Naïve Bayes classifiers

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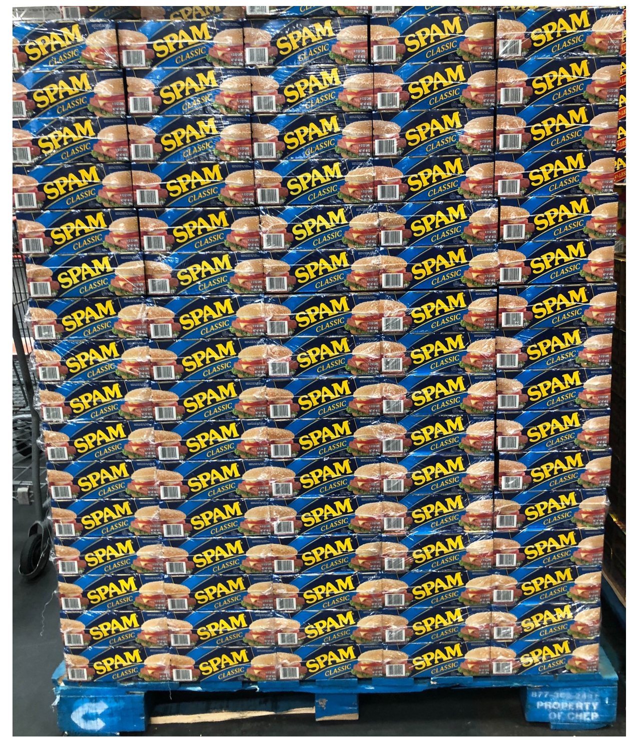
Common problem: detect spam tweets

- Q. Is the following tweet spam or ham (not spam)?

Viagra sale

- Without knowledge of content, what's the best we can do?



SPAM at big package store

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Common problem: detect spam tweets

- Q. Is the following tweet spam or ham (not spam)?

- Without knowledge of content, what's the best we can do?
- Use prior knowledge about relative likelihoods of spam/ham
- If *a priori*, we know 75% of tweets are spam, always guess spam
- (Note: this is solving same problem as, say, article topic classification)

Our base model

- If 75% of tweets are spam, always guessing spam gives us a baseline of 75% accuracy (which we hope to surpass)
- *Accuracy* has formal definition: % correctly-identified tweets
- A superior model must do better than 75% accuracy
- What if *a priori* spam rate was 99%? Model has 99% accuracy
- That hints that accuracy can be very misleading by itself and for imbalanced datasets[1]
- How can we do better than just the *a priori* probabilities?

[1] The Relationship Between Precision-Recall and ROC Curves
https://www.biostat.wisc.edu/~page/rocpr.pdf

Better model using knowledge of content

- If we can see tweet words, we have more to go on; e.g.,

Viagra sale | Buy catfood

- Given "Viagra sale", how do you know it's spam? Because we know:

$$P(\text{spam} \mid \text{Viagra} \cap \text{sale}) > P(\text{ham} \mid \text{Viagra} \cap \text{sale})$$

- We know that (partially) because "Viagra sale" is much more likely to appear in spam emails than in ham emails:

$P(\text{Viagra} \cap \text{sale} \mid \text{spam})$ is high
$P(\text{Viagra} \cap \text{sale} \mid \text{ham})$ is low

- Likewise, we know "Buy catfood" is **unlikely** to occur in spam email

Model based upon tweet likelihoods

- Predict **spam** if:

  $P(\text{Viagra} \cap \text{sale} \mid \text{spam}) > P(\text{Viagra} \cap \text{sale} \mid \text{ham})$

- Predict **ham** if:

  $P(\text{Viagra} \cap \text{sale} \mid \text{spam}) < P(\text{Viagra} \cap \text{sale} \mid \text{ham})$

- This model works great but makes an assumption by not taking into consideration what knowledge? The *a priori* probabilities; assumes equal priors

Model combining priors and content info

- Predict **spam** if:

$$P(\text{spam})P(\text{Viagra} \cap \text{sale} \mid \text{spam}) > P(\text{ham})P(\text{Viagra} \cap \text{sale} \mid \text{ham})$$

- Predict **ham** if:

$$P(\text{spam})P(\text{Viagra} \cap \text{sale} \mid \text{spam}) < P(\text{ham})P(\text{Viagra} \cap \text{sale} \mid \text{ham})$$

- We are weighting the content likelihoods by prior overall spam rate
- If spam-to-ham priors are .5-to-.5 the prior terms cancel out

Are these computations probabilities?

- Do these terms sum to 1.0 after weighting (covering all likelihood)?
  - $P(\text{spam})P(\text{Viagra} \cap \text{sale} \mid \text{spam}) + P(\text{ham})P(\text{Viagra} \cap \text{sale} \mid \text{ham}) \neq 1$

Imagine P(spam) = P(ham) = 0.5 and .7 for conditional probabilities, sum is 0.7!=1

- Nope. Must normalize term by dividing by (unconditional) probability of ever seeing that specific word sequence:

$$P(\text{Viagra} \cap \text{sale})$$

- Dividing by the *marginal* probability makes the terms fractions of the possibilities:

$$\frac{P(\text{Viagra} \cap \text{sale} \mid \text{spam})}{P(\text{Viagra} \cap \text{sale})}$$

- Answers "How much of unconditional does conditional cover?"

