Computational Neuroscience Exam
COMS30016

January 2021

Instructions to students

The exam has seven sections, corresponding to the seven weeks of teaching material. All questions are compulsory. You should consider it open-book in the sense that you can refer to the lecture notes.

Section 1: Brain basics (8 marks)

Q1. We can describe a single neuron's input-output function with the following equation: $y = \Theta(\sum_i x_i w_i)$. Explain what each of y, Θ, x_i and w_i represent and in each case, name which physical part of the neuron is involved. [2 marks]

Q2. Single neurons take their input signal from X neuron(s) and send their output signal to Y neuron(s) [2 marks]:
a) X = multiple, Y = a single
b) X = multiple, Y = multiple
c) X = a single, Y = multiple
d) X = a single, Y = a single

Q3. Which of the following are advantages for using fMRI relative to EEG when recording human brain activity? Tick all that apply. [2 marks]
a) Faster temporal resolution.
b) Can record brain activity from people as they walk around.
c) Cheaper.
d) Better spatial resolution.

Q4. Recurrent neural networks are believed to be better than feedforward neural networks for tasks involving temporal input signals. Offer an explanation for why. [2 marks]

Answers

Q1. y is the output, spikes along the axon. Θ is the spike threshold, done by the soma or axon. x is the input signals, corresponding to activity in presynaptic axons. And w is the synaptic weights, corresponding to the synapses. [2 marks total, give 1 mark for partial answer.]

Q2. answer b).

Q3. d) is only correct answer.

Q4. Recurrent neural networks are better for temporal tasks because the feedback connections let signals propagate around within the network, even when the input signal has gone. This implies they have some memory of past inputs and can therefore do temporal computations [1 mark for similar]. In contrast feedforward networks forget the input on the timescale of the single neuron membrane time constant [1 mark].

Section 2: Differential equations, leaky integrate-and-fire neurons (7 marks)

Question 1. Give the analytical solution to the following differential equation: $dy/dt = (2 - y)/3$ with $y(t = 0) = 6$. What value will y converge to after a long time, $t \to \infty$? [2 marks]

Question 2. For an integrate-and-fire neuron model with input resistance $R_m = 100$ MΩ, membrane time constant $\tau_m = 10$ ms, resting voltage $V_{rest} = -70$ mV, and threshold voltage $V_{th} = -50$ mV, what is the value of the minimum DC current needed to trigger a spike? [1 mark]

Question 3. For the Euler method of numerical integration, how does the error scale with the timestep size? Explain why. [2 marks]

Q4. When solving a differential equation numerically, the timestep size Δt should be chosen to be [2 marks]:
a) the same size as the typical timescale in the system, $\Delta t \sim \bar{\tau}$.
b) bigger than the slowest timescale in the system, $\Delta t > \tau_{max}$.
c) smaller than the fastest timescale in the system, $\Delta t < \tau_{min}$.

Answers

Q1: Solution is $y(t) = 2 + 4e^{-t/3}$. The steady-state value $y(\infty) = 2$.

Q2: Minimum current needed is $(V_{th} - V_{rest})/R_m = 0.2$ nA.

Q3: The error is $\mathcal{O}(\Delta t^2)$ [1 mark]. This is because Euler can be thought of as a truncated Taylor expansion of the function at each timestep, with the truncation including only the first two terms (the linear terms) in the Taylor series [1 mark].

Q4: answer c.

Section 3: Hodgkin Huxley, modelling neurons, analysing spiking data (7 marks)

Question 1. Which two ion channel types underlie the upswing and downswing of action potential in the Hodgkin-Huxley model, respectively? [1 mark]
a) potassium and calcium
b) potassium and sodium
c) sodium and potassium
d) sodium and calcium

Question 2. Describe in your own words how the two ion channels from the previous question jointly generate the action potential. [2 marks]

Question 3. The equation for the spike-triggered average is: $S(\tau) = \frac{1}{N}\sum_i^N s(t_i - \tau)$. What do s, τ and t_i represent here? And why does it make sense to call this quantity a "spike-triggered average"? [2 marks]

Question 4. The coefficient of variation (CV) and Fano Factor (FF) are two different measures of spike train variability. Define each measure and then comment on differences in which aspects of spike trains they are sensitive to. [2 marks]

Answers

Q1. c.

Q2. The sodium channels drive the upswing because they act like a positive feedback with the membrane voltage, becoming open with depolarisation and letting positive charge into the cell. Potassium channels on the other hand act like negative feedback, opening on depolarisation but passing a negative current into the cell. Sodium is faster than potassium so can drive the upswing before potassium 'reacts'.

Q3. s is the stimulus value, τ is time relative to the spike, and t_i are the individual spike times [1 mark]. The name makes sense because it calculates the average value of the stimulus (computed by the normalised sum) relative to the spike. It is the average of the stimulus preceding each spike, therefore 'triggering' each spike [1 mark].

Q4: The CV is the standard deviation of the interspike-interval divided by the mean, whereas the Fano Factor is the variance of the spike count divided by the mean [1 mark]. Since the CV operates on spike pair intervals, but the FF acts on spike counts over longer intervals, the CV is more sensitive to fine-timescale variability inherent in the data, whereas the FF can be sensitive to temporal correlations on experimentally-defined timescales [1 mark].

