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A scalable and P2P compatible index CRDT

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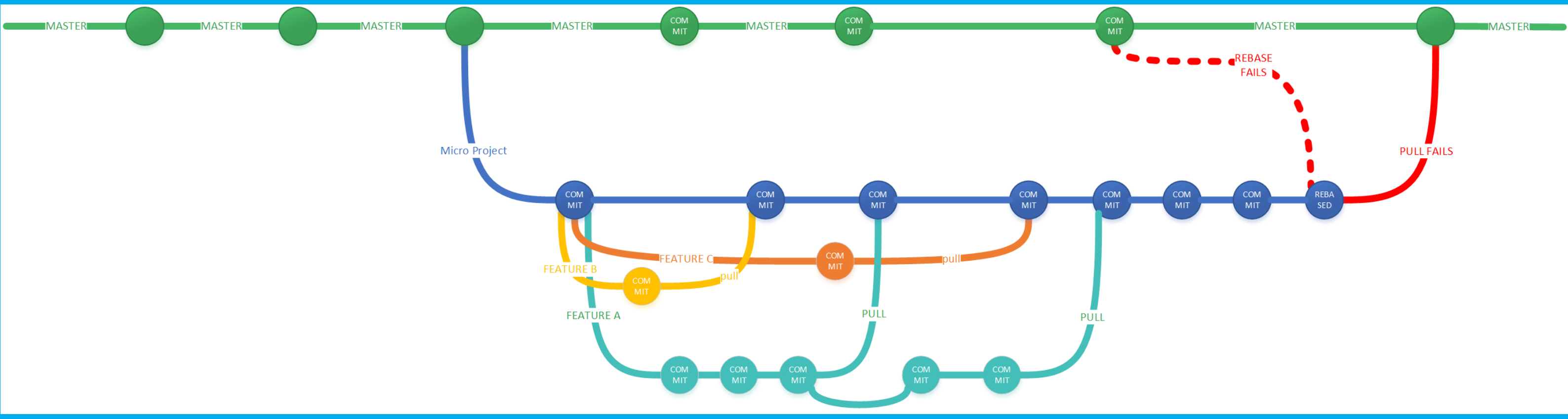

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by

E. M. Bongers

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Student number:	1188232	
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Thesis committee:	(dr.) ir. M. de Vos,	TU Delft, supervisor
	prof. dr. J. A. Pouwelse,	TU Delft
	prof. dr. S. Roos,	TU Delft

TU Delft

Abstract

In theory, CRDTs seem like a natural fit for open P2P networks, however there are obstacles to overcome. Many proposed CRDTs are grow-only because CRDTs may track deletions in permanent tombstone values or they may gather permanent information on every peer in the system. As such, CRDTs are not well adapted to open P2P networks. Many peers may come and go over time and having accurate information about all peers is infeasible. Also, some types of CRDT (mainly op-based CmRDT) require causal message delivery which is hard in open P2P networks. Lastly, CRDTs are typically built around the thought that all peers need all information and thus all data is fully replicated, even though a client may only be interested in a small subset.

A new state based CvRDT is proposed: BloomCRDT, which is a variation on the OR-set that replaces the standard R g-set with Bloom filters. It does not need knowledge of other peers, or their state, and avoids tombstones. This makes BloomCRDT compatible with open P2P networks. The grow-only aspect is vastly reduced compared to the standard OR-set. However, the BloomCRDT itself does not scale the number of contained items well enough to accommodate demanding applications. To address this, multiple BloomCRDTs can be combined to form a Conflict Free R-Tree (CFRT). Each node of the R-Tree is represented by a BloomCRDT. Concurrent tree modifications are allowed and, due to the characteristics of the R-tree, this does not result in an inconsistent data structure. A periodic optimization algorithm can be used to re-normalize the R-Tree, thus maintaining efficiency.

Preface

TODO ACKNOWLEDGEMENTS Thanks to M. de Vos for getting stuck with me.

E. M. Bongers
Schijndel, April 2021

Contents

1 Introduction

TODO: Introduction here, invoking emotions and reasons to keep reading

1.1. Local-First Software

The concept of Local-First Software[29] describes the perils of modern cloud based software development. The main argument against the use of such online collaborative services is the question of data ownership. Not so much in the legal sense (it's nothing [17]) but rather in terms of the users agency, the ability of the user to excersise control over digital artifacts. When using online collaborative services the user only has a local cache copy at best, not an authorative or complete copy. This devalues the data stored locally and leads to a continued reliance on the online service and thus the continued existence of said service. If Big Tech decides to discontinue the service the contents is read-only at best or moved to /dev/null at worst. The end result being the same, the users work stalls. Local-first Software proposes to steer software development clear of this danger by striving for the ”seven ideals for local-first software”.

1. Responsiveness and Availability, by having a local authorative copy of the user's data, applications do not have to wait for round trip calls to remote resources.

2. Data mobility, by not locking data in apps or on a specific device.

3. Optional network connectivity. Applications that rely on network connections loose their usefulness if the device is disconnected. Or put another way, global network coverage should not be an assumption.

4. Seamless collaboration, by not generating (version) conflicts during collaborative work.

5. Preservation, having all software locally enables users to access digital artifacts long after development stopped.

6. Security and Privacy, by using end-to-end encryption and keeping all private data on machines owned by the users.

7. User Agency, such that no other entity can decide what users can and cannot do in their creative process.

A narrative that is not yet described in the context of Local-First Software is that it allows the open source community to stand on equal footing with BigTech, not through bulk resources but through smart software. The trends towards cloud and SaaS invokes images of data centers, global networking and massive processing power, something most open source projects could not hope to finance, deploy and maintain. A good example is LibreOffice [18], it used to compete with MS Office (with varying levels of success) but with the increasing use of Google Docs, MS Office has evolved into Office360, but

LibreOffice is in danger of missing the boat when it comes to online collaboration. Applying the Local-First ideals can move open source forwards to enable the online collaboration features and workflows that many users are becomming accustomed to.

Chapter 2 introduces CRDTs and the relevant distributed systems bits to understand them in the context of this work. Next, chapter 3 identifies the exact problems with implementing CRDTs in open peer-to-peer systems. Then chapter 4 proposes a design that resolves the previously identified problems. Chapter 5 evaluates the design in several (adverse) circumstances, and explores the limits of the design. Lastly chapter 6 summarizes the work, discusses the contributions and examines possible ideas for further research.

2 Background & State-of-the-Art

A Conflict-free Replicated Data Type (CRDT) is an appealing primitive to use in designing scalable and decentralized applications. To this end, section 2.2 introduces the distributed systems concepts necessary to understand CRDTs. Section 2.3 explains CRDTs, their formal definitions and some examples. Section 2.4 lists and discusses some basic and advanced implementations of CRDTs. Lastly, section 2.5 discusses further research done on CRDTs and related research.

2.1. Terminology

TODO: find a nice/consise way to explain the exact use of these terms in this paper. The following terms (should) have an exact meaning in this work:

- node. from graphs import node. A node as used in graph theory, and in particular nodes of a tree structure.
- merge. The act of combining two tree nodes. Note that this is distinct from join, which deals with CRDT states.
- split. The act of splitting a tree node into two nodes.
- entry. A key-value pair stored in a node. In the case of interior nodes, the value may also be a pointer/edge to a child node.
- replica. A (local) copy of a CRDT.
- join. The function or act of combining 2 CvRDT (state based CRDT) replicas, leading to a new replica that contains all information in both input states. Or in other words, the resultant replica must order greater than both input states on the CvRDT's join-semilattice.

2.2. Distributed Systems and Consistency

In order to sufficiently comprehend the mechanisms and workings of CRDTs, some understanding of elementary Distributed Systems concepts is needed. Readers already versed in the core concepts of the CAP theorem and strong eventual consistency could skip ahead to 2.3.

2.2.1. Central Components and Decentralized systems

Some distributed systems rely on centralized components (such as central servers) to perform some critical function. These centralized components are a weak point for a distributed system. An example would be HTTP web pages. Through links it is an interconnected distributed system, it uses web servers as a central component and such servers are not able to perform their critical function when they are offline or disconnected.

The use of central components is often a design compromise. It's relatively straightforward to design a system with a central authorative truth or process executing in a controlled and trusted environment.

This can simplify designs greatly. A good example of this is MMO gaming, a distributed system where the actual game simulation happens in a controlled environment on a central server cluster. However distributed gaming without a central component has yet to be perfected, so for some distributed system designs a central component might be the only practical choice.

Centralized components have three aspects that make them fundamentally undesirable in a distributed system. Firstly from a technical point of view central components are a single point of failure, so the distributed system cannot function (fully) if a centralized component is unreachable or unresponsive. A second weakness is organisational: each instance of a central component is inevitably controlled by a single entity and, any entity is influencable by private and government action. Finally, central components also have a financial aspect: there is a real world cost associated with central components and the distributed system's user community has to provide for this in some way.

Distributed systems that, by design, contain no central components are called decentralized systems. In decentralized systems there is no central component to wield authority and make decisions. This makes all nodes in the distributed system equal peers and forming what is known as a peer-to-peer (P2P) system. In contrast to distributed systems containing a central component, peer-to-peer systems are already close to the Local-first Software ideals. At least they could be easily adapted to work that way. By necessity, a peer in a peer-to-peer system will be programmed with a local based world view in mind. Since a peer cannot depend on a central authority, it has to be its own authority, leading to a local world view.

2.2.2. The CAP theorem

Within distributed systems there is a well known theorem that bounds the capabilities of any distributed system: the CAP theorem[19]. The CAP theorem asserts that a distributed system cannot achieve *Consistency*, *Availability* and *Partition tolerance* at the same time. In this case Consistency should be taken to mean that the system should behave as if it where a single database [1]. Availability means that (user) operations don't block or wait until certain conditions are met, so the system should always be available for work. Partition tolerance means systems can recover from network interruptions. It is possible for a distributed system to achieve two of these conditions at any time, but not all three. An example of a system that gives up Consistency is the Domain Name System (DNS). DNS uses timers and caches when distributing data and thus achieves Availability and Partition tolerance. However it is possible that different results are returned to users depending on copies in intermediate caches. So the DNS system does not have strong consistency. Giving up Availability is exemplified by distributed locking, making some shared resources unavailable to prevent a lapse in Consistency. In addition, forfeiting Partition tolerance is a trade-off made by classical databases: they cannot recover from a loss of communication and, if networked, will typically be restricted to redundant locally networked clusters.

The author in [47] make some interesting observations about distributed systems that operate at internet scale. First off it is impossible to prevent partitions: links will need maintenance, smartphones will roam out of coverage, WiFi will be congested in urban areas, etc. So the system must absolutely be able to recover from partitions. Second, users nowadays have come to expect that systems are always Available, implying that Consistency should always be forfeited. While the argument for selecting Partition tolerance is certainly compelling, the argument for selecting either Consistency or Availability depends more on the application. Banks might well prefer Consistency over Availability in order to be safe from financial risks that could occur due to inconsistencies.

2.2.3. Strong Eventual Consistency

The CAP Theorem deals with the concept of *strong consistency*, where all identical queries to the distributed system as a whole return the same result. This can be thought of as all nodes in the system having an equivalent state. However if, in the context of the CAP Theorem, strong consistency is forfeited it can be replaced with a lesser consistency. Usually this takes the form of *eventual consistency* [47]: a delay is accepted that allows the consistency to propagate through the distributed system. If all updates or modifications to the distributed system were stopped, then after some time every node of the distributed system should respond with the same result. The DNS system is a good example. After updates have stopped, and all caches expire, each identical request should result in the same

[1]An ACID [23] database that is.

records being returned. Note however that this is not because all nodes of the DNS system have reached an equivalent internal state. Unfortunately the definition of eventual consistency is somewhat imprecise and varies among authors. There are many more classes of consistency, interested readers are referred to [47].

A variation on eventual consistency is that of *strong eventual consistency* [42]. This definition starts with a weak form of eventual consistency where all nodes in a distributed system are informed about all updates, eventually. In addition to that the Strong aspect requires that all nodes apply all updates in such a way that the final internal state of the nodes is equivalent. Note that this is indeed stronger than the previous definition of eventual consistency since that placed no restrictions on the internal state of the nodes.

2.3. Conflict-free Replicated Data Types

In attempts to mitigate the results of selecting both Availability and Partition tolerance from the CAP theorem some solutions have been proposed. A recent paradigm is that of Conflict-free Replicated Data Type (CRDT) as described in [42]. The idea of a CRDT is to achieve strong eventual consistency of a replicated data structure by structuring data and updates in such a way that no conflicts can arise when combining different versions. This then allows automatic merging of different versions or updates of the data structure. In terms of the CAP theorem this provides Availability and Partition tolerance, but with a strong formal guarantee that strong consistency will be reached eventually.

Proponents of Local-First Software envision a central role for Conflict-free Replicated Data Types (CRDTs) in applications that strive for the Local-First ideals. This is because CRDTs naturally embody the ideals of Local-First Software. It allows users to directly collaborate, without a centralized infrastructure and where any conflict introduced by concurrent updates of users can always be resolved later. To aid adoption of the Local-First ideals several projects have developed CRDT implementations for browser environments [28][21][30].

The description of CRDT in [42] provides two formal models for reasoning about CRDTs: the state-based Convergent Replicated Data Type (CvRDT) and Op-based Commutative Replicated Data Type (CmRDT). The two models are equivalent in expressive power but CvRDTs are more convenient for mathematical reasoning and CmRDTs are easier to implement. The following subsections present the two CRDT types in more detail.

2.3.1. State-based CRDT

A Convergent Replicated Data Type (CvRDT), often called a State-based CRDT, exchanges the full CRDT state between replicas. The CvRDT model is based on a join-semilattice, in this case it means a partial ordering on the set of all states held by all replicas, and a function (known as 'join' or 'least upper bound') for pairs in the set. The join function produces a state that orders strictly greater than its two inputs and is commutative, idempotent and associative. This implies that as long as states can be ordered they can always be joined, and the result monotonically proceeds up the partial ordering chain. It must therefore converge to a maximal element in the partial ordering, one where all initial states have been joined, and because of the state equivalence relation of strong eventual consistency any maximal element will do. Although in practice the maximal element could well be singular, the greatest element of the partial ordering. The CvRDT definition of a CRDT permits testing of a data type to determine if it is a CvRDT, but it leaves a lot to the imagination when it comes to designing and implementing one.

