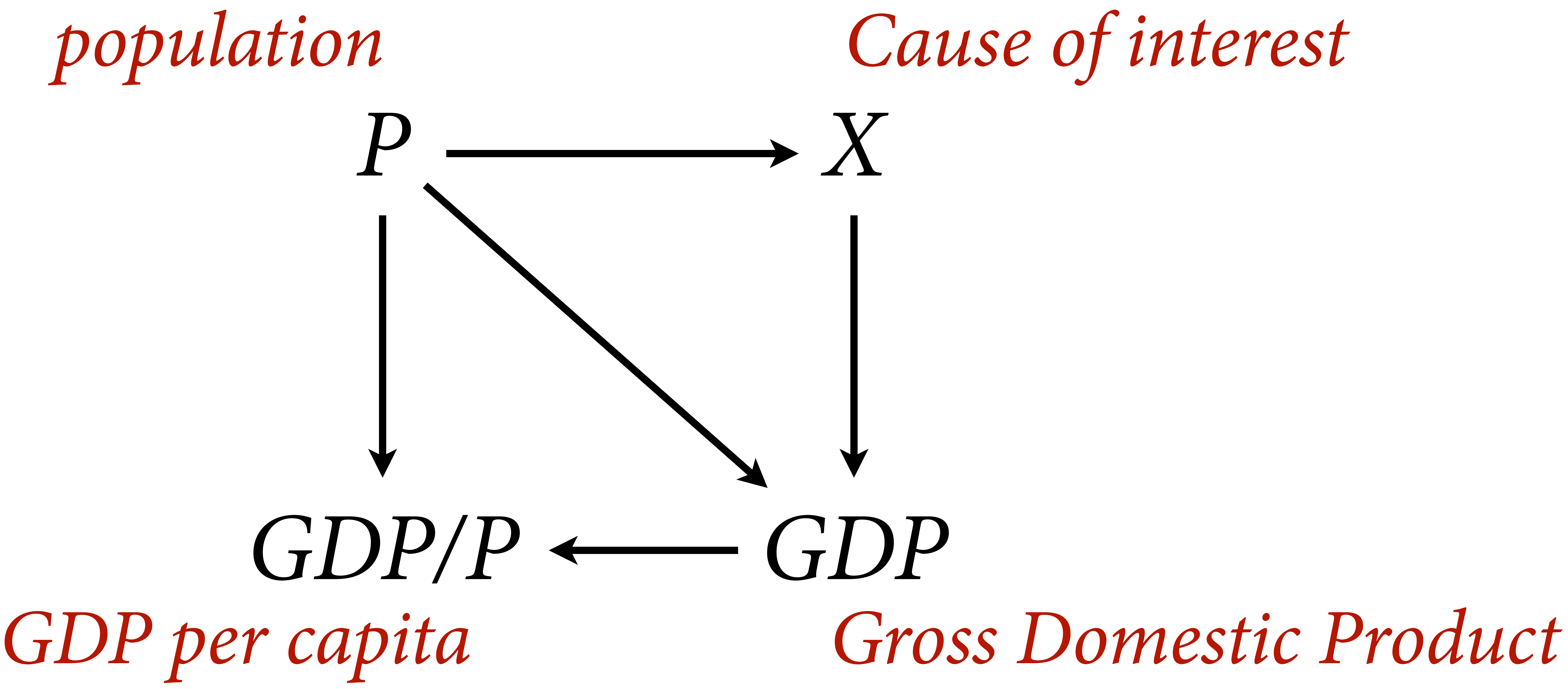
Example: Dividing GDP by population does not stratify by population size