Section 4: Synapses, synaptic plasticity (7 marks)

Question 1. A simple model of a synaptic current is $I_s(t) = \bar{g}s[E_s - V(t)]$. Which component of the model determines whether the synapse is excitatory or inhibitory? Explain why. [2 marks]

Question 2. Synapses are unreliable: sometimes a presynaptic action potential fails to cause a voltage response in the postsynaptic neuron. Where does this failure occur? [1 mark]
a) presynaptically
b) postsynaptically
c) both

Question 3. Given a neuron model $y = \sum_i x_i w_i$, explain why the Hebbian synaptic plasticity rule $dw/dt = \eta xy$ is unstable. [1 mark]

Question 4: The BCM model consists of two differential equations, one for the synaptic weights: $dw_i/dt = \eta_w x_i y(y - \theta)$, and one for the threshold: $d\theta/dt = \eta_\theta(y^2 - \theta)$. Explain why this model ensures stability. Given a linear neuron model $y = \sum_i x_i w_i$, calculate the steady state synaptic value of the weights, given a pair of inputs $x_1 = 2$ and $x_2 = 3$. Give your answer for w_1 in terms of w_2. Assume the weights are positive. [3 marks]

Answers

Q1: The E_s parameter determines whether the synapse is excitatory or inhibitory [1 mark]. If $E_s > V(t)$, then the current will be positive, and therefore increase/excite the voltage. In contrast, if $E_s < V(t)$, then the current will be negative and 'inhibit' the cell voltage [1 mark]
Q2: answer a.
Q3: Because of positive feedback: if $y = \sum_i x_i w_i$, then increases in the weights will lead to increases in y, which will lead to further increases in w, without bound.
Q4: The model ensures stability because the threshold must grow faster than the post-synaptic activity, causing the synaptic weights to decrease if they become too large [1 mark]. $w_1 = (1 - 3w_2)/2$ [2 marks]

Section 5: Hippocampus, Hopfield networks (7 marks)

Question 1. What is the tri-synaptic loop? [1 mark]
a) Dentate Gyrus → Subiculum → CA3 → CA1
b) Dentate Gyrus → CA3 → CA1 → Subiculum
c) Subiculum → CA3 → CA1 → Dentate Gyrus
d) Dentate Gyrus → CA3 → CA1 → Subiculum

Question 2. What are the weights for a 2 state Hopfield network with 2 input patterns $x_1 = (1, 1)$, $x_2 = (-1, 1)$? [3 marks]
a) (1 0.5; 0.5 1)
b) (1 0; 0 1)
c) (1 -0.5; -0.5 1)
d) (0.5 -0.5; -0.5 0.5)

Question 3. If we start at $x = (1, 1)$ and update both neurons simultaneously (synchronous update), what is the resulting output pattern (assume the threshold is zero)? [3 marks]
a) (1, 1)
b) (-1, 1)
c) (1, -1)
d) (-1, -1)

Answers

Q1: b
Q2: b
Q3: a

Section 6: Visual system, rate coding (7 marks)

Question 1. Which of these is NOT a potential benefit of sparse coding? [2 marks]
a) energy efficiency.
b) regularisation.
c) enhanced discriminability.
d) more information (in an information theoretic sense).

Question 2: V1 complex cells display a degree of: [2 marks]
a) spatial/phase invariance
b) frequency invariance
c) orientation invariance

Question 3. What stimulus would retinal ganglion cells respond strongly to? [1 mark]
a) a circular spot of light surrounded by darkness.
b) an oriented "edge".
c) a person's face.

Question 4. Which of the following is NOT a topographic map present in V1? [2 marks]
a) the spatial location of the edge stimulus
b) the eye preferred by the neuron.
c) angle of the edge stimulus.
d) the spatial frequency of the edge stimulus.

Answers

Q1: d
Q2: a
Q3: a
Q4: d

Section 7: Supervised learning, Cerebellum, temporal difference learning (7 marks)

Question 1. Which statement is FALSE? [1 marks]
a) Granule cells are the primary input to the Cerebellum.
b) Granule cells have a very large number of inputs (more than 1000).
c) Granule cell axons become parallel fibres.
d) Granule cells perform pattern separation.
e) Granule cells are the most numerous cell type in the brain.

Question 2. Define "inhibitory conditioning", what is the final outcome? [3 marks]

Question 3. Prove that in the partial reward setting (where there is one stimulus, $x_1 = 1$ that is always on, but the reward appears with probability α), the weight, w_1 converges towards the probability of reward, p. [3 marks]

Answers

Q1: b.

Q2: Alternate:
Light, no bell, $(x_1 = 1, x_2 = 0, r = 1)$ reward [1 mark]
Light, no bell, $(x_1 = 1, x_2 = 1, r = 0)$ reward [1 mark]

$w_1 = 1, w_2 = -1$ [1 mark]

Q3: solve for expected weight change = 0.
weight change for $r = 0$: $\Delta w(r = 0) = (r - wx) = -w$ [1 mark]
weight change for $r = 1$: $\Delta w(r = 1) = (r - wx) = 1 - w$ [1 mark]
$0 = E[\Delta w]$
$0 = -(1 - p)w + p(1 - w)$
$(1 - p)w = p(1 - w)$
$p = w$ [1 mark]