A Method for Simultaneous Image Reconstruction and Beam Hardening Correction

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***Abstract*— Beam hardening is a well known effect in CT scanners that is cause by a combination of a broad polychromatic source X-ray spectrum and energy-dependent material attenuation. When the object consists solely of a single material, the non-linear beam hardening effect can be corrected using sinogram pre-correction techniques. However, when multiple materials are present, it is impossible to fully compensate for the distortion using pre-correction.**

This paper presents a novel model-based iterative reconstruction algorithm, MBIR-BHC, for X-ray CT, which estimates and corrects for beam hardening distortions during the reconstruction process. The method is based on the assumption that the object is formed by a combination of two distinct materials that can be separated according to their densities. During iterative reconstruction, two separate forward projections are computed, one for the low density material and one for the high, and a polynomial forward model is estimated for the two component projection. The coefficients of the correction polynomial are estimated during the MBIR reconstruction process using an alternating optimization framework. Therefore, no additional system information such as spectrum or mass attenuation functions are needed in the algorithm and the correction is automatically adapted to the dataset being used. Using both the simulated and real dataset, we show the efficiency of MBIR-BHC in reducing streaking artifacts and improving image quality.

***Index Terms*— X-ray CT, model-based iterative reconstruction (MBIR), beam hardening correction, poly-energetic**

I. Introduction

X-RAY CT has been utilized in various applications, such as medical diagnosis and security inspection. One common source of image reconstruction artifacts is beam hardening, caused by a broad X-ray spectrum and the energy-dependent material attenuation. Due to the beam hardening effect, the standard measurement obtained from a CT scanner is generally not a linear function of the attenuation coefficients. Typically, systems will correct beam hardening effects for a single material using polynomial pre-correction of the raw sinogram. However, when multiple materials are present, such as plastic and metal, it is not possible to fully pre-compensate for this distortion by simply applying a single non-linear transform. In practice, beam hardening can contribute to artifacts such as streaking through metal objects in the reconstructed images.

Recent results in model-based iterative reconstruction (MBIR) [1]–[6] have demonstrated its ability to improve reconstructed image quality and several methods have been proposed to incorporate beam hardening correction into the iterative reconstruction process [3], [4]. However, these methods require additional knowledge of the X-ray spectrum and mass attenuation functions.

In this paper, we propose a novel model-based reconstruction method, MBIR-BHC, that works by simultaneously estimating the reconstruction and the non-linear beam hardening functional. Our method is based on the assumption that distinct materials can be separated according to their densities. We propose a simplified poly-energetic X-ray forward projection model which accounts for the beam hardening effect of two materials, one for low density and the other for high. Using this new forward model, the CT measurement is approximated by a correction polynomial of two separate material projections, one for low and one for high densities. We incorporate the forward model into the MBIR reconstruction framework and simultaneously estimate the coefficients of the correction polynomial during the iterative process using alternating optimization. Therefore, no additional system information such as spectrum or mass attenuation functions are needed in the algorithm and the correction is adapted to the dataset being used automatically. We present the reconstruction results using both the simulated and real datasets and demonstrate the efficiency of MBIR-BHC in reducing streaking artifacts due to beam hardening, and improving the image quality.

II. Model-Based Iterative Reconstruction with Simultaneous Beam Hardening Correction Algorithm (MBIR-BHC)

A. Model-Based Iterative Reconstruction

Let $x \in \mathbb{R}^N$ be the image vector representing the attenuation coefficients to be estimated, and $y \in \mathbb{R}^M$ be the vector of the tomographic measurements obtained from the CT scanner. In the Bayesian reconstruction framework, the reconstruction problem can be formulated as computing the maximum a posterior (MAP) estimate given by

$$\hat{x} = \arg\min_{x \geq 0} \{ -\log \mathbb{P}(y|x) - \log \mathbb{P}(x) \} \qquad (1)$$

where $\mathbb{P}(y|x)$ is the data likelihood and $\mathbb{P}(x)$ is the prior distribution of the unknown image. The positivity constraint

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is imposed on x.

In conventional CT systems, the measurement is generated by

$$y_i \triangleq -\log\left(\frac{\lambda_i}{\lambda_{T,i}}\right) \quad (2)$$

where λ_i and $\lambda_{T,i}$ are the photon measurements of the i-th projection from the target and air-calibration scans respectively. Assuming the measurement y is Gaussian, the log likelihood term can be approximated by a 2-nd order Taylor's expansion [7] given by

$$-\log \mathbb{P}(y|x) \cong \frac{1}{2}(y - \mathbb{E}[y|x])^T W (y - \mathbb{E}[y|x]) + c(\lambda) \quad (3)$$

where $W = \text{diag}\{w_1, \cdots, w_M\}$ is the diagonal inverse covariance of y, and $c(\lambda)$ is a function independent of x. Using the Poisson-Gaussian noise model [8], the entry w_i can be approximated as

$$w_i = \frac{\lambda_i^2}{\lambda_i + \sigma_e^2} \quad (4)$$

where σ_e^2 represents the variance of electronic noise in data acquisition.