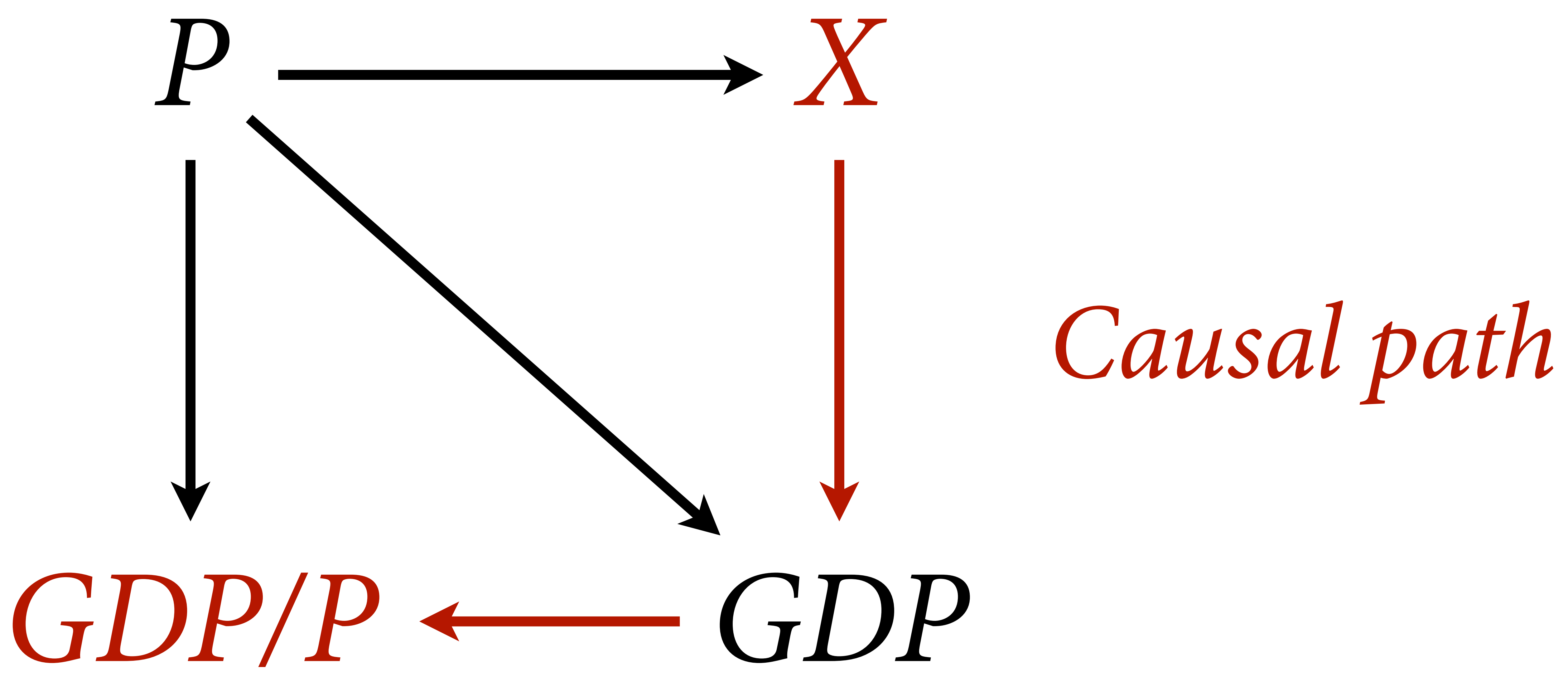
Now, using a more direct, but less fine-grained measure of economic development, log regional GDP per capita (mapped in Figure B.3.1), we verify the predicted association between kinship intensity and economic development. To remain consistent with the above analyses of nighttime luminosity, we estimate similar specifications, except that we now include year, year-continent, or year-country fixed effects in the models (because of the panel nature of the data), and do not include population density (since the dependent variable is already in per capita terms). As above, we cluster standard errors at the country level. We use the same set of control variables, constructed from the same data *population*

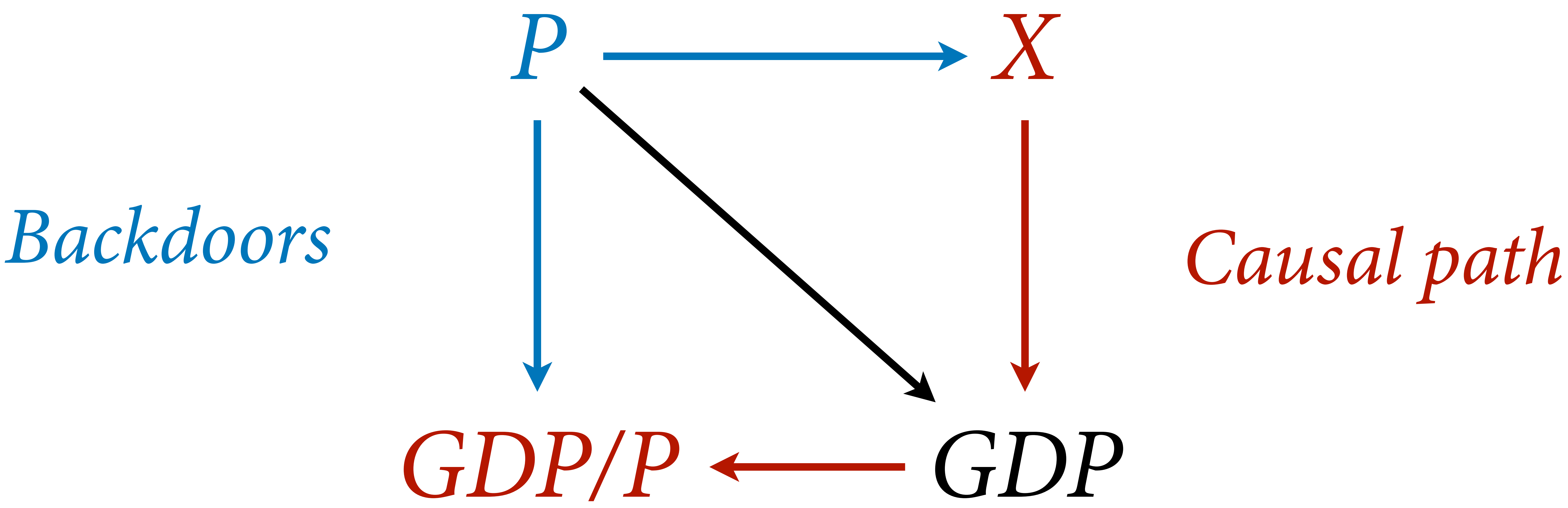
$$\begin{tikzcd} P \arrow[d] & \\ \mathrm{GDP}/P & \mathrm{GDP} \arrow[l] \end{tikzcd}$$