To construct an implementation of a CvRDT, nodes send out a copy of their local replica state to other nodes. These other nodes then check the partial ordering and join states that contain "new" information. An example of a CvRDT is shown in Figure 2.1. This figure shows the operation of a CvRDT that contains a grow-only set. There are three replica nodes each with an initial empty state. Nodes then send their state to other nodes after an update, in this example nodes take the union of their local state and a received state. In this way all nodes have reached strong eventual consistency as defined by the CvRDT definition.

Note that "taking an union" is a good example of why CvRDTs allow easy mathematical reasoning, but are not necessarily easy to implement. This requires comparing of two states to determine any differences and subsequently integrating those differences. Also, sending the entire state can be very inefficient: if the CvRDT state is large many bytes might be sent needlessly. These problems gave rise to a variation of the CvRDT, the δ-CRDT [1], where only a difference of state is communicated between

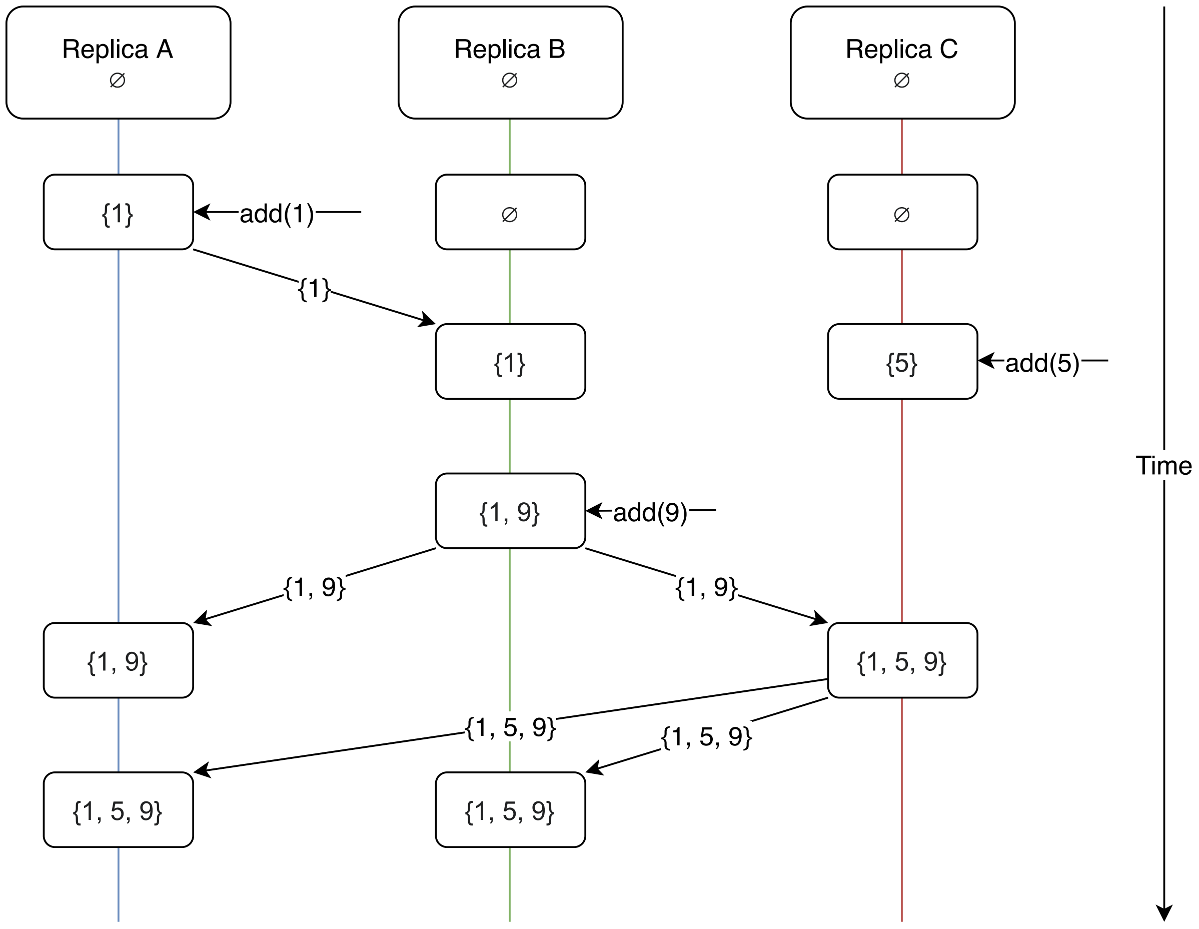


Figure 2.1: CvRDT operation. Replicas send full state messages to each other

replicas. An example of a δ-CRDT is TrustChain [35]. In this case a nodes state is the database of blocks that it knows about and new blocks are deltas on this state. Each node only needs to merge blocks it does not yet know about to advance its state towards a complete global state, so there is an ordering on states. Blocks can always be merged into the database independent of others, so the state of two nodes can always be merged. If all blocks where to be communicated to all other nodes, each node would end up with an equivalent (global) state.

2.3.2. Operation-based CRDT

The Operation-based style of CRDT known as a Commutative Replicated Data Type (CmRDT) is based around communicating the operations performed on the CRDT and having each replica apply these operations on their local state. The CmRDT model assumes a reliable causally ordered broadcast communication protocol and uses that to deliver a sequence of operations to all replicas. To each operation is bound a side-effect free precondition test that determines if an operation may be applied to the CmRDT's state. If at a replica two operations are pending, i.e. their preconditions are satisfied, applying either operation may not invalidate the preconditions of the other. This allows operations to be applied in any order once their preconditions are met, or put another way: all concurrent operations must be commutative. This ensures that operations can always be applied eventually, and thus lead to an equivalent state at each replica. An example of a CmRDT is shown in Figure 2.2. This is similar to the CvRDT example but instead of sending a full state to other nodes, the operations are sent. Each node then applies the operations on its local state. This is also distinct from a δ-CRDT which transports the change in state, that might not reflect the operation that was applied.

The CmRDT model is more suited to actual implementations since operations (e.g.: add, update, remove) on data occur naturally in programming and thus allow an easy transfer of theoretical concept

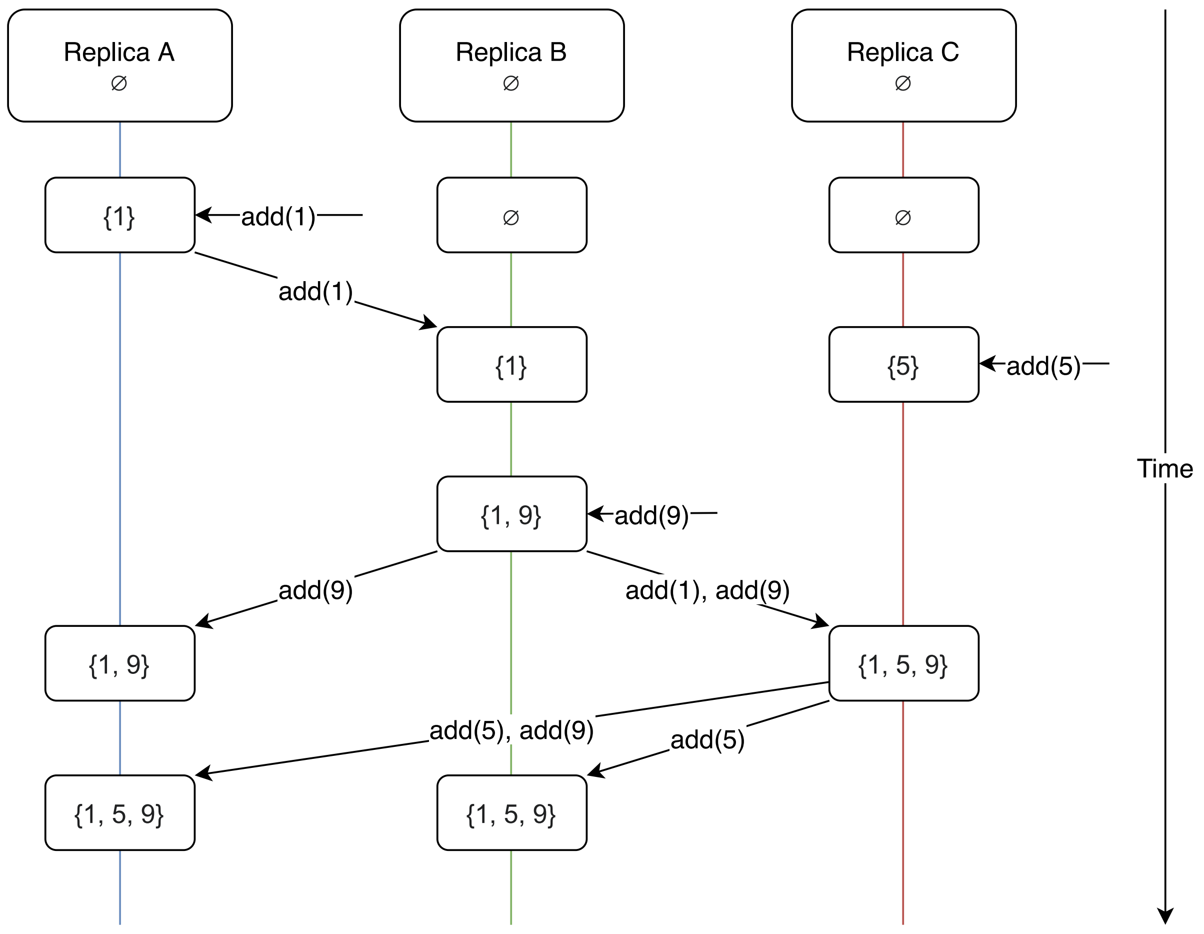


Figure 2.2: CmRDT operation. Replicas send update operations to each other

to practical implementation. This is a sharp contrast with the CvRDT model where the whole state of a replica arrives at another node, which then has to be ”merged”.

2.4. Known CRDTs and implementation designs

CRDTs have attracted interest from both the academic community and from software projects. Described in section 2.4.1 are some of the academic designs, and similarly section 2.4.2 presents the work done on applying CRDTs to practical situations.

2.4.1. Basic CRDT Types

There are several known data structures that meet the definition of a CRDT: such as a vector clock, monotonic counters and add-only sets. Compositions of these basic data structures develops more advanced data structures. For example using two monotonic counters P and N it is possible to create a non-monotonic counter by computing P - N. But more complex compositions allow for sets, dictionaries and directed graphs. Some of the basic CRDT data types are summarized in table 2.1.

In table 2.1 there are 4 common design elements that are used frequently when constructing CRDT data types. Firstly, there are monotonic operations, where the data only has one direction, such as the G-Counter, G-Set and monotonic DAG. Data only goes towards infinity and is never ”decreased”. This sidesteps combinations of concurrent add and remove operations that are not commutative. Secondly there is a frequent use of tombstones, special markers that indicate something used to be there but should be considered gone. This is frequently used in sets and ordered lists where concurrent updates could conflict. For example a `remove` and `addAfter` operation on the same element would conflict if the `remove` is processed first since the `addAfter` would then reference an invalid item. However

Name	Description	Ref.
G-Counter	A monotonic increasing counter.	[43]
PN-Counter	Two G-Counters, one for positive, one for negative. The value of the PN counter is calculated as P - N.	[43]
G-Set	A grow-only set.	[43]
2P-Set	Two phase set, akin to the PN-counter this set uses two G-Sets, one for items added and one for items removed (the tombstone set). The contents of the 2P-Set are the elements in the add set that are not also in the tombstone set	[43]
U-Set	A set that only adds and removes unique items. Combined with causally ordered messaging, this is enough to ensure no conflicts can arise.	[43]
LWW-element-Set	A set where the Last Write Wins in case of conflict.	[43]
PN-Set	A set where each element is paired with a PN-counter. Adding increments the counter, removing decreases the counter. An element is in the set if it associated counter value is greater than 0.	[43]
Observed-Remove Set	A set where elements are paired with a unique identifier. Before a remove can be issued the identifier needs to be observed, thus no concurrent add-remove conflicts can happen.	[43]
2P2P-Graph	A graph made up of vertices in a 2P set, and edges in another 2P set.	[43]
Add-only Monotonic DAG	An add-only graph that uses a simple edge direction following rule such that a DAG is formed.	[43]
Add-Remove Partial Order	A DAG graph that uses a 2P-Set with tombstones for vertices and a G-Set for edges. This combination ensures a new edge can always be found between two vertices if an intermediate vertex is removed.	[43]
Replicated Growable Array (RGA)	List based on linked list. Allows updates to the elements in the list. Clocks that allow tombstone garbage collection.	[39] [43]
WOOT	List with unique identified elements. Tombstone set to filter out deletes.	[34]
Logoot	List with unique identified elements, insertion by generating identifier between two others. No tombstones but potentially unbounded identifier length. Claims that practical use sees no such unboundedness.	[48]
TreeDoc	List/document based on prefix Trie for element identifiers. Trie may become unbalanced and requires rebalancing, which uses 2-phase commit involving all replica's. Uses tombstones.	[37]

Table 2.1: An overview of basic CRDT types

if the `remove` operation leaves a tombstone in the place of the element, the `addAfter` can still be processed. A third commonality is pairing list or set elements with unique or random identifiers. This can be an alternative to the use of tombstones in some cases. The principle is that removes must be causally ordered with respect to adds, since the paired identifier has to be observed first, before the remove of that particular element can be issued (see OR-set). Conversely a concurrent add of the same element produces two elements in the CRDT, each paired with an unique identifier. The fourth and last common design element is the use of clocks to mitigate the unbounded growth of tombstones in CRDTs. Often a vector clock is used since that allows a node to deduce if all other nodes have seen a particular tombstone and if so, discard such a tombstone from the CRDT.