Yay! You've just reinvented *Bayes Theorem*

- Normalized likelihood decision rule:

$$\frac{P(\text{spam})P(\text{Viagra} \cap \text{sale} \mid \text{spam})}{P(\text{Viagra} \cap \text{sale})} \gtrless \frac{P(\text{ham})P(\text{Viagra} \cap \text{sale} \mid \text{ham})}{P(\text{Viagra} \cap \text{sale})}$$

- Says how to adjust *a priori* knowledge of spam rate with tweet content evidence

Maximum *a posteriori* classifier

- Choose class/category for document *d* with max likelihood:

$$c^* = \underset{c}{argmax}\ P(c|d)$$

- Substitute Bayes' theorem:

$$c^* = \underset{c}{argmax}\ \frac{P(c)P(d|c)}{P(d)}$$

Bayes' theorem

$$P(c|d) = \frac{P(c)P(d|c)}{P(d)}$$

- You will often see the classification decision rule called the *Bayes test for minimum error*:

$$\mathrm{P}(c_1 \mid d) \gtrless \mathrm{P}(c_2 \mid d)$$

Simplifying the classifier

- P(*d*) is constant on both sides so we can drop it for classification:

$$c^* = \underset{c}{argmax}\ P(c)P(d|c)$$

- If P(*c*) is same for all *c* OR we don't know P(*c*), we drop that too:

$$c^* = \arg\max_{c} P(d|c)$$

Training the classifier

- We need to estimate P(*c*) and P(*d* | *c*) for all *c* and *d*
- Estimating P(*c*)? The number of documents in class *c* divided by the total number of documents; e.g., frequency of spam docs
- Estimating P(*d* | *c*)? E.g., we need P(Viagra ∩ sale | spam)
- That means considering all 2-word combinations (*bigrams*); *n*-grams grow exponentially with length *n*!
- For 10 word tweet we need to estimate probability of a 10-gram; considering all 10-grams is intractable

The naïve assumption

- Naïve assumption: conditional independence; Estimate P(Viagra ∩ sale | ham) as P(Viagra | ham) x P(sale | ham)

$$P(d|c) = \prod_{w \in d} P(w|c)$$

- So, our classifier becomes

$$c^* = \underset{c}{argmax}\ P(c) \prod_{w \in d} P(w|c)$$

- where *w* is each word in *d* <u>with repeats,</u> not *V* (vocabulary words)

Fixed-length word-count vectors

- Rather than arbitrary-length word vectors for each document *d*, it's much easier to use fixed-length vectors of size |*V*| with word counts:

$$c^* = \underset{c}{argmax}\ P(c) \prod_{w \in d} P(w|c)$$

becomes:

$$c^* = \underset{c}{argmax}\ P(c) \prod_{w \in V} P(w|c)^{n_w(d)}$$

(If *w* not present in document, exponent goes to 0, which drops out P(*w*|*c*) for that *w*)

Estimating P(*w* | *c*)

- Use the number of times *w* appears in all documents from class *c* divided by the total number of words (including repeats) in all documents from class *c*:

$$P(w|c) = \frac{wordcount(w, c)}{wordcount(c)}$$

- Or, use num docs with *w* divided by number of docs (which could be better with really short docs like tweets)

What if *w* never used in docs of class c? Laplace smoothing

- If P(*w* | *c*) = 0 then the entire product goes to zero. Ooops!

$$c^* = \underset{c}{argmax}\ P(c) \prod_{w \in V} P(w|c)^{n_w(d)}$$

- To avoid, add 1 to each word count in numerator and compensate by adding |V| to denominator (to keep a probability)

$$P(w|c) = \frac{wordcount(w,c) + 1}{wordcount(c) + |V|}$$

- (We have added +1 to **each** word count in *V* and there are |V| words in each word-count vector)

Dealing with “mispeled” or unknown words

- Laplace smoothing deals with *w* that is in the vocabulary *V* but not in class *c: i.e., when* P(*w* | *c*) = 0 such as P(viagra|ham)=0
- What should *wordcount*(*w,c*) be for a word not in *V* when classifying new doc? Zero doesn’t seem right; OTOH, if *wordcount*(*w,c*)=0 for all classes, classifier is not biased
- Instead: map all unknown *w* to a wildcard word in *V* so then *wordcount*(unknown,c)=0 is ok but |V| is 1 word longer
- Likely not a huge factor…
- Store count of unknown words in word vector at index 0; the *wordcount*(unknown,*c*) is same for all *c* so not biased
- Likelihood of any unknown word is small: 1 / (*wordcount*(*c*) + |*V*| + 1)

$$P(w|c) = \frac{wordcount(w,c) + 1}{wordcount(c) + |V| + \boxed{1}}$$

Avoiding floating point underflow

- In practice, multiplying lots of probabilities in [0,1] range tends to get too small to represent with finite floating-point numbers
- Take log (a monotonic function) and product becomes summation

$$c^* = \underset{c}{argmax}\ P(c) \prod_{w \in V} P(w|c)^{n_w(d)}$$

$$c^* = \underset{c}{argmax} \left\{ log(P(c)) + \sum_{w \in V} n_w(d) \times log(P(w|c)) \right\}$$

