A crash course in binary trees

We'll revisit in MSDS689 but we need binary trees for projects now

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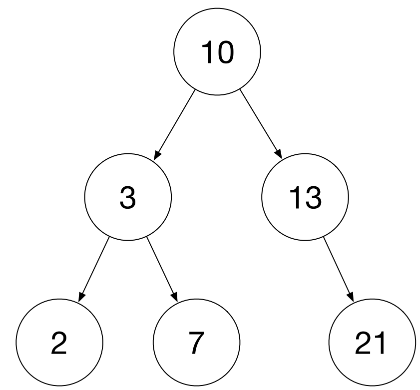
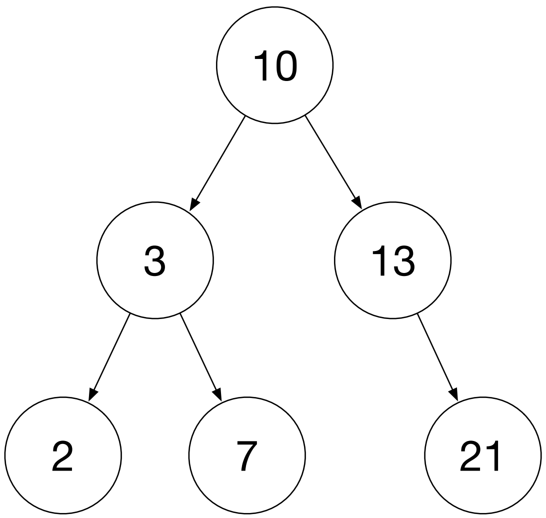
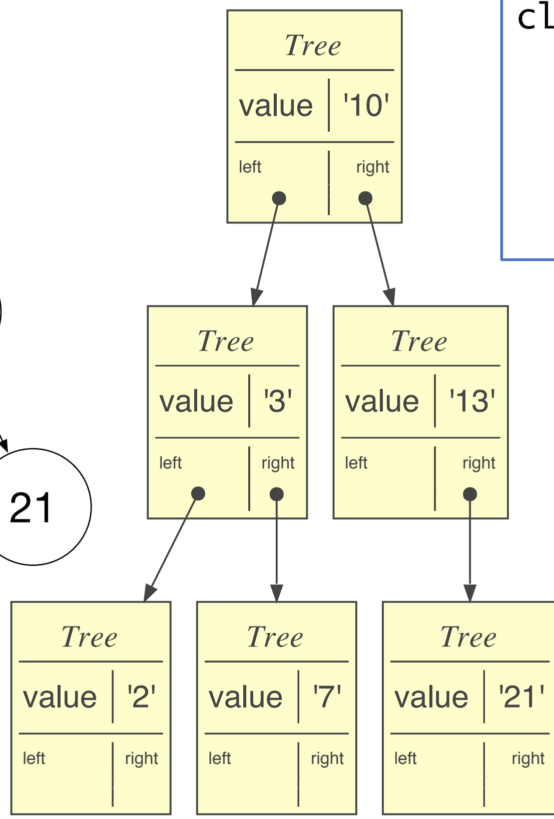
Binary tree abstract data structure



- A *directed graph* with *internal* nodes and *leaves*
- No cycles and each node has at most one parent
- Each node has at most 2 child nodes
- For n nodes, there are n -1 edges
- Nodes have payloads (values) that can be anything
- A *full* binary tree: all internal nodes have 2 children
- Height of full tree with n internal nodes is about $\log_2(n)$
- Height defined as number of edges along path root→leaf
- Level 0 is root, level 1, …
- Warning: binary tree doesn't imply *binary search tree*

Concrete binary tree using pointers





Drawn with https://github.com/parrt/lolviz

```
class TreeNode:
  def __init__(self, value,
                 left=None, right=None):
    self.value = value
    self.left = left
    self.right = right
```

For our purposes here, you can think of a class definition as defining a dictionary that maps fields/members to values. Objects are instances of a class and can act like dictionary objects. Compare:

```
o = dict()              o = TreeNode()
O['value'] = 3          o.value = 3
```

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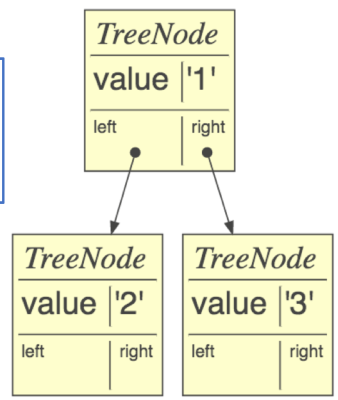
Building binary trees