2.4.2. Applied CRDTs

In addition to research on basic CRDT structures, some software projects have also applied CRDTs to real world situations. Table 2.2 lists some of them. Two types of application are typical among the applied uses of CRDTs. One is no-sql or key-value store databases, and the second is collaborative editing.

Name	Description	Ref.
Redis	Redis is an in-memory key-value store that can use CRDTs to implement multi master replication.	[38] [49]
Riak	Eventual consistent key-value store based on CRDTs.	[4] [12]
Roshi	SoundCloud uses Roshi, a CRDT that uses LWW-set in combination with garbage collection. Based in part on Redis.	[44]
Akka	Actor based programming language which uses CRDTs for its replicated data types.	[24]
Scalable XML Collaborative Editing with Undo	Applies CRDTs to XML document editing in a collaborative setting. Garbage collection using vector clocks.	[30]
Conflict-Free Replicated Relations for Multi-Synchronous Database Management at Edge	Applies CRDT to traditional RDBMs'.	[50]
Designing a Planetary-Scale IMAP Service with Conflict-free Replicated Data Types	Replicated maildirs over IMAP using CRDT to sync geo replicas. Uses a CRDT map/dictionary internally.	[25]
A Study of CRDTs that do Computations	CRDTs that result in a computation being performed in the CRDT state. Comparable to the join step in parallel processing.	[33]

Table 2.2: An overview of CRDT applied uses

2.5. Other work related to CRDTs

There are many interesting works that further explore CRDTs, their limits and ways in which they could be improved. Such as [6] which aims to "Constraining the Eventual in Eventual Consistency". In order to achieve this leases are added to CRDTs such that operations can timeout and be canceled. This provides the same consistency but with a bound on the "eventual" part of the consistency, at the cost that some operations might eventually produce an error. There are also dead ends in CRDT research as explained in [27]. This work examines a common problem in collaborative editors, moving a range of characters. This operation turns out to be particularly difficult to capture in CRDTs since some combinations of concurrent operations are non commutative. In [5] the authors examine the problem of ever growing metadata in CRDTs, particularly unbounded growth of tombstones and operation histories.

The field of CRDTs is closely related to the much older field of Operational Transformation (OT). In OT as proposed by [16], concurrent operations on a replica are serialized to a predictable order, and then applied to the state sequentially. After an operation is executed, all pending operations are adjusted to ensure they reflect an operation against the current state. In other words, the pending operations are transformed to work against an updated version of the state. The difficult part in the OT scheme is deciding the order of operations. Popular products that use OT such as Google Docs use a centralized component, a server in this case, to decide the order of operations. This ensures that all clients reach the same state. When the internal state is designed properly it is even possible to create a hybrid OT/CRDT system [31], thus allowing choice about the mechanism to use.

Also closely related to CRDTs are Cloud Types [13]. These are based on a similar idea of always allowing divergent versions of a value to be be recombined without much conflict. In the case a conflict does arise Cloud Types defer to a centralized replica (in the cloud somewhere) to handle arbitration.

In contrast to CRDTs the use of a centralized component allows Cloud Types to use non-commutative concurrent operations. On the other hand this means that the replicas held outside of this master copy are not authorative. Cloud Types also use a variation of a vector clock to ensure values cannot be recombined to older versions of themselves. In short, Cloud Types are aimed at enterprise scenarios with centralized components, which is exactly what Local First Software tries to avoid.

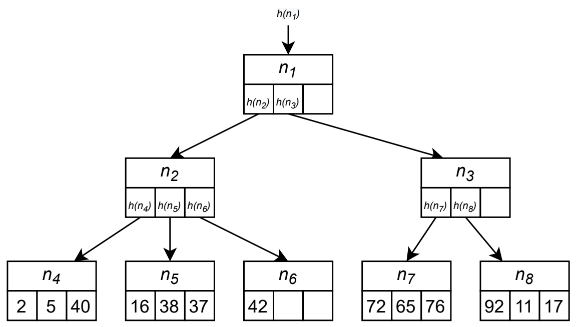
Another slightly more generic branch of research that is closely related is research on anti-entropy algorithms. The main idea here is to use an algorithm to synchronize peers, and thus reduce entropy in the distributed system. Conflicts are resolved using Last-Writer-Wins rules or are left unresolved as multiple versions of a value. The DottedDB[20] is a distributed key-value database that uses a double clock mechanism to garbage collect causality information that is no longer usefull. To avoid unbounded grow DottedDB introduces a watermark set, a method for detecting what other peers know and discarding information is accepted by a quorem. This scheme is tolerant of peer churn, but use of a quorem indicates the peers are a limited set that are well connected.

Lastly there are CRDT designs Vegvisir[26] and Merkle-CRDT[40] which apply to low power IoT devices and IPFS respectively. Both build on the idea of using a Directed Acyclic Graph (DAG). Each operation on a CRDT is represented by a node in the DAG, and each such node has an edge directed at one or more previous operation-nodes. The edges of the DAG thus encode the causal relation of the updates and are sufficient to allow CRDTs to work on this.

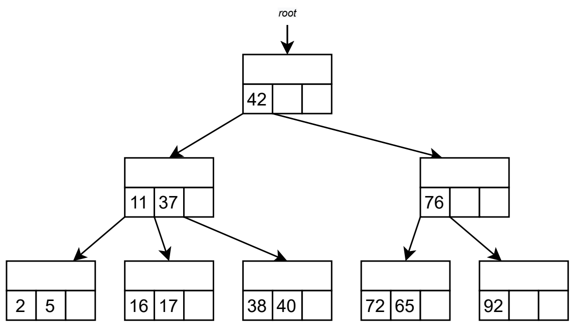
2.6. CRDTs embedded in Merkle Trees

Merkle Trees are data structures that can be used to efficiently find differences in large sets of elements. The idea begins with hashing each element, then a certain number of these hashes are concatenated and hashed again forming the leaves of a tree structure where each level towards the root repeats the concatenation and hashing (as depicted in figure 2.3a). The root of a Merkle Tree is a single hash that identifies this particular tree. With this scheme any change in an element produces a different hash for all its parents, up to and including the root. This way a hash can compare subtrees or indeed a whole tree with one message exchange. Many practical applications use Merkle Trees including ZFS, bitcoin, git and Riak to name a few. While a Merkle Trees can detect detect differences in large element sets, it does not provide direction over a key space, a Merkle Tree is not an indexing structure.

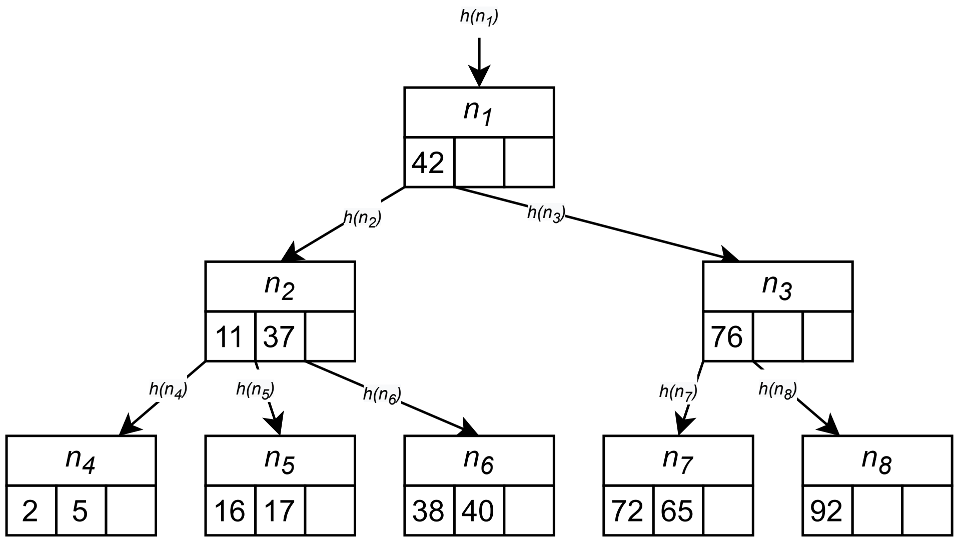
The Merkle Search Tree (MST, [3]) aims to unify Merkle Trees and indexing structures. Since a Merkle Tree is indifferent to exactly how the children of each node are selected, the MST can exploit this freedom and proposes to compose elements into a tree structure similar to a B-Tree[8] (see figure 2.3b). It starts by ordering all elements and hashing them. Each element is then assigned a level (height or distance from leaf layer) based on the number of leading zeros in its hash. This height combined with the ordering results in a tree structure, to which the Merkle Tree concept can be applied (see figure 2.3c). Replicas of the MST can be compared using the techniques developed for Merkle Trees, in this case gossiping of the current root hash and subsequent rounds of obtaining missing nodes/hashes. Of special note is that the MST is formed deterministically from the set of elements. Given the same set of elements, each replica will form an identical tree. This ensures there is no infinite varaiation on hash due to different causal paths, leading to a converging state and an eventually consistent MST. The MST proposes using key/value pairs as entries and using CRDTs in the leaf values to resolve concurrent updates, much like Riak and Redis. The paper presenting the MST hints at, but is not clear on, the mechanism of removing elements from an MST. After inquiry with the first author[2] it is clear that the MST supports deletes in a limited cappacity. Any local delete would be considered a missing value in any subsequent gossiping round and be restored. Thus the best the MST can offer is a special tombstone value in the key/value pair, making it a grow-only structure.



(a) Example of a Merkle Tree. Concatenating node contents and hashing repeatedly to create a tree. Terminating in a single root hash. Note that the leaf nodes do not need any specific ordering to construct an Merkle Tree. In practice however it is usually favourable to have an ordering since the Merkle Tree expresses a permutaion of elements and ordering reduces that to a combination of elements.



(b) A B-Tree[8] is a generalization of a binary search tree that allows more than one value in each node. Entries are added to leaf nodes that split when they become full. Because splits propagate up from the leaf layer and creates only sibblings, the tree is kept balanced. Adding a new layer at the root when needed.



(c) The Merkle Search Tree combines the Merkle Tree with the B-Tree and allows searching, but also efficient comparison of (sub)trees.

Figure 2.3: The Merkle Search Tree combines the concepts of a Merkle Tree with those of a regular indexing structure. n_x indicates a tree node and $h(n_x)$ indicates the hash of a node's contents and pointers.

3 Problem Description

The previous chapter describes the model of a CRDT, and its properties. The sections on applications show there is no significant use of CRDTs in open P2P systems. Most applications are collaborative editors for users and key/value data stores. These are controlled situations with a limited number of replicas where new replicas need permission to join and all replicas are informed about each other. This in contrast to open P2P systems and hinders the ideals of Local First software. This chapter describes the problems that arise when applying CRDTs to open P2P systems. Section 3.1 describes these problems in three subsections. Next, section 3.2 describes how these problems impact a real world application.

3.1. Limits and Problems of current CRDTs

Explained in the next sections three inherently limiting issues are explained that, at minimum, hinder a wider adoption of CRDTs in open P2P systems. This section uses Git[14] as an example since this software is very familiar to most readers and displays many characteristics of CRDTs[1].

3.1.1. Data structure Scalability

In theory, the basic definition of a CRDT does not place any hard limits on state size. However, when applying theory to practice there will always be real world considerations. Authors have found that the size of the CRDT's state was a topic for future research [5][28], as it is a fundamental limitation on CRDT applicability and adoption. In Git an example would be many or large binaries being added which leads to a large working copy. In many CRDTs a similar issue occurs with grow-only structures, and more specifically tombstones inflating the CRDTs state. In Git it suffices to delete the binaries to reduce the size of the working copy. However, the grow only nature of many CRDTs means it is not possible to simply remove tombstones without introducing conflicts. So CRDTs that rely on tombstones require other solutions to keep their state manageable. One solution is to ignore the problem if the CRDT is of an ephemeral nature. Since the current applications of CRDTs are in collaborative software, it is reasonable to assume the collaboration will cease eventually. If the CRDT's state has not grown so much that it becomes impractical then there is no problem to solve in practice.

If the CRDT is not ephemeral but of a more persistent nature, ignoring a grow-only state or the state size in general is a strategy that will fail eventually. One solution that has been suggested is a distributed garbage collection scheme to remove tombstones that no longer serve a purpose. As briefly discussed in section 2.4.1, the go-to solution is to use a vector clock and attach a timestamp to each tombstone. This allows each node to reason about the causality at other nodes. Once it can be inferred that all replicas have seen the tombstone, it can be removed safely. The problem with a vector clock is that it:

[1]In fact Git could be considered a CRDT itself. It has states that can be merged towards eventual consistency. However in such a view of Git the join function is a human that resolves conflicts. Git itself is not aimed at being conflict free, just being able to resolve the conflicts that happen. Furthermore, the use of a human to do something that pure mathematics can not is questionable. In essence the join function is an oracle machine; useful for mathematical reasoning but not very practical.

1. Requires knowledge about the existence of other replicas. When replicas are not know to each other, they are not included in the vector clocks. This prevents replicas from correct causal reasoning about operations.

2. Limits the number of replicas in practice. If many replicas join, then a vector clock becomes unwieldy and timestamps grow in size. A simple calculation shows that a standard Ethernet frame would overflow with just 188 replicas, assuming 64-bit counters being used.

3. Grows as replicas come and go. Since it is impossible to tell what replicas are partitioned from the network and what replicas have left the network, the only safe option is to assume a partition has happened and thus save all required metadata (see also section 3.1.2).

To demonstrate how infeasible the standard vector clock is in an open P2P setting consider trying to impose a vector clock on the Linux kernel git repository. The first step would be to discover, world wide, all replicas of the Linux kernel repository. If there are many, then the clock state will become large and require special consideration in software. If all replicas have been found, the clock must keep track of the clock states of all replicas, even of replicas that have been removed. Since removal of replicas is indistinguishable from a network partitioning and there is no limit on the duration of the network partitioning, their clock states should be held indefinitely. As such a vector clock on the Linux kernel Git repository would become a grow-only data structure. Recurs to section 3.1.1 to read about the problems of grow-only data structures.

