

Stereographic Projection

Using similar triangles we have

$$q = \frac{\sin \theta}{1 + \cos \theta}.$$

Solving for sin and cos yields

$$\cos \theta = \frac{1 - q^2}{1 + q^2}, \quad \sin \theta = \frac{2q}{1 + q^2}.$$

Given $\dot{\theta} = f(\theta, x)$, we have

$$\dot{q} = \frac{1 + q^2}{2} f(\theta, x),$$

which can be written as

$$e(q)\dot{q} = p(q, x),$$

with $e()$ and $p()$ polynomial in q .