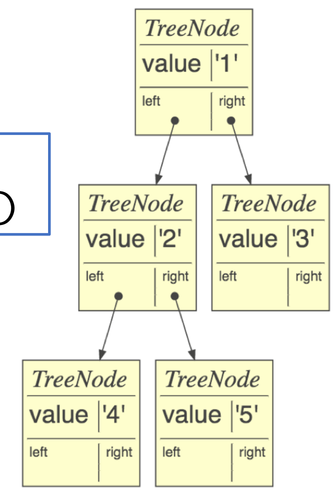
```
class TreeNode:
  def __init__(self, value,
               left=None, right=None):
    self.value = value
    self.left = left
    self.right = right
```

- Manual construction is a simple matter of creating nodes and setting left/right child pointers or passing kids to init method

```
root = TreeNode(1)
root.left = TreeNode(2)
root.right = TreeNode(3)
```




```
root.left.left = TreeNode(4)
root.left.right = TreeNode(5)
```



or

```
root = TreeNode(1, TreeNode(2), TreeNode(3))
```

https://github.com/parrt/msds621/blob/master/notebooks/trees/basics.ipynb

Recursion detour

Math recurrence relations ⇒ recursion

- Factorial definition:
  - Let 0! = 1
  - Define $n! = n * (n-1)!$ for $n \geq 1$
- Recurrent math functions become recursive functions in Python
- Non-recursive version is harder to understand and less natural

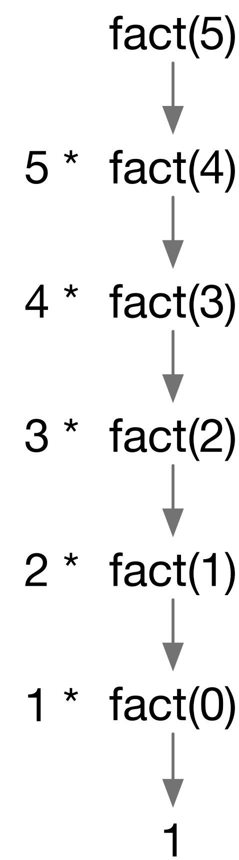
```
def fact(n):
    if n==0: return 1
    return n * fact(n-1)
```

```
def factloop(n):
    r = 1
    for i in range(1,n+1):
        r *= i
    return r
```

Recursion traces out a call graph

- Think of each call to the function as node in chain or graph of calls
- Result of each function call is a piece of the result and each call combines subresult(s) to create more complete answer and pass it back

```
def fact(n):
    if n==0: return 1
    return n * fact(n-1)
```



Formula for writing recursive functions

def **f**(input):

1. check termination condition
2. process the active input region / current node, etc…
3. invoke f on subregion(s)
4. combine and return results

Steps 2 and 4 are optional

```
def fact(n):
    if n==0: return 1
    return n * fact(n-1)
```

Terminology: *currently-active region* or *element* is what **f** is currently trying to process. Here, that is argument **n** (the "region" is the numbers 0..**n**)

Don't let the recursion scare you

- Just pretend that you are calling a different function
- Or, as you write the function, pretend that you are calling the same function except that it is already complete
- We call this the *recursive leap of faith*
- Follow the “Formula for recursive functions” and all will be well!

Recursive tree procedures

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An analogy for recursive tree walking

- Imagine searching for an item in a maze of rooms connected by doors (no cycles)
- Each room has at most 2 doors, some have none
- Search procedure that works in ANY room:

```
def visit(room):
    if item in room: print(“rejoice!”)
    if room.left exists: visit(room.left)
    if room.right exists: visit(room.right)
```

- This approach is called *backtracking*

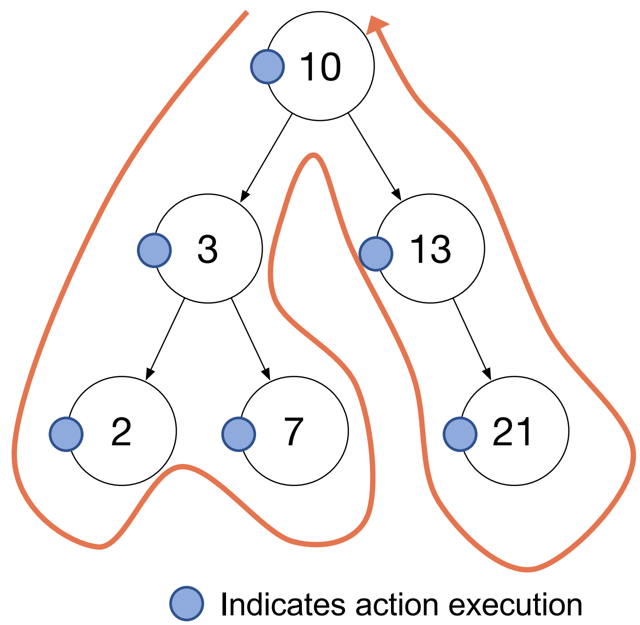
Recursive tree walk is natural

```
def f(input):
    1. check termination condition
    2. process the current node
    3. invoke f on subregion(s)
    4. combine and return results
```