The problems the vector clock encounters in the above scenario stem from the openness of Git: anyone can come along and decide to create a replica without being forced to register this replica. Previous works on CRDTs have either been theoretical, or have applied CRDTs to controlled settings where the participants in the distributed system are known, reasonably limited in number and no churn occurs. This means that CRDTs have not found their way into P2P distributed systems, even though CRDTs would seem to be a natural fit for P2P networks at first glance (some notable exceptions being Vegvisir[26] and Merkle-CRDT[40]).

3.1.2. Metadata Scalability

CRDTs consist of two types of data: application state is used by the application built on top of the CRDT, and metadata is needed to make the CRDT function but has no direct value to the using application. Git also clearly shows this distinction: the working copy can be considered an application state while the commit history is metadata kept by Git to make the users workflow possible. Just as unbounded growth of the application state is problematic, so is unbounded growth of a CRDT's metadata. Op-based CRDT implementations are a prime example of this unbounded metadata growth because they require that all operation messages contain causality information. This is not a problem by itself but usually this causality information is only informative in the context of the complete history of operations on the op-based CRDT. In other words, operations refer to the operations that came before them. Also, keeping the full history of a CRDT ensures that a replica recovering from a network partition can be supplied with the operations it still has to perform.

At first glance state-based CRDTs should not have the issue of grow-only metadata, since state-based CRDTs do not require a causal ordering. However, implementations may require this anyway in order to determine the join of two states. The state-based CRDT needs to determine what state is newer, or has components that have not yet been observed. In terms of Git an example would be a merge (or rebase), that requires a common ancestor state to determine how two states have diverged. Changes since such a common ancestor state can then be compared and a joined state emerges. The definition of the state-based CRDT is much looser than this Git example would suggest. Nonetheless, it highlights that even state-based CRDTs can be reliant on persisting metadata of previous states.

3.1.3. Forced Convergence Assumption

A fundamental assumption of many CRDT implementations is that they aim for an active full state synchronization among peers. For the small and closed communities that most CRDTs target this works well. However, there is no requirement in the definition of CRDTs that replicas are *forced* to converge. Put another way, the delay after which eventual consistency is reached could be at infinity. CRDTs where initially designed for applications with the assumption that it is desirable to actively converge all replicas to a globally equivalent state. This is what many projects have implemented and indeed this has so far proven a good fit for collaborative editor applications and key-value databases. However, this *forced convergence assumption* has some disadvantages. Most importantly, a replica might not need the complete data structure in order to function. This is a key design insight that allows for example ConTrib[15] [2] to function efficiently. ConTrib expressly avoids globally distributing block chains. Secondly, CRDT applications that aim for an actively converged state assume that all replicas are known. This allows CRDT implementations to push new states or operations to other replicas, forcing them to converge towards a globally equivalent state. However, as with vector clocks in section 3.1.1, keeping track of all replicas is not trivial. Lastly, the active convergence assumption does not consider replicas on heterogeneous hardware. If a receiving replica is low-powered, then new updates that are pushed towards this replica force it to use a lot of power to keep up. That is, if the low-powered device is capable of keeping up at all. Especially in the case where only a small fraction of the whole data structure is needed, the burden on this replica might be disproportional compared to the benefit for the user.

3.2. Applying CRDTs to open P2P systems

The previous section describes several problems that CRDTs must overcome to successfully work in open P2P systems. This section describes a specific practical application that would benefit from a CRDT that is capable of working in such an open P2P system. This practical application concerns the Channels feature of Tribler. Tribler[36] is a BitTorrent client created by the Distributed Systems department of the TU Delft. Tribler has been downloaded by over 1.8 million users. For over a decade Tribler has enabled researchers to investigate a variety of P2P topics including distributed content indexing and searching, video streaming and anonymous downloading. What makes Tribler unique as a research vehicle is the fact that it is used by thousands of real world end users. New features get put to the test in the real world, not just academic benchmarks.

One of the distinguishing features of Tribler is the ability for users to create and manage their own channels with content. Users can subscribe to channels and after doing so Tribler will start collecting all torrents in the subscribed channel. Figure 3.1 shows a client with channel subscriptions. Each torrent consists of a.o. a name, magnet link, tracker info, thumbnail(s) and other metadata. In the current deployment only the user that created a channel can add or remove content torrents. This situation seems perfectly suited for a CRDT. In essence Tribler's Channels are collaborative adds and removes on a set. However, Tribler is an open P2P system and as such naive CRDT implementations would suffer from the problems discussed in section 3.1.

Since the release of Tribler's Channels feature some channels have grown beyond a million torrents. Assuming 2.1KB per torrent, an estimate for the application state size of a CRDT supporting such a channel would be over 2.1GB. On top of that any add/remove would leave a tombstone, and since the channels are more permanent than ephemeral, the tombstones would eventually dominate the CRDT. It could be argued that the size of the CRDT would become impractically large to handle as a single CRDT. In addition to that, such a huge CRDT would need to be either an op-based or a δ-CRDT, since sending the full state between replicas for each update is not a sustainable approach in terms of network usage. Both models of CRDT keep a full history of operations or deltas, causing a further metadata scalability problem over time. Lastly, it is hard to imagine that any user would have a need for every single torrent in such a channel. Downloading all the torrents and consuming all content would be infeasible. Therefore, its reasonable to assume each user is only interested in a small subset of the full channel. Thus if such a large channel would be implemented as a CRDT, the forced convergence assumption would not only force nodes to know and track all the other nodes, but would also force users to acquire the full application state and metadata of the CRDT and to keep up with any changes. This is a heavy burden for a user that is only interested in a small subset of the full state. Since all of the problems described in section 3.1 apply, it is clear that applying a CRDT to the Tribler open P2P network is not trivial and warrants further research.

[2] The ConTrib structure can be interpreted as a δ-CRDT, on the condition that all nodes behave correctly. The state of a replica in this case is the local database of blocks that a replica holds. There is only one update function: `addBlock`. Which translates to a δ-state message containing the new block. This message is distributed to other replicas and merged into the state (persisted in the database). The merge operation always produces a consistent new state that is further along the path to global convergence. Thus it has all the attributes of a state-based CRDT and is an example of a CRDT that does not force convergence of replicas.

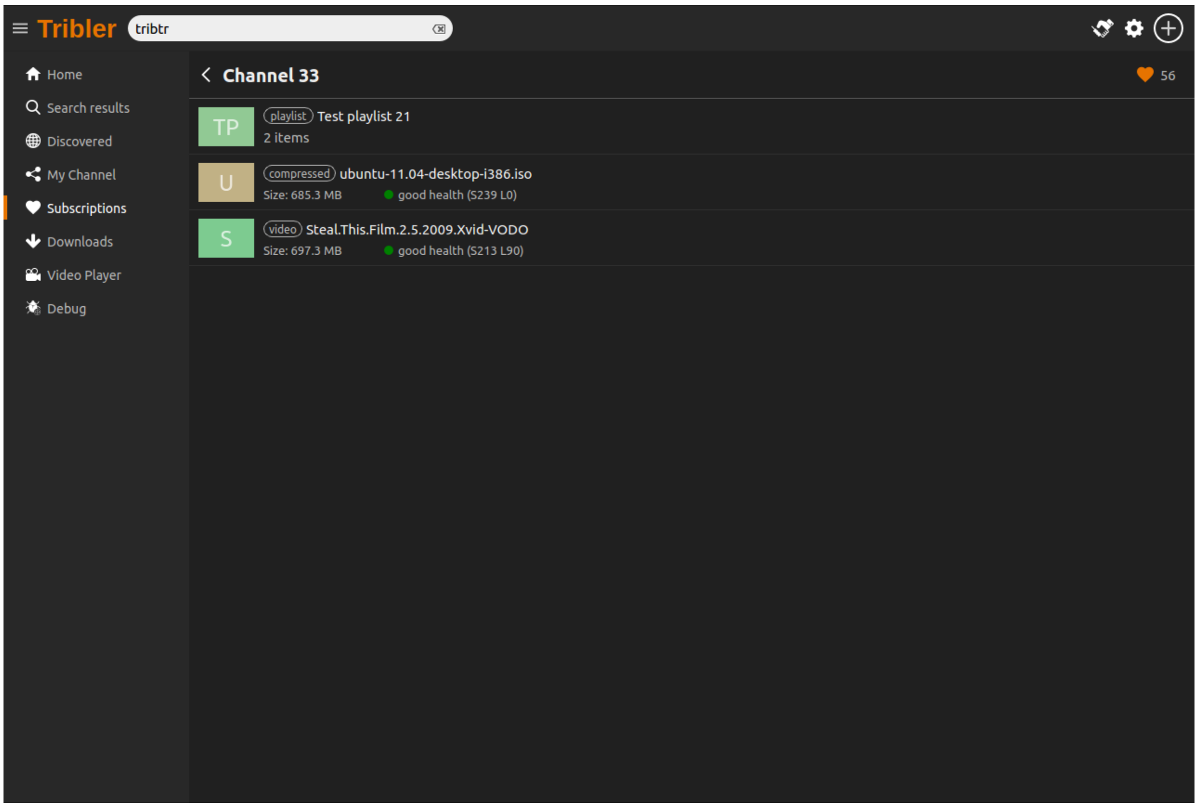


Figure 3.1: Tribler client subscribed to 3 channels

3.3. Requirements of solution

TODO: fill

- Must keep crdt property of commutative updates (or state merges).
- Must not show unbounded growth in CRDT or history (in practice. Logoot shows theoretical unbounded growth does not need to materialize)
- Must pull updates, not push them to avoid overwhelming nodes. (Must converge to full crdt state when nodes decide to pull everything) (push can be allowed as optimization, but nodes must be allowed to discard and pull again when they are ready)
- Must not require full knowlege of all replicas
- Must not require impractical clock protocol.

4 The Conflict Free R-Tree

Two concepts are presented that overcome the problems described in the previous chapter. First, BloomCRDT an innovation on the standard OR-set that vastly reduces the unbounded growth and operates in an open P2P environment. Second, the Conflict Free R-Tree is a composition of BloomCRDTs into an index data structure that scales far beyond what a single BloomCRDT could practically contain.

4.1. System model and assumptions

The previous chapter describes the problems faced by CRDTs in open P2P systems. These problems are modeled by the following assumptions:

- Message delivery is imperfect, messages may arrive out of order or not at all.
- Peers and network links may fail. This could even lead to network partitions.
- Over time, large numbers of peers may join and leave the network and interact with a BloomCRDT or CFRT instance.

Moreover, there are several basic communication features on which BloomCRDT and CFRT are based. It is assumed that there exists a communication library such that:

- Messages are checked for tampering using cryptographic signatures. For example a system where a peer's identity is a public key which is used uses to verify messages.
- Groups can be resolved from global identifiers. For CFRT it is necessary to form identifiable groups of peers that exchange messages. Knowing the ID of a group should allow a new peer to join that group.
- Each peer can message a (small) subset of other peers in joined groups.

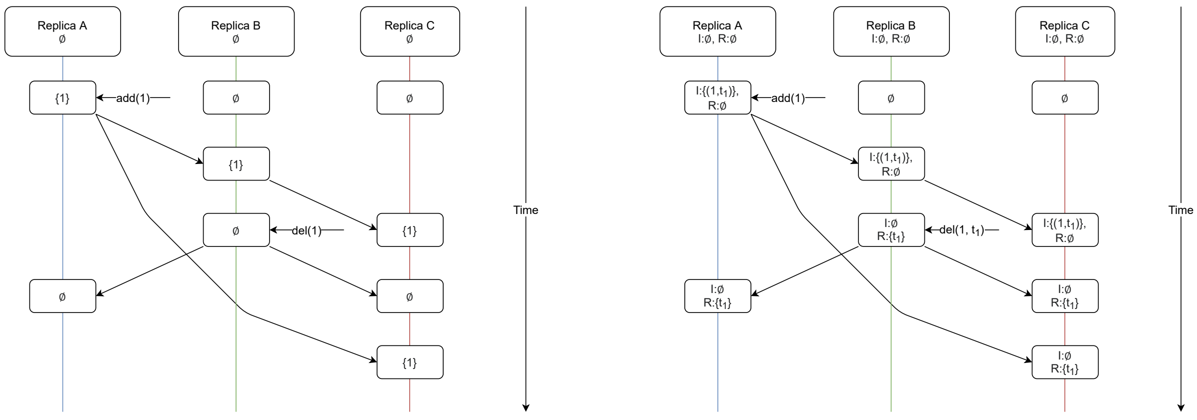
4.2. BloomCRDT

Presented here is BloomCRDT, a novel CRDT that provides set semantics in an open P2P environment. BloomCRDT is a state-based CRDT that behaves like a set and is based on the traditional OR-set[43]. BloomCRDT does not rely on causal message delivery nor does it need knowledge of other peers or their state, and finally does not show unbounded growth over time in practice. There appears to be no CRDT described in related work that matches these characteristics.

4.2.1. The Observed-Remove Set

BloomCRDT is a modification of the Observed-Remove set (OR-set), which should be explained before presenting the modified version. An OR-set tags each added element with a random number or *tag*, this effectively makes each element unique and unpredictable. Internally, the OR-set consists of a grow-only set of inserted elements (denoted I) and a grow-only set of removed elements (R). The contents of the OR-set from the user's perspective is the relative complement $I \setminus R$. New elements (e)

are paired with a tag (t) and added to I. To remove an element, the (e, t) pair is added to R. Since t is unpredictable a replica must first *observe* a (e, t) pair in I before the pair can be added to R to remove it from the OR-set, hence the name Observed-Remove set. The required observation step ensures any remove must causally follow the add.



(a) Traditional set demonstrating the overtaking delete problem.

(b) OR-set solution to the overtaking delete problem

Figure 4.1: The overtaking delete problem

To emphasize the problem that the OR-set solves, consider the $add()$ and $delete()$ operations on a traditional set. In the case of state-based CRDTs that have non-causal message delivery, messages can get reordered as demonstrated in figure 4.1. So the message that signals the deleted state might arrive before the message that carries the added state. This is known as the overtaking delete problem [32]. In this case Replica C observes the delete of an item before it observes the add. As shown in sub figure 4.1a a traditional set is not guaranteed to be eventually consistent. Sub figure 4.1b shows how the OR-set solves the problem of an overtaking delete. In the last step at Replica C, the R set reveals that the element was already deleted.

