Normalized Online Learning Tutorial

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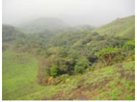
Motivation: Covertype Data Set



54 total features

Name	Units
Elevation, Distance to X	meters
Aspect, Slope	degrees
Hillshade at time t	"hillshade index" (0-255)
Wilderness Area	$\{0, 1\}^4$
Soil Type	$\{0, 1\}^{40}$

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- This can be made arbitrarily bad in only two dimensions by scaling one of the dimensions while leaving the other fixed. <u>Not an artifact of the analysis.</u>

Example

- Generate data like this

$$
\begin{aligned}
x_1 &\sim N(0, 1) \\
x_2 &\sim N(0, \sqrt{s}) \\
z &\sim N(x_1 + \frac{1}{s} x_2, 1)
\end{aligned}
$$

- Do squared-loss prediction of z.
- NB: x_2 is statistically identical to x_1 scaled by s.

Example

Demo

Summary of Demo

- Un-normalized learning
  - Lots of fiddling with learning rate.
  - Slow convergence at extreme scales.
- Normalized learning
  - No fiddling with learning rate.
  - Same convergence across different scales.

On "Non-Demo" Datasets

- Un-normalized learning
  - Lots of fiddling with learning rate.
  - Slow convergence at extreme scales.
- Normalized learning
  - ~~No~~ Less fiddling with learning rate.
  - ~~Same~~ Similar convergence across different scales.

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- Ergo:
  - Algorithm keeps track of $\max_{s<t} |x_i^{(s)}|$ for each j.
  - When $|x_i^{(t)}| > \max_{s<t} |x_i^{(s)}|$, rescale w_i via

$$w_i \leftarrow w_i \frac{\max_{s<t} |x_i^{(s)}|}{x_i^{(t)}}.$$

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  - Intuition behind N_t: if this is an example with small x_i, prediction is not changing very fast because gradient is normalized by scale.

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- If the scale suddenly gets very large near the end of the input sequence, the scale estimates have been poor for most of the updates.
- Theorems are driven by $\Delta_i = \frac{\max_{t \in 1:T} |x_{ti}|}{|x_{t_0^i i}|}$.

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  ...will you give vanilla AdaGrad without normalization.