- *Depth-first search* is how we walk (visit) through nodes
- *Pre-order traversal*: executing an action at discovery time, before visiting kids

Follows formula for recursive functions

```
def walk(p:TreeNode):
    if p is None: return
    print(p.value) # preorder
    walk(p.left)
    walk(p.right)
```



Think of launching a minion to walk the left subtree and then another to walk the right

https://github.com/parrt/msds621/blob/master/notebooks/trees/basics.ipynb

How can walk() remember where it has visited?

- "Where to return" is tracked per function **call** not per function **definition**
- Function **f** calls **g** calls **h** and Python remembers where each was called from
- Each function call saves its place like keeping a finger on the call statement; `return` statement uses that as location to resume after invoked function returns
- Just imagine that **f**, **g**, and **h** are the same function and you'll see that recursive calls also remember where they came from

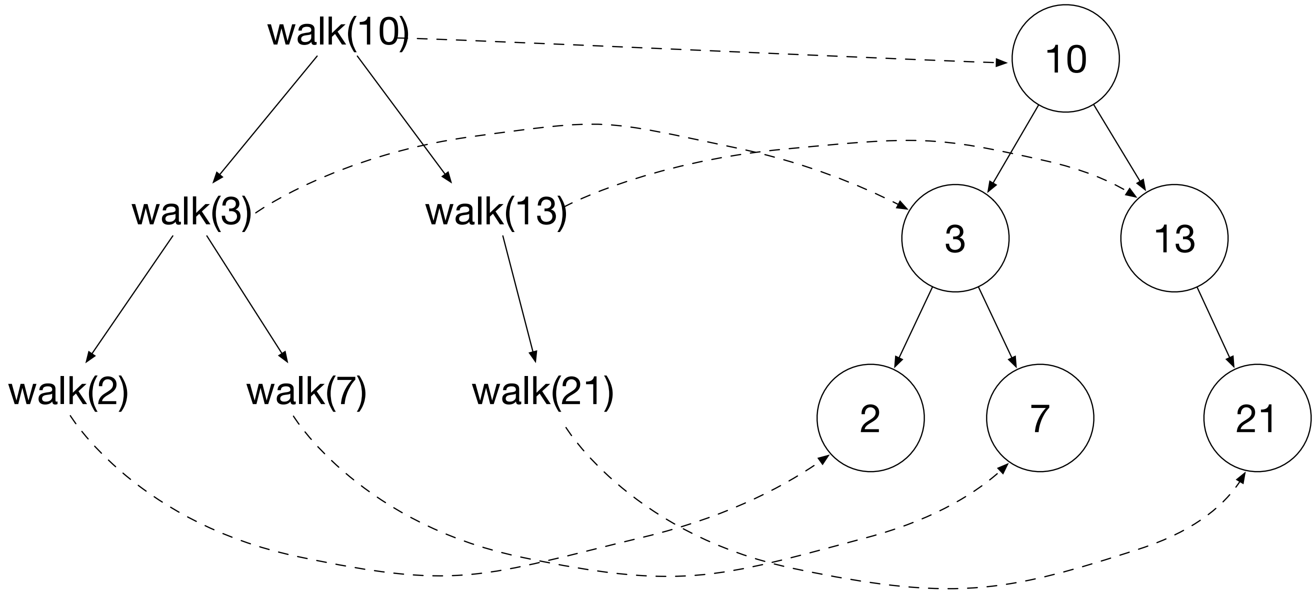
```python
def f():
    g()
    print("back from g()")

def g():
    h()
    print("back from h()")

def h():
    print("hi I'm h!")
```

```python
f()
print("back from f()")
```

```
hi I'm h!
back from h()
back from g()
back from f()
```

Recursion call tree vs tree

```
def walk(p:TreeNode):
    if p is None: return
    walk(p.left)
    walk(p.right)
```



Exhaustive search of all nodes

Searching in binary tree

- Let's modify the tree walker to search for an element and compare to unrestricted depth-first tree walk

```
def walk(p:TreeNode):
    if p is None: return
    print(p.value)
    walk(p.left)
    walk(p.right)
```

```
def search(p:TreeNode, x:object):
    if p is None: return None
    if x==p.value: return p
    q = search(p.left, x)
    if q is not None: return q
    q = search(p.right, x)
    return q
```