Some research has been done to improve upon the OR-set, mostly to address its grow-only nature. The Optimized Conflict-free Replicated Set[9] proposes the OptORSet, a modified OR-set that includes information about the state of other peers. This allows reasoning about what state has been propagated globally and thus what state can be discarded. To do this the OptORSet uses causal message delivery and keeps a per peer state. These concepts are not suited to an open P2P environment. The Optimized OR-set Without Ordering Constraints[32] improves on this because it does not require causal message ordering and makes a compelling argument for its interval version vectors. However, this solution also uses per peer state. In the open P2P environment this will show unbounded growth over time.

4.2.2. Structure of the BloomCRDT

In an OR-set, I does not actually need to be a grow-only set. Once a (e, t) pair is in R it can be removed from I. The relative complement $I \setminus R$ will still compute the OR-set contents. However there is no obvious way to remove the grow-only aspect of R. In a more theoretical sense, R provides information for the join function such that the result state is ordered greater on the join-semilattice than either input state. Or viewed another way, R encodes the history of an OR-set such that a join function can move forward and will not regress. In R it is especially the unique tags t that are of interest, since those are what make the elements unique and force the observed relationship. If only there where a space efficient method to encode set membership of many elements without having to persist the members.

Luckily Bloom filters[10] can encode set membership without having to persist the members and are a suitable replacement of the R set in an OR-set. Bloom filters start as a list of bits, each set to 0. When an element is added to the Bloom filter a number of hashes is computed on the added element, and each hash function produces an index in the Bloom filter's bits. At the calculated indices the bit is set to 1. To test if an element is in the Bloom filter a queried element is hashed with the hash functions and if the bit at any of the computed indices is zero, then the queried element is not in the Bloom filter. Else it most likely in the Bloom filter, however it could also be a false positive if other elements flipped the relevant bits. The idea is to choose the number of bits and number of hash functions in such a way that the probability of a false-positive is small enough for the application.

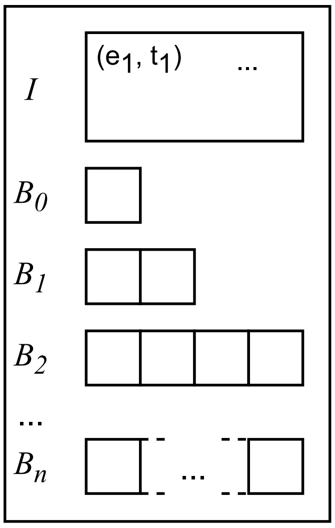


Figure 4.2: Structure of a BloomCRDT. Each entry is tagged with a unique tag. One or more Bloom filters of increasing size are added as needed.

Using a Bloom filter, it is possible to change the OR-set into a BloomCRDT. To do this R is replaced with a Bloom filter, and coupled with the previous observation that I need not be grow-only, the BloomCRDT is constructed (see figure 4.2). While this is the basic premise of the BloomCRDT, there are some details that require further explanation.

- **Bloom Filter alternatives** Instead of a Bloom filter there are more recent alternatives to consider such as Cuckoo filters and XOR filters. However there are specific conditions that the R replacement must handle. For example, Cuckoo filters require storing fingerprints of elements in the set which would not amount to a net savings since the t tags are similar to the required fingerprints. The R replacement should result use less space compared to the size of R as a grow-only set. The more recent XOR filters are built from all elements prior to filtering and are not updatable. But R is not something that can be computed a priori, it updates over time. This covers the two most frequent reasons other techniques cannot be used. In addition to this, the R replacement must be able to join efficiently as will be explained in 4.2.3. In the end, Bloom filters are very simple and can show a net space saving over R after adding just a few entries.

- **False Positives** A major problem for Bloom filters is their probabilistic nature: there can be no false negatives but there is a chance of a false positive. When applied as a replacement of R, this means there is a chance that an element is falsely considered as removed. This is only a problem in exceptional circumstances[1]. Moreover Bloom filters allow the implementer to select the probability of false positives which should be customized to fit the risk of the application. The specific conditions required for a false positive to adversely effect the BloomCRDT coupled with the low probability of actually creating a false positive in the first place, means this event can reasonably be excluded during further discussion.

- **Parameter selection** The only further consideration for Bloom filters is that they require an a priori estimation of the number of elements that will be added in order to guarantee the probabilistic bounds. This presents a problem for long lived BloomCRDTs, the number of elements added to the Bloom filter cannot be fixed in advance. To address this, BloomCRDT actually uses a list of Bloom filters (B_i where i is the index in the list of Bloom filters). When it is estimated that a Bloom filter is nearing saturation, a larger Bloom filter is added to the list of Bloom filters. This unfortunately reintroduces an unbounded growth in the theoretical sense, albeit with a much

[1]Elements are only checked against the Removed Bloom filter during a join, and only if they are only in one of the two input states. Thus it requires a removal in one replica that creates an update in the Bloom filter that collides with a concurrent addition that is still propagating among replicas.

reduced magnitude. Section 4.3.3 discusses the composition of BloomCRDTs into a CFRT and includes a mitigation strategy for this potentially unbounded growth.

4.2.3. Joining two BloomCRDTs

The traditional OR-set has a simple algorithm for the CRDT join function, but for BloomCRDT this is slightly more complicated. Assume that a join function has two inputs named l and r, and an output state named j. The traditional OR-set calculates its new states as $I_j = I_l \cup I_r$ and $R_j = R_l \cup R_r$. The BloomCRDT join is slightly more complex and must consider joining the Bloom filters and a correct handling of the difference in I_l and I_r. The Bloom filters of j can be computed as $B_{n,j} = B_{n,l} \mid B_{n,r}$ where $B_{n,.}$ is considered as all zeros if the index is undefined. Since Bloom filters only change their bits from 0 to 1, a bit wise-or suffices to combine them. Initially $I_j = I_l \cap I_r$, and each element in $I_l \triangle I_r$ that has a tag that is not in B_j is also added to I_j. The idea is that if a (e, t) pair is in one of I_l or I_r, then it is either a new element or it was removed. Since removed elements should be present in B_j, they are omitted from I_j. The bit wise-or of two Bloom filters implies the condition that both Bloom filters are based on identical parameters. So when growing the list of Bloom filters, there should be a deterministic process to set the parameters of the next Bloom filter. This ensures that when two or more replicas concurrently add a new Bloom filter, the new Bloom filters are compatible for merging. A simple strategy could be to double the capacity compared to the last Bloom Filter.

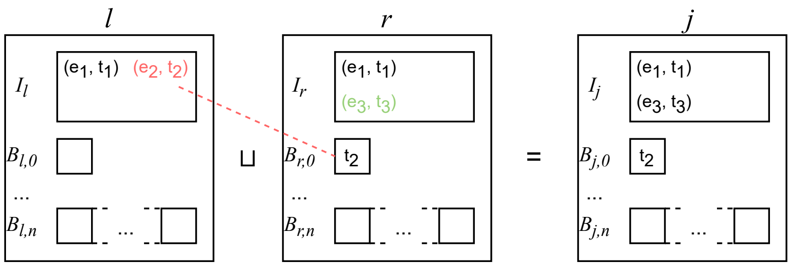


Figure 4.3: Example of joining two BloomCRDT states.

An example of a BloomCRDT join is shown in figure 4.3. Here BloomCRDTs l and r are joined and result in BloomCRDT j. The symmetric difference $I_l \triangle I_r$ reveals that (e_2, t_2) and (e_3, t_3) are not in both states and should thus be checked against the Bloom filter. In this case (e_3, t_3) is a new element since its tag is not in the Bloom filters and its pair is thus added to I_j. However t_2 is in the Bloom filter, so its pair is omitted from I_j. The resulting state j captures all information in both input states and is thus a join in the CRDT semi-lattice sense. The state j orders greater on the semi-lattice than both l and r.

With the join algorithm as described, there is a complicating factor that need to be considered when using BloomCRDT. If many deletes are being processed concurrently it is possible that a subsequent join pushes a Bloom filter over its designated capacity. Thus each BloomCRDT should have a soft limit on the number of elements in each of the Bloom filters. The soft limit should be the floor of designed capacity of the Bloom filter minus the global rate of distinct elements removed multiplied by the time to global convergence. For example, if a Bloom filter is designed to hold 20 elements, the global removal rate is 0.2Hz and the time to global convergence is 17 seconds, then the soft limit would be $\lfloor 20 - 0.2 * 17 \rfloor = 16$. When reaching this soft limit a BloomCRDT should stop adding to the Bloom filter and add a new one, with the idea that the remaining space in the Bloom filter could be filled up by deletes that are still propagating. Estimations of the variables are obviously implementation dependent, and could be set at design time or set at run-time dynamically. Moreover exceeding the designed capacity of a Bloom filter is not automatically a problem it only slightly increases the chance of a false positive.

4.3. The Conflict Free R-Tree

The Conflict Free R-Tree (or CFRT) is a novel data structure that is composed of many BloomCRDTs, allowing it to scale far beyond what a single BloomCRDT could practically contain. The BloomCRDT behaves much like a typical set with linear computational complexity on insert, lookup and remove operations. To support larger datasets a lower complexity is required, which implies ordered elements. There are several possible directions that could work. For example BloomCRDTs could be composed to form a DHT, or a skip list. However, when composing BloomCRDTs the most difficult part is correctly maintaining the links between the BloomCRDTs. A classic index tree structure provides ordering on the contained elements and a low number of links compared to other solutions.

4.3.1. The R-Tree

The R-Tree[22] is similar to the well known B-Tree, as used in Merkle Search Tree described in section 2.6, but has different characteristics. The R-Tree is an index on a key space $\mathbb{K}$ such that it can efficiently $add(k)$, $lookup(k)$ and $remove(k)$ where $k \in \mathbb{K}$. The central idea is to build a tree that labels its edges with a range on $\mathbb{K}$ that indicates the range of keys that can be found in the sub tree that the edge points to. Walking the tree from root towards leaf nodes should result in an ever narrower range. The R-Tree nodes typically contain many edges towards children, but the fan out factor is implementation dependent. Figure 4.4 shows an example of a R-Tree.

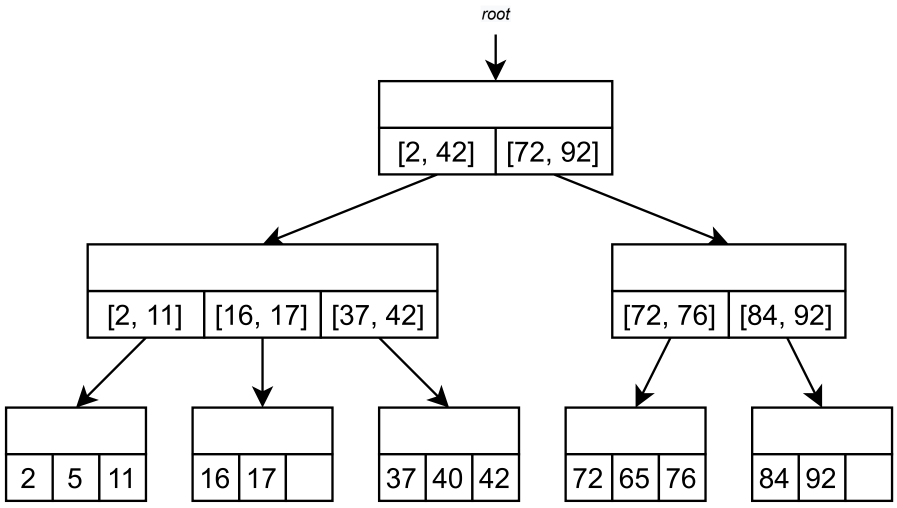


Figure 4.4: The diagram shows an example R-Tree where each edge is labeled with a range to indicate the range of keys that are contained in the child .

The R-Tree starts as a single node. Entries get added and at some point the node decides it is too full and needs to split. So there is a limit on growth after which a node will be considered full. When a node detects that it is full it splits into two nodes and informs the parent of this change. The parent then adds an entry to register the new child. This in turn can cause the parent to become too full and split. This splitting can proceed up the tree all the way to the root node. If the root node splits, it creates a new node that becomes the new root of the tree and the parent of the old root. The inverse of splitting can also happen: if a node contains only a few entries, it can be merged with a sibling. A sibling is selected and the entries are transferred to this sibling. The node is removed and the parent is updated to remove the corresponding child entry. If the root node has only 1 child, that child becomes the new tree root and the old root is removed. If enough entries are removed, the tree can collapse all the way down to a single node.

This scheme is very robust. It does not matter *how* a node is split, its entries could be randomly distributed over the old and the new node and the R-Tree will still be consistent. Similarly the sibling selected for merging does not matter, any sibling is valid and results in a consistent tree. The cost of this imprecise work is efficiency. The random split and merge strategies result in children overlapping the range of the parent almost fully. This requires a lookup to visit most of the nodes in the R-Tree, effectively a complicated scheme for sequential scan. The average fraction of overlap between children is a well known (and studied) performance characteristic of R-Trees. Any overlap that can be avoided will improve the efficiency of a lookup.

The mechanic of splitting and merging is all very similar to a regular B-Tree so what justifies the storage cost of ranges on $\mathbb{K}$ as opposed to just elements from $\mathbb{K}$? The R-Tree can, in contrast to the B-Tree, contain children whose ranges overlap without affecting the consistency of the tree as a whole. In a B-Tree any key in an interior node serves to direct algorithms that traverse the tree to one of the children at either side of the key. This implicitly bounds the key space range of children, but cannot express children with overlapping ranges. The R-tree explicitly tracks the range of children and is thus able to express overlapping children, at a cost in storage since each child requires two bounds from $\mathbb{K}$.