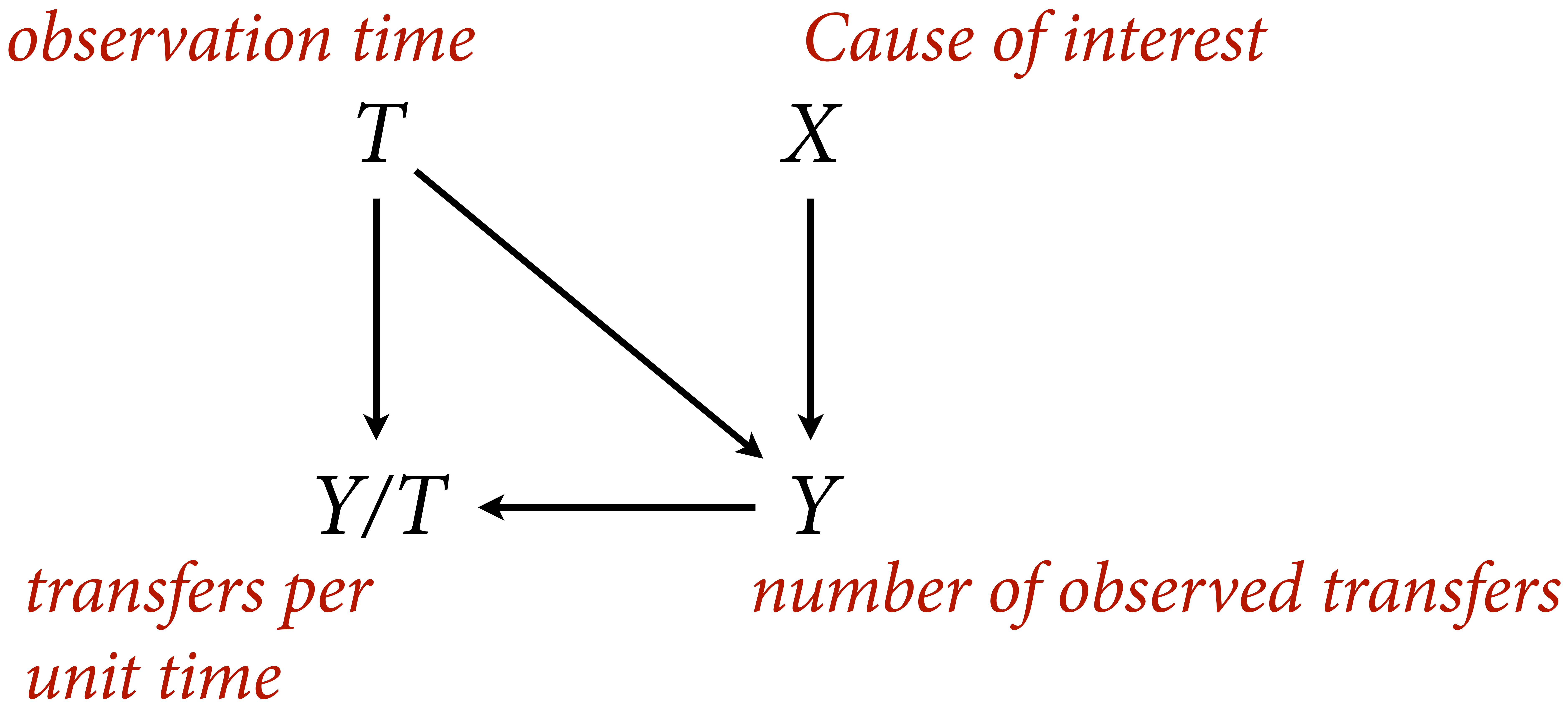
GDP per capita *Gross Domestic Product*


population
P
influence does not
have to linear
GDP/P
GDP
GDP per capita
Gross Domestic Product