Decision tree stumps

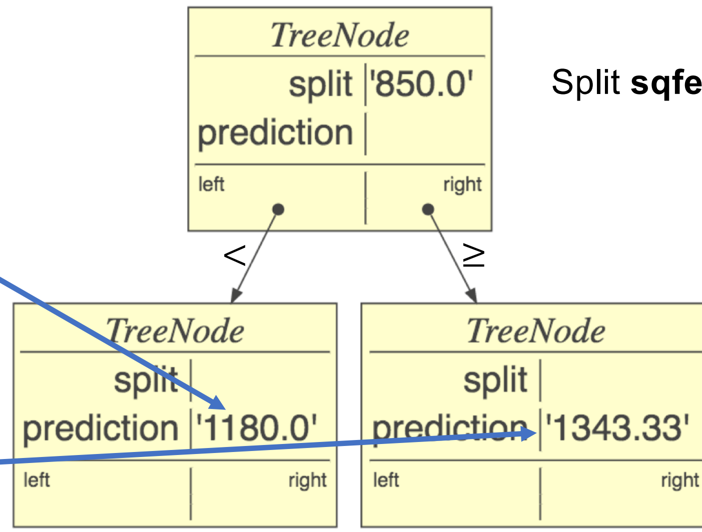
Stumps

- A stump is a 2-level tree w/decision node root & 2 predictor leaves
- Used by gradient boosting machines as the "weak learners"
- If node has field **split**, it's a decision node else it's a leaf

```
# Define a single-node class for simplicity
class TreeNode:
  def __init__(self, split=None, prediction=None,
               left=None, right=None):
    self.split = split
    self.prediction = prediction
    self.left = left
    self.right = right
```

Sample stump that picks midpoint as split

sqfeet	rent
750	1160
800	1200
850	1280
900	1450
950	1300



Split **sqfeet** feature space at 850

Average in right region

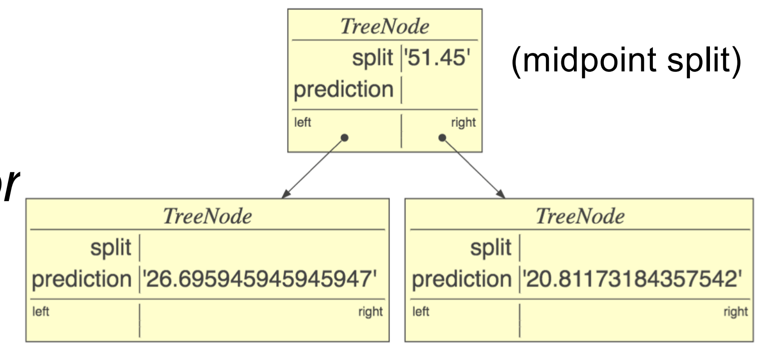
Creating decision tree stumps

- For demonstration purposes only, let's split x always at midpoint between min/max:

```
def stumpfit(x, y):
    if len(x)==1 or len(np.unique(x))==1:
        # if just one or unique x value, create & return a leaf
        return TreeNode(prediction=np.mean(y))
    split = (min(x) + max(x)) / 2 # split at x midpoint
    t = TreeNode(split)
    t.left  = TreeNode(prediction=np.mean(y[x<split]))
    t.right = TreeNode(prediction=np.mean(y[x>=split]))
    return t
```

The magic of recursion

- Demo converting stumpfit() to treefit()
- See "*Regression tree midpoint split for Boston dataset*" in notebook
- In treefit(x,y), simply convert



(midpoint split)

```
t.left = TreeNode(prediction=np.mean(y[x<split]))
```

Create node

to

```
t.left = treefit(x[x<split], y[x<split])
```

Create subtree

Notebook: https://github.com/parrt/msds621/blob/master/notebooks/trees/decision-trees.ipynb

In practice, better to use two classes

- See notebook for 1D decision tree implementation
  https://github.com/parrt/msds621/blob/master/notebooks/trees/decision-trees.ipynb

```
class DecisionNode:
    def __init__(self, split, left=None, right=None):
        self.split = split  # split point chosen from x
        self.left = left
        self.right = right

class LeafNode:
    def __init__(self,y):
        self.y = y
```

Key takeaways

- Binary tree: acyclic tree structure with at most two children, constructed by hooking nodes together (e.g., `root.left = TreeNode(2)`)
- Self-similar data structures built and walked with recursion
- Each recursive call does a piece of the work and returns its piece combined with results obtained from recursive calls
- Recursion traces out a call tree that's like a tree data structure
- Recursive call in treefit() returns newly-constructed subtree
- Remember the recursive function template!
- Depth-first-search visits each node through backtracking
- Study these recursive tree functions!