4.3.2. Structure of CFRT

After adding a few thousand elements the BloomCRDT becomes very inefficient. By the nature of a state-based CRDT, the entire state needs to be transferred to communicate even the smallest change. So a natural idea is to split the elements over multiple BloomCRDTs and link them together. Since the R-Tree is tolerant of overlapping children, this provides a suitable data structure to organize the linking of BloomCRDTs. Each node of the Conflict Free R-Tree is backed by a BloomCRDT. It is reminiscent of the MST, but the MST is not based on a CRDT for the actual tree nodes and is grow-only. Moreover the MST is modeled after a B-Tree but that is an unsuitable choice in the context of concurrency. If replicas independently decide to split a tree node then without coordination there is no consensus on how the elements are divided. Since building consensus is contrary to the idea of CRDTs, the only other solution is to deal with the conflicting splits that will happen. The B-Tree is not able to express such conflicting splits, but the R-Tree is uniquely suited to this situation.

In order to use a BloomCRDT for each tree node, there must be a mapping of R-Tree node *entries* to BloomCRDT set *elements*. Entries are be formed as triples of the form $e = (k_{min}, k_{max}, v)$. The k_{min} and k_{max} indicate a range in $\mathbb{K}$ covered by the entry. When $k_{min} = k_{max}$ this indicates a leaf value, and when $k_{min} \neq k_{max}$ this indicated a child reference. In the case of a leaf value, v is the value associated with the key k_{min}. In the case of a child reference, v is a global identifier of another BloomCRDT that can be resolved to produce a local replica. Furthermore, a special entry is added to each BloomCRDT of the form $e = (''parent'', ''parent'', p)$ where p is the global identifier of the node's parent. Figure 4.5 shows an example of R-Tree entries mapped to BloomCRDT set elements for each node.

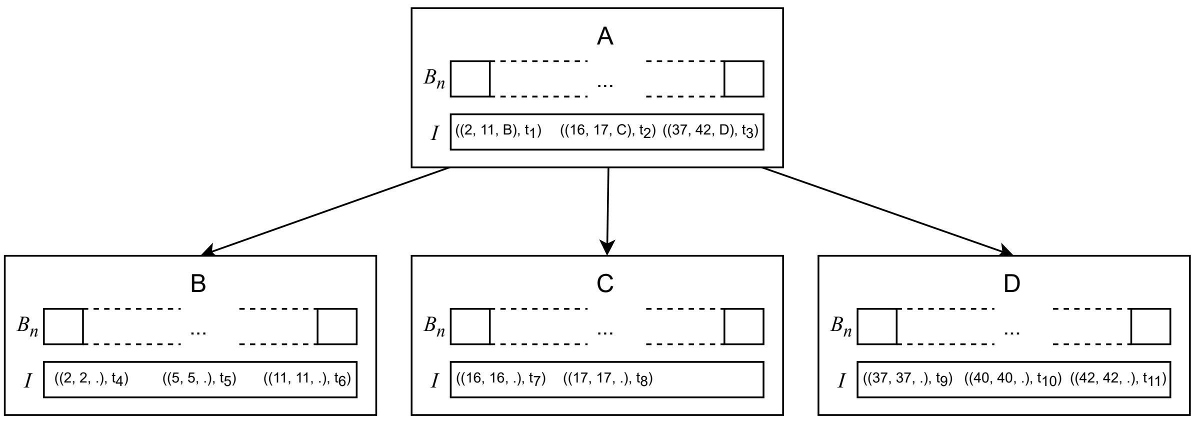

A
B_n
...
I
((2, 11, B), t_1) ((16, 17, C), t_2) ((37, 42, D), t_3)
B
B_n
...
I
((2, 2, .), t_4) ((5, 5, .), t_5) ((11, 11, .), t_6)
C
B_n
...
I
((16, 16, .), t_7) ((17, 17, .), t_8)
D
B_n
...
I
((37, 37, .), t_9) ((40, 40, .), t_{10}) ((42, 42, .), t_{11})

Figure 4.5: Example structure of a CFRT, showing the mapping onto BloomCRDTs

In addition to mapping the R-Tree node entries onto BloomCRDT set elements, the CFRT also needs a specification for its three basic operations: $add(k, v)$, $lookup(k)$ and $remove(k)$, and for its internal operations $split()$ and $merge()$

- $lookup(k)$ The lookup function is straightforward. Given a tree node, for each entry (k_{min}, k_{max}, v) that satisfies $k_{min} \leq k \leq k_{max}$, return v if it is a leaf value and otherwise recurs to the BloomCRDT identified by v.

- $add(k, v)$ The add function descends the CFRT identical to the lookup function, but it can run into the situation that zero or multiple entries satisfy $k_{min} \leq k \leq k_{max}$. If zero entries satisfy the condition the add function *could* select any child to descend, but to reduce the potential for overlap and the resulting reduced performance, the child that *should* be selected has the smallest increase in range to accommodate k. If multiple entries satisfy the condition, choose one. When at a leaf node, add entry (k, k, v) to the node. While unwinding the recursion stack, each node should update its entry in its parent to reflect any changes to the range of the child.

- $remove(k)$ The remove function descends the CFRT identical to the lookup function. If in any node no entry satisfies $k_{min} \leq k \leq k_{max}$, then k is not in the CFRT. When at a leaf node, remove any entry that satisfies the condition. While unwinding the recursion stack, each node should update its entry in its parent to reflect any changes to the range of the child.

- $split()$ To split a node (n), select the median (m) of all k_{min} and k_{max} values of all entries in n. For each entry in n, decide if it orders less or greater than m. For entries that overlap m, so $k_{min} \leq m \leq k_{max}$, if m is closer to k_{min} than to k_{max} the entry orders less, otherwise it orders greater. If the distances are equal, choose less or greater at random. Create a new CFRT node n' and add to it all entries that ordered greater than m. Update the parent of n with the range and id of n'. Remove from n all entries that ordered greater than m. Update the parent of n with the new range of n. If the transferred entries where not leaf values, update the parent references of the nodes identified by those entries from n to n'.
- $merge()$ To merge two sibling nodes (n and n'), add all entries from n' to n. Update the parent of n with the new range of n. From the parent of n' remove entries where v identifies n'.

There is however one caveat with the system outlined above. Assume a node splits (or merges) and moves a portion of its children to a new node, then the children will need to be informed of their new parent. However there is no way to atomically update all the replicas of each child node, since the parent reference is stored in a BloomCRDT entry. In fact, in the case that the parent is split by multiple nodes concurrently the child's parent pointer is updated concurrently and points to multiple new parents. There are two solutions to manage the parent/child relationship: one is to work without parent pointers at all, and the second is to simply allow multiple parents for each node. When working without a parent pointer, the parent node would need to observe the child for modifications and update its entries accordingly. However after a node splits, the parent has no way to discover the newly created siblings and would thus require sibling pointers on each node to aid in discovery. These sibling pointers have problems similar to the parent pointer. Also without parent pointers the immediate propagation of information towards the root is interrupted. Suppose a split would propagate all the way up to the root, then at each layer it has to pass the propagation has to wait for the parent to observe the split. The second way to manage the parent/child relationship is to simply allow the child to point to multiple parents. Both mechanisms result in the child being referred to by multiple parents. This type of R-Tree is called a R+-tree[41], and even though this forms a Directed Acyclic Graph this does not violate any R-Tree constraints. This does obviously introduce overlap, since all parents must include the range of the child.

4.3.3. Checking for optimizations

So far the definition of the CFRT had been straightforward, but the observant reader might have already wondered, what if two replicas concurrently $split()$ a node? The parent will contain three or more entries that represent overlapping child ranges. This is where the R-tree has a clear advantage over the B-tree, since it can express overlapping ranges of children while remaining consistent. The flow of events and the resulting state is exemplified in figure 4.6. A concurrent $add()$ leads to a concurrent $split()$. The adds have introduced entries at different positions in the key space, so the split will not use the same median. Since the CFRT is composed of BloomCRDTs at each tree node, the BloomCRDTs will join their states and eventually settle. After the tree nodes have joined their states the CFRT contains overlapping child ranges in the root node, but the data structure is still consistent.

The CFRT could be left in this state with the hope that adds/removes will, over time, naturally reduce the overlap. However reduced overlap can also be actively pursued by means of a periodic $check()$ function. This active restructuring of a R-Tree is the principle behind the R*-tree[7] and can result in improved efficiency. Active restructuring also means that each BloomCRDT is likely to be removed at some point, thus preventing the infinite growth of its Bloom filters. Furthermore, the $check()$ function can test for some other structural optimizations that can be applied to the CFRT.

- **Check parent-child links** As explained in section 4.3.2, child nodes can have multiple parents. After the joining process of BloomCRDTs the parent/child links can become outdated. Removing superfluous parent and child links helps to reduce the overlap between nodes by shrinking ranges.
- **Check merge/split threshold** R-Tree nodes in traditional databases have a hard limit on their size, often aligned to a memory or disk page size. Thus an $add()$ that overflows a node immediately triggers a split. For the CFRT there is no such limit, allowing a decoupling of the decision

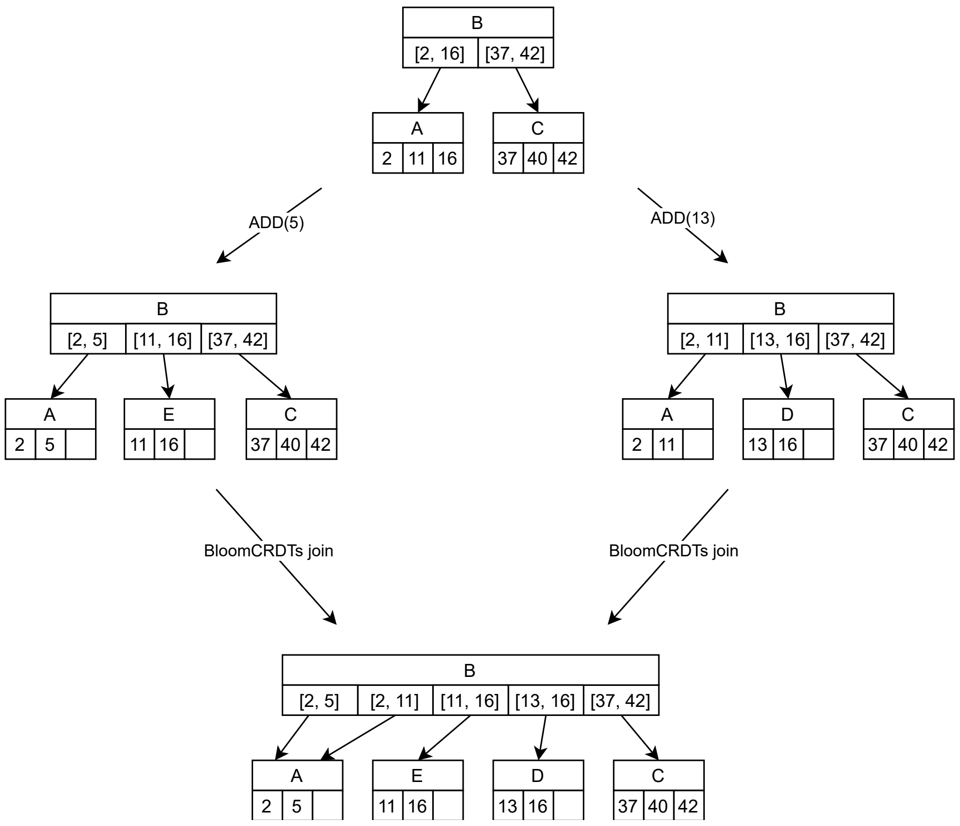


Figure 4.6: Example of CFRT concurrent $split()$.

to split/merge from the add/remove logic. The $check()$ function is an excellent place to do this since its periodic execution allows batching additions and removes.

- **Check entry in parent** A concurrent split can result in multiple entries in a single parent pointing to the same child node. During the periodic check, a child node should check for and correct this condition.
- **Check (large) overlapping children** If there are entries in a node that have an overlapping range then these could be merged. This is a natural result of a concurrent split of a child node, that will likely produce multiple siblings that overlap a lot due to duplicate contents. Merging the siblings that overlap the most removes duplicated entries. There is however also the situation that the overlap in range is large, but the number of duplicate items is low. In such cases, the merge can result in a node that is over the split threshold and will split again, with a more favorable split.

As an example of how the $check()$ function fixes inconsistencies consider figure 4.7. It starts with the last state from figure 4.6. This state shows a double reference for node A and shows that ranges $[11, 16]$ and $[13, 16]$ overlap. The only reason that the splits are not identical is that the two concurrent adds affected the selection of the median. In practical implementations a node contains more than three entries, where as in this example entries 2 and 16 represent larger sequences of entries. So in practice, the overlap resulting from concurrent updates would be far more dramatic. The check function fixes the double pointer and merges node D into node E. The result is an optimized CFRT where overlap is avoided.

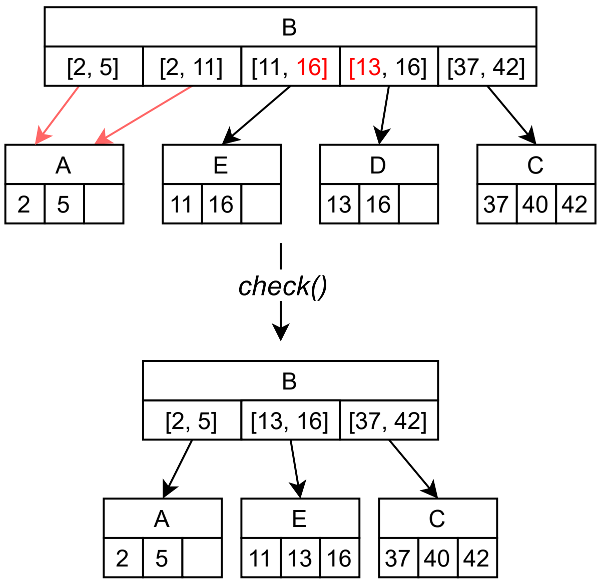


Figure 4.7: Example of CFRT $check()$ function correcting overlap in the ranges spanned by its entries.

5 Evaluation

This chapter provides empirical evidence that confirm the claimed properties of BloomCRDT and the CFRT.

5.1. Implementation

The system model (see section 4.1) requires the use of a communication middleware that provides the required features. In this case PyIpv8[46] was selected because it not only provides the required features but is very flexible and works well with the Gumby[45] experimental framework. This also automatically means the experiments are programmed in the Python programming language. To shorten development time a search was performed for Python compatible CRDT libraries and off-the-shelf components.