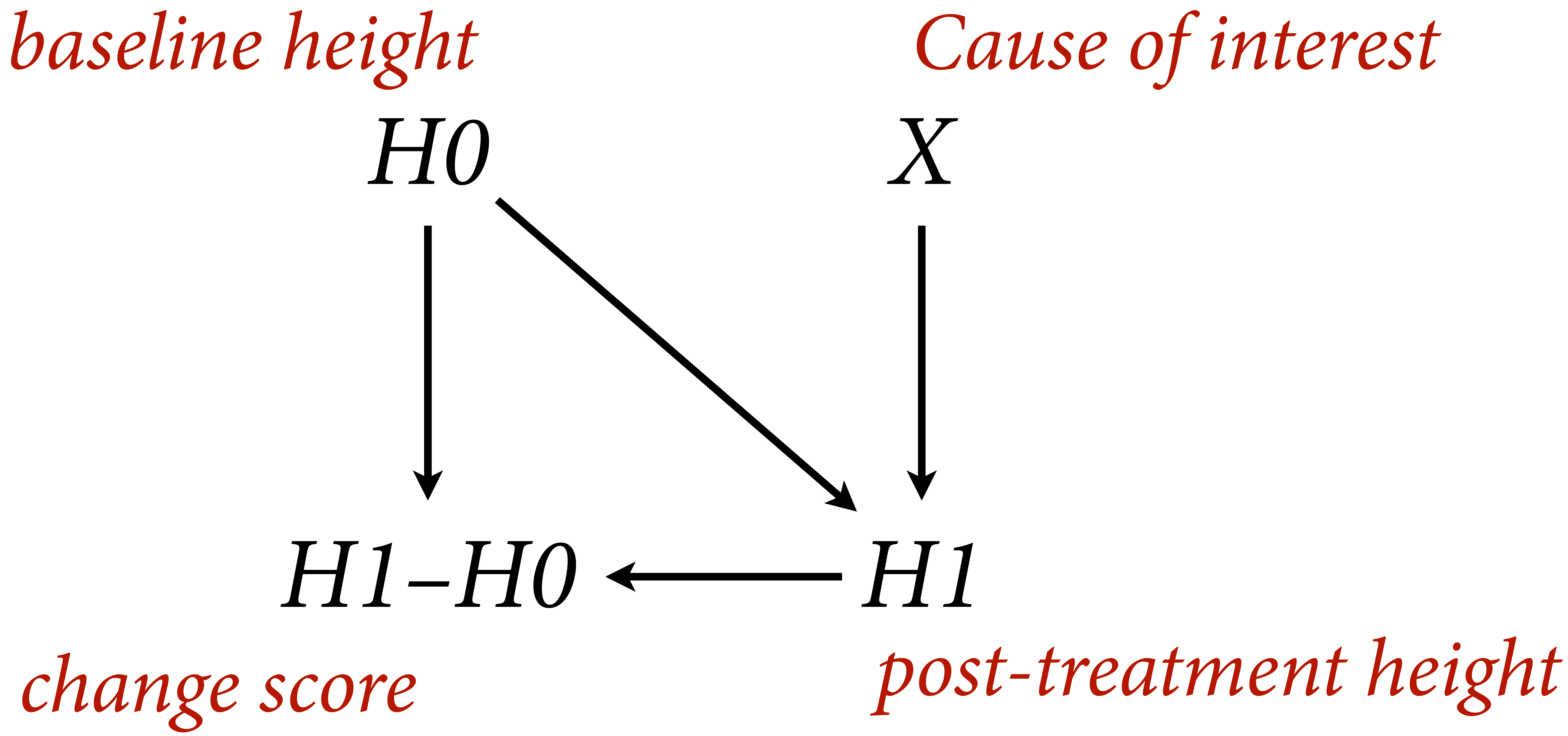

population
Cause of interest
P
X
GDP/P
GDP
GDP per capita
Gross Domestic Product


P
X
Causal path
GDP/P
GDP


P
X
Backdoors
Causal path
GDP/P
GDP



Does not account for differential precision of each Y/T



Requires linear relationship, no floor/ceiling

Constructed Variables Are Bad

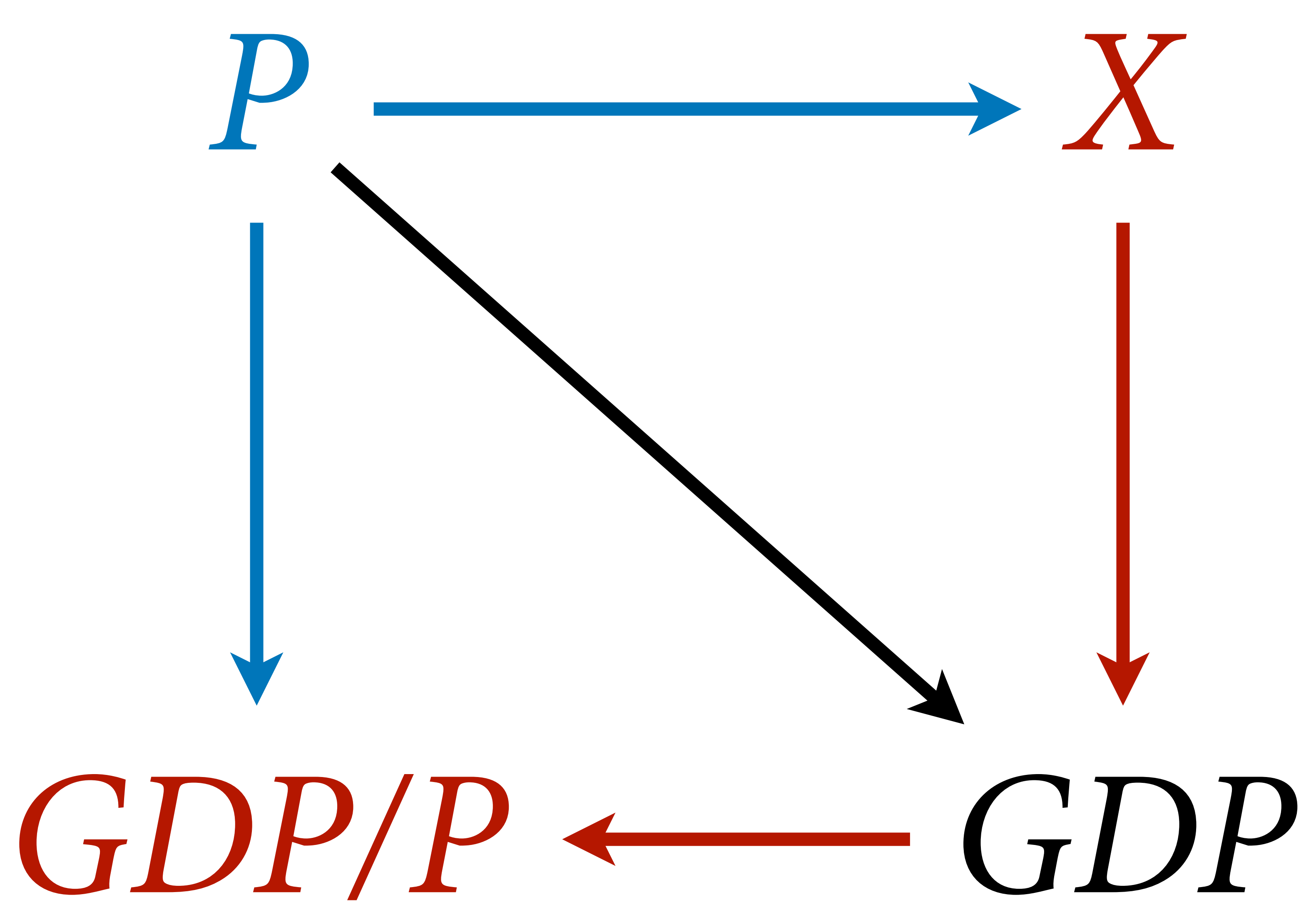
Arithmetic is not stratification

Assumes a fixed relationship, when you should estimate

Ignores uncertainty, e.g. rates

Similar: Do not use model predictions (residuals) as data

Do: Use causal logic, justify, test



Adhockery

Long tradition of *adhockery*: *ad hoc* procedures, intuition as justification

"we expect a correlation"

ad hoc procedures not justified by probability theory go wrong

Simple rule: Model what you measure