An example

Documents as word-count vectors

- One column per vocab word, one row per document

```
d1 = "sale viagra sale"
d2 = "free viagra free viagra free viagra free"
d3 = "buy catfood, buy eggs"
d4 = "buy eggs"
```

Priors:
P(spam) = 2/4
P(ham) = 2/4

column for unknown words

X matrix

spam or ham

Document	unknown	buy	catfood	eggs	free	sale	viagra	spam
0	0	0	0	0	0	2	1	1
1	0	0	0	0	4	0	3	1
2	0	2	1	1	0	0	0	0
3	0	1	0	1	0	0	0	0

Note:
|V|=6
+1 for unknown

Estimating probabilities: P(*w* | spam)

- 1st, get total word count in spam category: sum across rows or cols then sum that result
- *wordcount*(spam) = `spam.sum(axis=1).sum()`

spam

	unknown	buy	catfood	eggs	free	sale	viagra	
0	0	0	0	0	0	2	1	3
1	0	0	0	0	4	0	3	7

wordcount(spam) = 10

$$P(w|c) = \frac{wordcount(w,c)+1}{\boxed{wordcount(c)}+|V|+1}$$

Estimating probabilities: P(*w* | spam)

- 2nd, get total count for each word in spam docs, *wordcount*(w,spam)

spam

	unknown	buy	catfood	eggs	free	sale	viagra	
0	0	0	0	0	0	2	1	
1	0	0	0	0	4	0	3	
		$\frac{0}{10}$	$\frac{0}{10}$	$\frac{0}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$\frac{4}{10}$	$\frac{wordcount(w\|c)}{wordcount(c)}$

$$P(w|c) = \frac{\boxed{wordcount(w,c)} + 1}{wordcount(c) + |V| + 1}$$

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Estimating probabilities: P(*w* | spam)

- 3^{rd}, compute P(*w*|spam) w/smoothing & unknown word adjustment
  - *wordcount*(*w*,spam) + 1 =

unknown	buy	catfood	eggs	free	sale	viagra
1	1	1	1	5	3	5

  - *wordcount*(spam)+|*V*|+1 = 10+6+1=17

P(*w* | spam) →

unknown	buy	catfood	eggs	free	sale	viagra
0.058824	0.058824	0.058824	0.058824	0.294118	0.176471	0.294118

$$P(w|c) = \boxed{\frac{wordcount(w,c)+1}{wordcount(c)+|V|+1}}$$

Estimate P(*c*|*w*) w/o P(*d*) normalization

- Dot product of *X* matrix with log of P(*w*|*c*) vector, add log(P(*c*))

`log(0.5) + 1*log(0.294118) + 2*log(0.176471)`

	P(spam|d)	P(ham|d)
d1 = "sale viagra sale"	$\boxed{-5.386125}$	-8.387995
d2 = "free viagra free viagra free viagra free"	$\boxed{-9.259575}$	-18.647793
d3 = "buy catfood and buy eggs"	-12.026001	$\boxed{-6.388596}$
d4 = "buy eggs"	-6.359574	$\boxed{-3.338139}$

$$c^* = \underset{c}{argmax}\left\{ log(P(c)) + \sum_{w \in V} n_w(d) \times log(P(w|c)) \right\}$$

Key takeaways

- Naïve bayes is classifier applied to text classification; e.g., spam/ham, topic labeling, etc…
- Less often used these days with rise of deep learning
- Fixed-length "bag of words" vectors are the feature vector per doc
- Bayes theorem gives formula for P(*c*|*d*)
- Naïve assumption is conditional independence $\longrightarrow P(d|c) = \prod_{w \in d} P(w|c)$
- Training estimates P(*c*), P(*w*|*c*) for each *w* and *c*
- P(*c*) is ratio of docs in *c* to overall number of docs
- P(*w*|*c*) is ratio of word count of *w* in *c* to total word count in *c*
- Classifier: $c^* = \underset{c}{argmax}\ P(c) \prod_{w \in V} P(w|c)^{n_w(d)}$

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Implementation takeaways

- Avoid vanishing floating-point values from product; take log:

$$c^* = \underset{c}{argmax} \left\{ log(P(c)) + \sum_{w \in V} n_w(d) \times log(P(w|c)) \right\}$$

- Avoid P(*w*|*c*)=0 via Laplace smoothing
  - add 1 to all word counts
  - adjust P(*w*|*c*) denominator with |V| since every doc now has every word
  - this is for missing words where *w* not in *c* but in *V*
- Treat test doc words *w* not in *V*, unknown words, as likelihood: 1 / (*wordcount*(*c*) + |*V*| + 1)

Lab time

- Exploring Naïve Bayes
  https://github.com/parrt/msds621/blob/master/labs/bayes/naive-bayes.ipynb