- https://github.com/ericmoritz/crdt, up to 2013. Based on python2, where as pyipv8 is python3. The CRDT base class only offers 2 convenience methods / prototypes for obtaining the ”payload” state.
- https://github.com/kishore-narendran/crdt-py, up to 2016. Uses CRDTs in Redis as multi valued registers. This is not the intended application domain of BloomCRDT and CFRT.
- https://github.com/anshulahuja98/python3-crdt, current. Active. No bases classes to inherit for BloomCRDT and CFRT.
- https://github.com/merchise/xotl.crdt, up to 2020 (dec). Offers a base class that has 3 extra lines to (un)pickle CRDTs.

All the modules that provide Python CRDT implementations are simple implementations of the CRDT primitives from the original CRDT tech report: the G-Set, Counter, 2P-Set, 2P2P-graph, etc. These primitives are not directly usefull for BloomCRDT nor CFRT, any further usefullness is a few lines of code to (un)pickle CRDT state. Using the investigated modules would add a dependency but are unlikely to accelerate development.

The python code written for this work consists of several python modules. Figure 5.1 shows the architecture and relations between these modules.

- **CRDT set primitives** This part only implements the algorithm for in process use.
  - **BloomCRDT** Including an implementation of the classic Bloom filter. The BloomCRDT algorithm is fairly simple and the expressiveness of python means BloomCRDT uses just 127 Lines-of-Code (LoC), slightly under half of which is the Bloom filter implementation. The initial implementation was very straightforward but inefficient in computing the hashes for the Bloom filter. This became a problem during the experimental phase, since BloomCRDT would become CPU-bound much sooner than other algorithms. After profiling the problem was identified and a more efficient method of computing hashes was implemented.

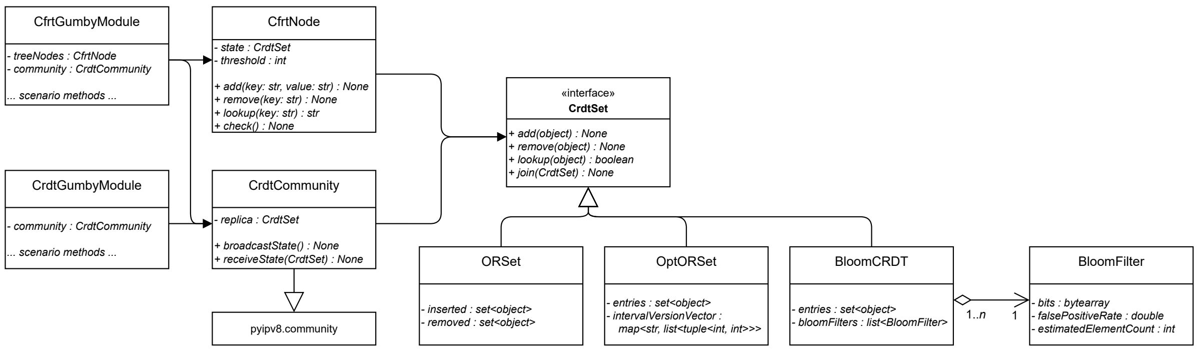


Figure 5.1: Diagram of implementation and experimentation architecture.

  - **Classic OR-Set** as described in [43] just 43 LoC.
  - **OptOR-Set** Or the Optimized OR-Set Without Ordering Constraints as described in [32]. 88 LoC

- **CFRT Node** Uses one instance of BloomCRDT for each R-Tree node. The implementation is more complex than BloomCRDT and the CRDT set primitives at 328 LoC. Of particular note are the separation of $check()$ and the randomized split/merge thresholds. The $check()$ method is only directly executed when absolutely nessecary. Not only because it is not cheap to execute but also because it offers the chance to coalesce multiple $split()$ and $merge()$ operations. Each instance of a CFRT node is provided with a randomized threshold for merging and splitting. The rationale is that this reduces the probabillity that a peer concurrently $split()$ or $merge()$ a node. While CFRT is tolerant of such an event, it is more efficient to reduce this occurence. The ranges for the random thresholds are set such that a split produces two nodes that cannot cross the merge theshold of another peer and vice versa.

- **CRDT PyIPv8 Community** This module is responsible for sending messages to other peers. All CRDT set implementations (BloomCRDT, OR-Set and OptOR-Set) are state based CRDTs that expose a common set of methods. This allows the CRDT community to be agnostic as to the actual type of CRDT being used. The CRDT community thus holds a referrence to an instance of any of the CRDT set implementation. When requested it will serialize (pickle) this instance and send it to up to 10 peers. When a message arrives from a peer the CRDT community deserializes the CRDT set instance contained in the message and provides it as an argument to the $join()$ function of the local replica. During experimentation it was immediately obvious that the 64KByte limit on UDP packets used by PyIPv8 might need to be exceeded. To overcome this limit a very rudimentary fragmentation scheme was added to the CRDT community. This raises the message size limit enough to allow the experiments to run. This brings the total LoC for the CRDT community to 110.

- **CRDT Gumby Module** manages the CRDT PyIPv8 Community based on Gumby scenario directives. It exposes several methods to control concurrent tasks in a Gumby scenario, $add()$, $remove()$ and $join()$, with configurable parameters to simulate different workloads. Note that it is the $join()$ task that ultimately initiates the CRDT Community to broadcast its state. An alternative would be to have the CRDT Community as an observer of its CRDT set, but this precludes the option of bunching several updates in a single state broadcast. A final task $stats()$ outputs statistics gathered by the CRDT community that are later plotted in graphs. The CRDT Gumby module also allows switching the type of CRDT set used by the CRDT Community.

- **CFRT Gumby module** leverages the CRDT PyIPv8 Community to synchronize the BloomCRDT instances contained in CFRT Nodes. It is very similar to the CRDT Gumby module however; it tracks different statistics, manages the referrence to the CFRT root node and modifies the CFRT with key/value pairs as opposed to random elemens like the CRDT Gumby module uses.

The code as used in the experiments is public and can be found at [11]. Unless otherwise noted it is assumed that BloomCRDT initialized the Bloom filter with $p(falsepositive) = 1^{-8}$ and an expectation of 500 elements. The CrdtCommunity aims to maintain 10 connections to other peers for broadcasting replica states. These values are suitable to demonstrate the effectiveness of BloomCRDT in an experimental setting.

5.2. BloomCRDT Experiments

Section 4.2 predicts two advantages of the BloomCRDT that should be experimentally confirmed.

- Does BloomCRDT provide a reasonable space complexity when the workload includes $delete()$? The space complexity is very important since, as with all state-based CRDTs, the whole state has to be communicated to other peers in order for changes to propagate. Thus a lower space complexity requires less bandwidth and can reduce the need for message fragementation.
- Does BloomCRDT provide a reasonable space complexity when the network has peer identity curn? If BloomCRDT is affected by peer identity churn then it could grow over time in a P2P environment.

To answer these questions BloomCRDT is compared to the OR-Set and the OptOR-Set under various conditions. The OR-Set was chosen since it is the design that BloomCRDT is derived from. The OptOR-Set was chosen because; it has set semantics as opposed to the list semantics of many other CRDTs, it has a state-based CRDT mode of operation, is is the state-of-the-art in OR-Set design, it is also a derivative of the OR-Set, and as such it shows some algorithmic similarity to BloomCRDT in the $join()$ function.

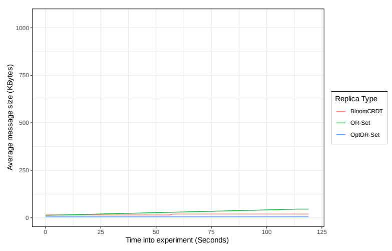
5.2.1. BloomCRDT $delete()$ workload storage cost

The hypothesis is that BloomCRDT has an acceptable space complexity for a workload that includes $delete()$. Since BloomCRDT does not keep the full elements (nor tags) of deleted elements it should have a lower space complexity compared to the OR-Set. In fact for the given false positive probability a Bloom filter should use ≈38.34 bits per element. An OR-Set would need a pointer to each deleted element, on current hardware this uses 64-bits. This is almost double the bits used by a BloomFilter, and does include the actual bytes that comprise the element. So the BloomCRDT should be superior, but by how much? On the other hand the Optimized OR-Sets Without Ordering Constraints[32] (OptOR-Set) actively works to reduce the size of the state and its space complexity should be constant with respect to the number of $delete()$ opertations performed.

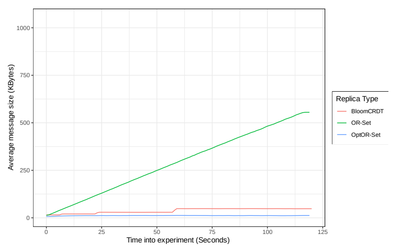
To test the space complexity of BloomCRDT, the standard OR-Set and the OptOR-Set with respect to the number of $delete()$ operations, an experiment is setup as follows. A variable number (n) of peers is started. Each peer holds one replica of a BloomCRDT, OR-Set or OptOR-Set. All sets are preloaded with 250 randomly generated elements, in this case 128-bit integers. The sets are allowed to reach eventual consistency. After that, for 110 seconds, each replica starts two processes to concurrently add and remove random elements. Every two seconds replicas exchange and $join()$ their state. After the add and remove processes have stopped peers have 5 seconds to join their states and reach eventually consistency again. During the experiment a record is kept of: the number of bytes used to store a replica, the time taken to $join()$ exchanged replica states tie time taken for (de)serialization of the replicas.

The add and remove processes aim to keep the number of elements in each set around 250. This is to ensure that the size measurement only measures the effect of element churn in each CRDT set type and not increased or decreased element count. The number of bytes used is measured with python's pickle module. This is because it is difficult to have python produce an accurate count of memory bytes used for a given object graph. The pickle methods are not a perfectly accurate measure of the memory bytes used for an object graph since pickle will dereferrence pointers and insert back references in the byte stream if it encounters a pointer to a previously serialized object. Pickle can also eliminate padding that is used for object member alignment. This means the number of bytes produced by pickle will likely be less than actual memory bytes used. On the otherhand the pickle methods are also used to serialize replica state when communicating with other nodes. Thus using the pickle methods gives an accurate measure of message space complexity in regards to the most limiting factor: message size.

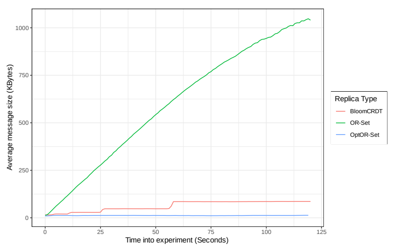
Figure 5.2 shows the space complexity results of the experiment for various values of n. Each graph shows experiment time in seconds progressing on the horizontal axis and the average mea-



(a) $n = 1$

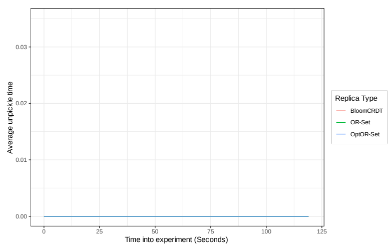


(b) $n = 10$

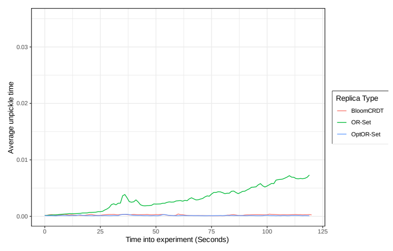


(c) $n = 25$

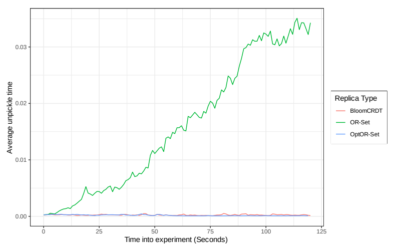
Figure 5.2: Space complexity of BloomCRDT, OR-Set and OptOR-Set



(a) $n = 1$



(b) $n = 10$



(c) $n = 25$

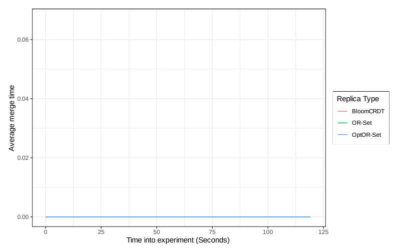
Figure 5.3: Time complexity (Deserialization) of BloomCRDT, OR-Set and OptOR-Set

sured replica size in KBytes on the vertical axis. Within each graph a distinction is made between BloomCRDT, the OR-Set and the OptOR-Set. The results for $n = 10$ and $n = 25$ clearly show the expected benefit of BloomCRDT over the OR-Set. In this case the difference in message complexity is around 1 order of magnitude. However a practical use of CRDTs might very well use more than 16 bytes of information per element as was used in this experiment. So this result is indicative of the lower bound on space complexity improvement of BloomCRDT over the OR-Set. The OptOR-Set shows the expected behaviour and is able to collapse metadata and remain at a constant space complexity with respect to the number of elements deleted.

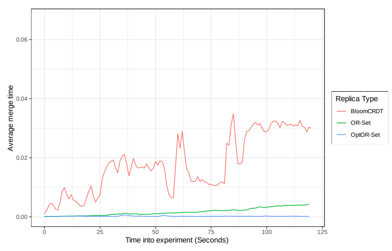
In the interval between broadcasting replica states the CRDT Community can expect to receive one replica state from each peer it is connected to. As decribed this number is set to 10 in these experiments. The different steps of receiving and joining a CRDT state should be efficient enough to keep up. Figure 5.2 shows the deserialization time complexity results of the experiment for various values of n. Each graph shows experiment time in seconds progressing on the horizontal axis and the average time in seconds taken for replica deserialization on the vertical axis. Within each graph a distinction is made between BloomCRDT, the OR-Set and the OptOR-Set. The effect of a reduced space complexity of BloomCRDT and the OptOR-Set compared to the OR-Set is also apparent in the time used to deserialize replica states. This is in part due to to simply more bytes to process for the OR-Set instances, but also the structure of the OR-Set. It consists almost entirely of small objects like tuples and (random) integers, each such entity will add extra overhead to the deserialization time. In contrast the BloomCRDT set has a structure where most bytes are in large arrays that can be efficiently processed. Even though the state is bigger, it still deserializes as fast as the OptOR-Set. Because the OptOR-Set is able to collapse its metadata state it is able to keep a very small state. Even though its state is also composed of many small entities, it is still very efficient when deserializing.

