Computational Neuroscience Exam
COMSM0039

January 2021

Instructions to students

The exam has seven sections, corresponding to the seven weeks of teaching material. All questions are compulsory. You should consider it open-book in the sense that you can refer to the lecture notes.

Section 1: Brain basics (8 marks)

Q1. We can describe a single neuron's input-output function with the following equation: $y = \Theta(\sum_i x_i w_i)$. Explain what each of y, Θ, x_i and w_i represent and in each case, name which physical part of the neuron is involved. [2 marks]

Q2. Single neurons take their input signal from X neuron(s) and send their output signal to Y neuron(s) [2 marks]:
a) X = multiple, Y = a single
b) X = multiple, Y = multiple
c) X = a single, Y = multiple
d) X = a single, Y = a single

Q3. Which of the following are advantages for using fMRI relative to EEG when recording human brain activity? Tick all that apply. [2 marks]
a) Faster temporal resolution.
b) Can record brain activity from people as they walk around.
c) Cheaper.
d) Better spatial resolution.

Q4. What is the relationship between machine learning, deep learning, and computational neuroscience? [2 marks]

Answers

Q1. y is the output, spikes along the axon. Θ is the spike threshold, done by the soma or axon. x is the input signals, corresponding to activity in presynaptic axons. And w is the synaptic weights, corresponding to the synapses. [2 marks total, give 1 mark for partial answer.]

Q2. answer b).

Q3. d) is only correct answer.

Q4. Deep learning is between machine learning and computational neuroscience [1 mark]. Machine learning is about computer algorithms that improve after training on data. Deep learning is a subfield of machine learning that uses artificial neural networks, which are brain-inspired. Computational neuroscience in contrast is about understanding the brain [1 marks for similar].

Section 2: Differential equations, leaky integrate-and-fire neurons (7 marks)

Question 1. Give the analytical solution to the following differential equation: $dy/dt = (2 - y)/3$ with $y(t = 0) = 6$. What value will y converge to after a long time, $t \to \infty$? [2 marks]

Question 2. For an integrate-and-fire neuron model with input resistance R_m, membrane time constant τ_m, resting voltage V_{rest}, threshold voltage V_{th}, and DC input current I_e, analytically calculate the time interval between spikes. [3 marks]

Question 3. For the Euler method of numerical integration, how does the error scale with the timestep size? Explain why. [1 mark]

Q4. When solving a differential equation numerically, the timestep size Δt should be chosen to be [1 mark]:
a) the same size as the typical timescale in the system, $\Delta t \sim \bar{\tau}$.
b) bigger than the slowest timescale in the system, $\Delta t > \tau_{max}$.
c) smaller than the fastest timescale in the system, $\Delta t < \tau_{min}$.

Answers

Q1: Solution is $y(t) = 2 + 4e^{-t/3}$. The steady-state value $y(\infty) = 2$.

Q2: Interspike interval is given by $T = \tau_m \log \left[\frac{R_m I_e}{V_{rest} + R_m I_e - V_{th}} \right]$.

Q3: The error is $\mathcal{O}(\Delta t^2)$ [1 mark]. This is because Euler can be thought of as a truncated Taylor expansion of the function at each timestep, with the truncation including only the first two terms (the linear terms) in the Taylor series [1 mark].

Q4: answer c.

Section 3: Hodgkin Huxley, modelling neurons, analysing spiking data (7 marks)

Question 1. Which two ion channel types underlie the upswing and downswing of action potential in the Hodgkin-Huxley model, respectively? [1 mark]
a) potassium and calcium
b) potassium and sodium
c) sodium and potassium
d) sodium and calcium

Question 2. Describe in your own words how the two ion channels from the previous question jointly generate the action potential. [2 marks]

Question 3. The equation for the spike-triggered average is: $S(\tau) = \frac{1}{N}\sum_i^N s(t_i - \tau)$. What do s, τ and t_i represent here? And why does it make sense to call this quantity a "spike-triggered average"? [2 marks]

Question 4. Name two advantages that the integrate-and-fire neuron model has over the Hodgkin-Huxley model. Explain how these advantages follow from the models' description. [2 marks]

Answers

Q1. c.

Q2. The sodium channels drive the upswing because they act like a positive feedback with the membrane voltage, becoming open with depolarisation and letting positive charge into the cell. Potassium channels on the other hand act like negative feedback, opening on depolarisation but passing a negative current into the cell. Sodium is faster than potassium so can drive the upswing before potassium 'reacts'.

Q3. s is the stimulus value, τ is time relative to the spike, and t_i are the individual spike times [1 mark]. The name makes sense because it calculates the average value of the stimulus (computed by the normalised sum) relative to the spike. It is the average of the stimulus preceding each spike, therefore 'triggering' each spike [1 mark].

Q4: Possible advantages of integrate-and-fire: fewer parameters to set, faster to simulate, do mathematical analysis, quite generic [1 mark for naming any two]. Fewer parameters from simple equation describing dynamics; faster to simulate as fewer ODEs to solve and fewer operations to compute per timestep; simple enough to do some mathematical analysis; generic because could plausibly be any cell type in the brain, whereas hodgkin huxley is a squid giant axon [1 mark for two correct explanations].