Figure 5.4 shows the time complexity results of the actual merge algorithm for various values of n. Each graph shows experiment time in seconds progressing on the horizontal axis and the average time in seconds taken for the join algorithm on the vertical axis. Within each graph a distinction is made between BloomCRDT, the OR-Set and the OptOR-Set. The OptORSet takes the cake! Clearly the join of two replica states is more complex to compute for BloomCRDT than for OR-Set and OptOR-Set. This is because the BloomCRDT has to compute hashes for new elements. The OR-Set also shows some increase in the time taken to join two states. This is due to the nature of the python $set()$ primitive which, for each added element, has to check if the element is already a member of the set.

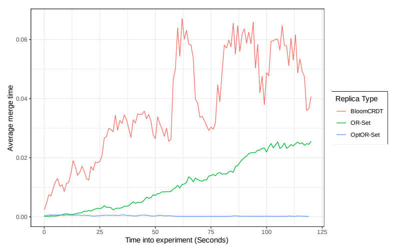
Figure 5.4 shows the sum of figures 5.4 and 5.3 for various values of n. Both the deserialization and join have to be performed once for each message received. After examining the steps separately



(a) $n = 1$



(b) $n = 10$



(c) $n = 25$

Figure 5.4: Time complexity (Join) of BloomCRDT, OR-Set and OptOR-Set

(a) $n = 1$

(b) $n = 10$

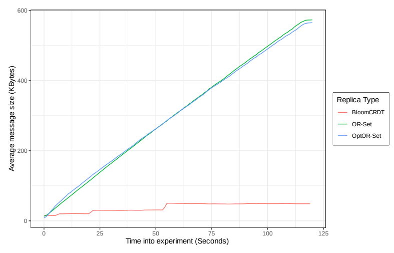
(c) $n = 25$

Figure 5.5: Time complexity total of BloomCRDT, OR-Set and OptOR-Set

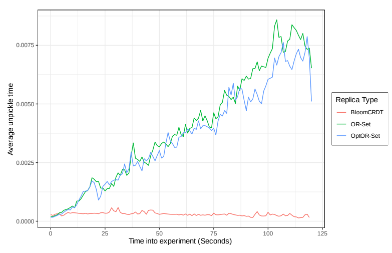
this graph shows the combined result. Depending on the situation BloomCRDT and OR-Set have a comparable time complexity, with BloomCRDT being more favourable as more deletes happen over time.

5.2.2. BloomCRDT P2P tolerance

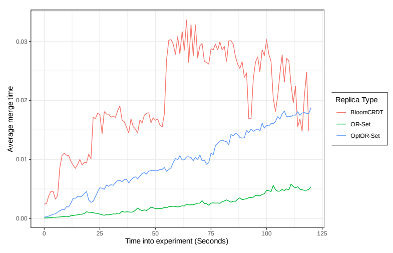
Given the previous experiment one could conclude that the OptOR-set is a superior solution that performs better than BloomCRDT and the OR-Set. Other solutions like, Redis[38], Riak[4] and DottedDB[20] would show results similar to the OptOR-Set. All these solutions require either a limited number of peers, or keep persistent information about other peers in the network. How does BloomCRDT compare to such systems when there is peer churn in a P2P network? Peer (identities) come and go over time, and keeping track of them all could inflate a replica's state size. Using the OptOR-Set as a stand-in for systems that keep per peer state, an experiment was performed to investigate the effect of peer identities over time on the replica state size. The experimental setup is identical to section 5.2.1 with the only change being an additional process that changes the identity of each replica every second to simulate peer churn. In this experiment n is not varied and fixed at $n = 15$.



(a) Storage/Message Cost of CRDTs



(b) Deserialization time of CRDTs



(c) $join()$ time of CRDTs

Figure 5.6: Experiment comparing BloomCRDT, OR-Set and OptOR-Set with network churn

Figure 5.6 shows the results of the experiment. As can be seen in figure 5.6a, the OptOR-Set trades the regular OR-Set's linear space complexity with respect to the number of deletes for a linear space complexity with respect to the number of peer identities encountered. In this experiment the OptOR-Set encounters $15 * 110 = 1650$ identites and shows a space complexity for this similar to the regular OR-Set. The time complexity of the OptOR-Set also shows the same trends as the OR-Set. P2P networks can have millions of identities over the lifetime of the network. So clearly the OptOR-Set is not well suited to a P2P environment. On the other hand figure 5.6 shows that BloomCRDT is unaffected by the number of encountered peer identities. So BloomCRDT is suitable for P2P networks with peer churn that require a CRDT with set-semantics that also supports deletes.

5.3. CFRT Experiments

The motivation for the design of CRFT is the lineair scalability of BloomCRDT. However, BloomCRDT is atomic and it can only be distributed as an indivisible state. So if, for example, a use-case arises where an application wants to store a million torrents and magnet links in a BloomCRDT, then this would work but only in theory. In practice users will probably not want to wait while a gigabyte sized BloomCRDT state is downloaded. However, the CFRT has some storage overhead for each key compared to BloomCRDT. At what replica size is CFRT a preferable choice over BloomCRDT?

5.3.1. Practical limits of BloomCRDT and CFRT

This experiment explores the practical limits of BloomCRDT and CFRT. Especially how they function from the perspective of a new peer joining the network with the aim of adding a specific key/value pair. To complete the $add()$ the new peer will have to fetch a BloomCRDT or several CFRT nodes. The number of fetched bytes directly translates to the time it takes to execute the operation. As already found in experiment 5.2.1 a singleton BloomCRDT should show lineair growth. The CFRT, by nature of any n-ary lookup tree, should show log_k behaviour for fetched bytes where k is the fanout factor of the tree nodes.

The setup of this experiment is as follows. A number $(n = 15)$ of peers is started. Each peer starts out with a replica of an empty BloomCRDT and an empty root CFRT node. To be able to test BloomCRDT in an experiment with CFRT, the BloomCRDT replica is represented by a single CFRT node with infinite bounds. Since a single CFRT node contains a single BloomCRDT set the infinite bounds will keep the CFRT node from splitting while the CFRT code provides the nessesary logic to allow key/value pair manipulation in a BloomCRDT. For 110 seconds, each replica starts a process to add random key/value pairs. Every two seconds replicas perform a $check()$ on CFRT nodes and subsequently exchange and $join()$ all BloomCRDTs. During the experiment a record is kept of: the number of bytes used to store a BloomCRDT replica, the number of and size of CFRT nodes visited during each $add()$, the number of $split()$ and $merge()$ calls, the number of nodes that require a $check()$.

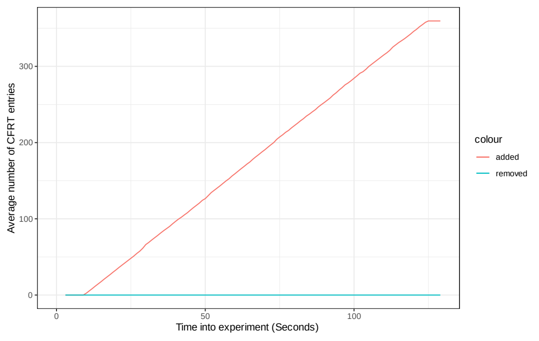
Figure 5.7 shows the results of the experiment. The left column shows results for BloomCRDT, the right side column shows the results for CFRT. All figures have the time into experiment in seconds on the horizontal axis. The first two figures, 5.7a and 5.7b show total number of entries added to the data structure under test on the vertial axis. The graphs show that both the BloomCRDT and CFRT contain the same number of entries. Next, figures 5.7c and 5.7d show statistics relating to the number of nodes in the tree. The BloomCRDT is obviously limited to 1 node, no splits and merges happen. The CFRT shows a lineair growth in the total number of nodes, as is to be expected when adding at a constant rate and a limited number of entries per node. The split graph also shows lineair growth, which is also to be expected since it counts the number of splits that have happened, which is once for every new node. The number of merges is zero, since no keys are removed during the experiment. The number of nodes fetched is the number of nodes that a new peer with no prior information would have to retrieve to add a new value to the data structure. In the case of CFRT it indeed shows a log_k relation to the number of nodes in the datastucture, and by extension to the number of entries in the datastructure. The number of nodes that are check()ed as a result of all add operations is also plotted, it too seems to show a log relationship to the number of nodes in the tree. The last two figures 5.7e and 5.7f show the byte size corresponding to the number of nodes fetched and checked. Again the reflect the log nature of the CFRT, and the linear nature of the BloomCRDT. In the long run it should be expected that CFRT scales much better than BloomCRDT with respect to the number of entries.

5.3.2. Fault tollerance of CFRT

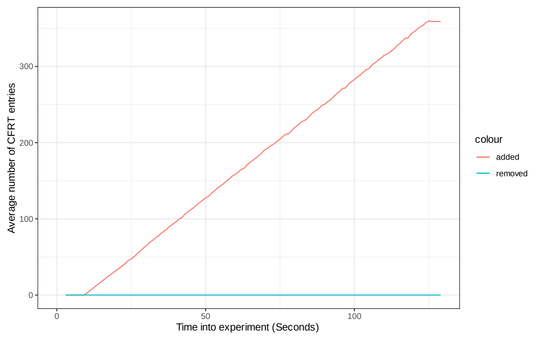
The CFRT should be eventually consistent, even in the face of adverse network conditions. Experiment to show this using reduced/fluctuating number of peer connections. Measure if convergence happens at all, and the time to convergence for different conditions.

5.3.3. Practical workloads of CFRT

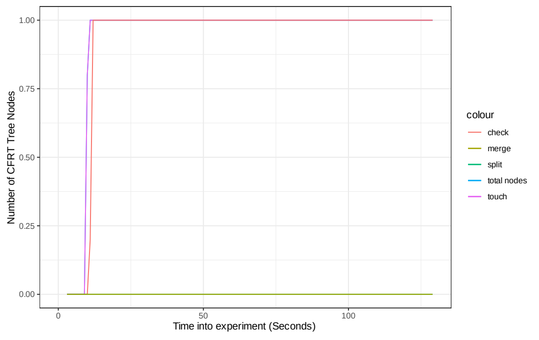
Nice results sofar, but is it good enough to support a real world workload? Try the Channel Index application. What metrics to measure?



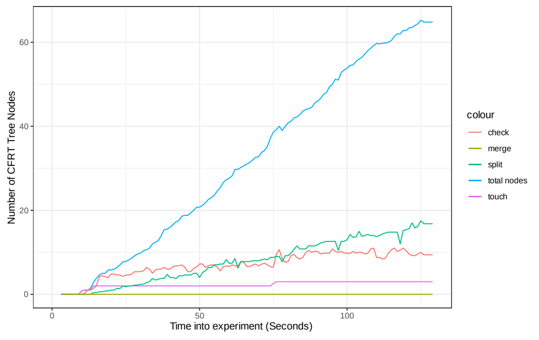
(a) Total number of key/value entries in BloomCRDT



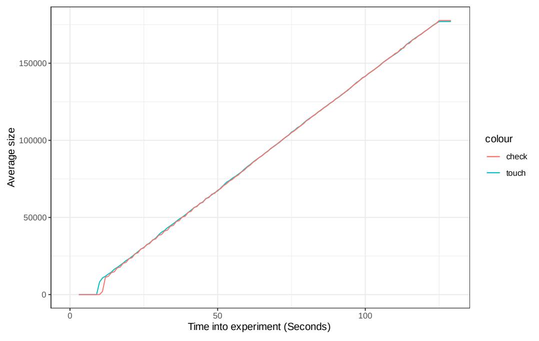
(b) Total number of key/value entries in CFRT



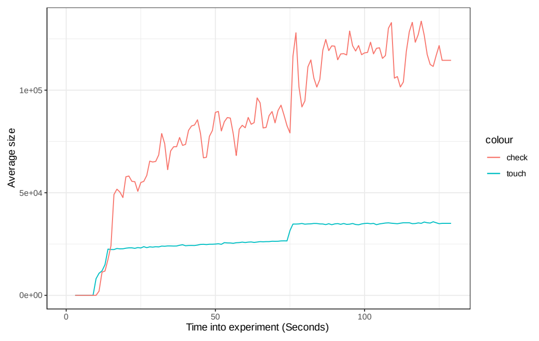
(c) Node counts in BloomCRDT



(d) Node counts in CFRT



(e) Average sum of node sizes touched and checked in Bloom-CRDT



(f) Average sum of node sizes touched and checked in CFRT

Figure 5.7: Scaling comparison of BloomCRDT to CFRT

5.4. Old Thoughts

About BloomCRDTs In the case of Channel Index the bloomfilter false positive probability was set to $p(falsepositive)10^-12$. It is likely that channels will be filled with either a manualy curated selection of content, or that the channel will be filled by some automated process. In the case of manual content, the chances of having a false positive will be very very low given the limited number of elements to expect. In the case of an automated process, it can check the original to see if some element might require resynchronisation.

Open questions:

- How to assure availability if everyone leaves a community? (last one to leave has to merge into a neighbouring state or something? That leaves the last one to leave the whole CRDT with a pretty large cheque)
- Propagation time. How fast does information travel through the network? The worst case is tied to graph diameter and the best case is tied to graph radius. Find expression for both from graph size and

6
Conclusion and further work

TODO: leader

TODO: structure of chapter

6.1. summary

TODO: write about what was.

TODO: write about what has been done.

TODO: write about what is.

6.2. Contributions

TODO: write what has been contributed to science

6.3. Ethical dilemmas?

TODO: if any?

6.4. Good engineering

TODO: Did we do good engineering? Does that need verification/justification?

6.5. Further Reserach

TODO: write about future research

Contrary to B-Trees, R-Trees can contain multi-dimensional keys, but does that affect the validity or use in a CFRT?

Can we refactor the bloom filter list into a tree? That could strongly reduce the chance of a false positive in the case there are many bloom filters, since the check won't be against all the filters.

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