Section 4: Synapses, synaptic plasticity (7 marks)

Question 1. Explain how synaptic short term depression can act as a low-pass filter. [2 marks]

Question 2. Synapses are unreliable: sometimes a presynaptic action potential fails to cause a voltage response in the postsynaptic neuron. Where does this failure occur? [1 mark]
a) presynaptically
b) postsynaptically
c) both

Question 3. Given a neuron model $y = \sum_i x_i w_i$, explain why the Hebbian synaptic plasticity rule $dw/dt = \eta xy$ is unstable. [1 mark]

Question 4: The BCM model consists of two differential equations, one for the synaptic weights: $dw_i/dt = \eta_w x_i y(y - \theta)$, and one for the threshold: $d\theta/dt = \eta_\theta(y^2 - \theta)$. Explain why this model ensures stability. Given a linear neuron model $y = \sum_i x_i w_i$, calculate the steady state synaptic value of the weights, given a pair of inputs $x_1 = 2$ and $x_2 = 3$. Give your answer for w_1 in terms of w_2. Assume the weights are positive. [3 marks]

Answers

Q1: If two presyaptic spikes arrive nearby in time, then the synaptic resource (e.g. vesicles) will be depleted for the second spike, so the response will be small. In contrast, if two spikes arrive far apart in time, the synaptic resource will have recovered to full strength. So high-frequency pre-synaptic signals will result in small postsynaptic response, whereas low-frequency signals will be unattenuated [2 marks for similar, 1 mark for partial answer].
Q2: answer a.
Q3: Because of positive feedback: if $y = \sum_i x_i w_i$, then increases in the weights will lead to increases in y, which will lead to further increases in w, without bound.
Q4: The model ensures stability because the threshold must grow faster than the postsynaptic activity, causing the synaptic weights to decrease if they become too large [1 mark]. $w_1 = (1 - 3w_2)/2$ [2 marks]

Section 5: Hippocampus, Hopfield networks (7 marks)

Question 1. What is path integration? What is the main disadvantage of path integration? [2 marks]

Question 2. What are the weights for a 2 state Hopfield network with 2 input patterns $x_1 = (1, 1)$, $x_2 = (-1, 1)$? [2 marks]
a) (1 0.5; 0.5 1)
b) (1 0; 0 1)
c) (1 -0.5; -0.5 1)
d) (0.5 -0.5; -0.5 0.5)

Question 3. If we start at $x = (1, 1)$ and update both neurons simultaneously (synchronous update), what is the resulting output pattern (assume the threshold is zero)? [3 marks]
a) (1, 1)
b) (-1, 1)
c) (1, -1)
d) (-1, -1)

Answers

Q1: Path integration is a navigational strategy in which you combine the start position with an estimate of velocity and direction to give a final position [1 mark]. Disadvantage: estimate of position gets worse over time due to noise in direction/velocity estimate [1 mark].
Q2: b
Q3: a

Section 6: Visual system, rate coding (7 marks)

Question 1. When estimating the firing rate of a neuron, what issues can arise when using a too-small bin-width/bandwidth? What about when using a too-large bin-width/bandwidth? [2 marks]

Question 2: V1 complex cells display a degree of: [2 marks]
a) spatial/phase invariance
b) frequency invariance
c) orientation invariance

Question 3. What stimulus would retinal ganglion cells respond strongly to? [1 mark]
a) a circular spot of light surrounded by darkness.

b) an oriented “edge”.
c) a person’s face.

Question 4. Which of the following is NOT a topographic map present in V1? [2 marks]
a) the spatial location of the edge stimulus
b) the eye preferred by the neuron.
c) angle of the edge stimulus.
d) the spatial frequency of the edge stimulus.

Answers

Q1: If there is a consistent, low firing rate and the bin-width too small, it will look like there is a large firing rate in a very narrow window around spikes [1 mark for similar]. If there were bursts in the data with fast onset and offset of spiking, and the bin-width is too large, it will smooth out the onset and offset of those peaks [1 mark for similar].
Q2: a
Q3: a
Q4: d

Section 7: Supervised learning, Cerebellum, temporal difference learning (7 marks)

Question 1. Which statement is FALSE? [1 mark]
a) Granule cells are the primary input to the Cerebellum.
b) Granule cells have a very large number of inputs (more than 1000).
c) Granule cell axons become parallel fibres.
d) Granule cells perform pattern separation.
e) Granule cells are the most numerous cell type in the brain.

Question 2. Which of these arises in the inferior olive and provides feedback signals to Cerebellum? [1 mark]
a) Granule cells
b) Purkinje cells
c) Climbing fibres
d) Parallel fibres

Question 3. Define “inhibitory conditioning”, what is the final outcome? [2 marks]

Question 4. Prove that in the partial reward setting (where there is one stimulus, $x_1 = 1$ that is always on, but the reward appears with probability α), the weight, w_1 converges towards the probability of reward, p. [3 marks]

Answers

Q1: b.

Q2: c.

Q3: Alternate:
Light, no bell, $(x_1 = 1, x_2 = 0, r = 1)$ reward [1 mark]
Light, no bell, $(x_1 = 1, x_2 = 1, r = 0)$ reward [1 mark]
$w_1 = 1, w_2 = -1$ [1 mark]

Q4: solve for expected weight change = 0.
weight change for $r = 0$: $\Delta w(r = 0) = (r - wx) = -w$ [1 mark]
weight change for $r = 1$: $\Delta w(r = 1) = (r - wx) = 1 - w$ [1 mark]
$0 = E[\Delta w]$
$0 = -(1 - p)w + p(1 - w)$
$(1 - p)w = p(1 - w)$
$p = w$ [1 mark]