The conditional mean of the measurement $\mathbb{E}[y|x]$ is the forward model, reflecting the generation of the measurement y from the image x. In the traditional mono-energetic X-ray model, y is assumed to be an un-biased estimator of the line integral of attenuation, resulting in

$$\mathbb{E}[y|x] = Ax \quad (5)$$

where $A \in \mathbb{R}^{M \times N}$ is a linear forward projection matrix, whose entries $A_{i,j}$ represent the length of intersection of the i-th ray with the j-th pixel. However, it is well-known that due to beam hardening, $\mathbb{E}[y|x]$ will not be a linear function of x in general and furthermore, when multiple materials are present, it is impossible to fully linearize the nonlinear effect by simply applying a single pre-correction polynomial on y.

B. A Simplified Poly-energetic X-ray Forward Model

In general, the attenuation coefficients depend on the energy. If we denote $\mu_j(E)$ to be the energy-dependent attenuation coefficient of the j-th pixel, a more accurate forward model which accounts for the energy-dependency is given by

$$\mathbb{E}[y_i|\mu] = -\log\left(\int_{\mathbb{R}} S(E) e^{-\sum_{j=1}^{N} A_{i,j}\mu_j(E)} dE\right) \quad (6)$$

where $S(E)$ represents the normalized X-ray spectrum. However, this complex expression involving logarithm, integral and exponential is hard to incorporate in the iterative optimization process directly. Here we propose a simplified poly-energetic X-ray forward model which decouples the location and energy as follows. Our model is based on the assumption that different materials can be separated by their densities. Mathematically, we model $\mu_j(E)$ as a combination of two basis materials given by

$$\mu_j(E) = x_j\left((1 - b_j) r_L(E) + b_j r_H(E)\right) \quad (7)$$

where the image x_j here represents the average attenuation coefficient of the j-th pixel, $r_L(E)$ and $r_H(E)$ are two energy-dependent basis functions of "low" and "high" density materials, and $b_j \in \{0, 1\}$ is the binary variable indicating which one of materials the j-th pixel is of. Plugging (7) into (6), we obtain our simplified poly-energetic X-ray forward model as

$$\mathbb{E}[y_i|x] = -\log\left(\int_{\mathbb{R}} S(E) e^{-r_L(E) p_{L,i} - r_H(E) p_{H,i}} dE\right) \quad (8)$$

where for the i-th projection, we implicitly form two separate energy-independent material projections as

$$p_{L,i} = \sum_{j=1}^{N} A_{i,j} x_j (1 - b_j) \ , \quad (9)$$

$$p_{H,i} = \sum_{j=1}^{N} A_{i,j} x_j b_j \ . \quad (10)$$

We then further parametrize $\mathbb{E}[y_i|x]$ as a joint polynomial of $p_{L,i}$ and $p_{H,i}$ given by

$$\mathbb{E}[y_i|x] \triangleq h(p_{L,i}, p_{H,i}) \quad (11)$$

$$h(p_{L,i}, p_{H,i}) = \sum_k \sum_l \gamma_{k,l} (p_{L,i})^k (p_{H,i})^l \ , k, l = 0, 1, \cdots \quad (12)$$

where $\gamma_{k,l}$'s are the coefficients of the correction polynomial. We showed in [9] that $\gamma_{0,0}$, $\gamma_{1,0}$ and $\gamma_{0,1}$, which are determined by physics, are given by

$$\gamma_{0,0} = 0 \ , \quad (13)$$

$$\gamma_{1,0} = \gamma_{0,1} = 1 \ , \quad (14)$$

and in practice we will fix the order of the polynomial and estimate the other unknown coefficients during the iterative process. In this work, we use a 2-nd order joint polynomial and the unknown coefficients to be estimated are listed in Table I. The binary indicator b_j is calculated by segmenting the initial reconstruction $x^{(\text{init})}$ as

$$b_j = \delta(x_j^{(\text{init})} \geq T) \quad (15)$$

where T is the user-defined segmentation threshold to separate the "low" and "high" density pixels.

TABLE I: The Unknown Coefficients of The 2-nd Order Correction Polynomial

model order	coefficients to be estimated
2-nd order model	$\gamma_{1,1}$, $\gamma_{0,2}$

C. Image Prior Model

For the image prior, we use a Markov random field model with the form

$$-\log \mathbb{P}(x) = \sum_{\{s,r\} \in \mathcal{C}} a_{s,r} \rho(x_s - x_r) \quad (16)$$

where $\mathcal{C}$ is the set of all pairwise cliques, $a_{s,r}$ is the weight for the neighboring pixel pair (s, r), and ρ is the positive symmetric potential function on pixel difference. We choose the potential function to be the q-generalized Gaussian MRF (q-GGMRF) [1] given by

$$\rho(x_s - x_r) = \frac{|x_s - x_r|^p}{1 + |(x_s - x_r)/c|^{p-q}}, \; 1 \le q \le p = 2 \quad (17)$$

The parameter c balances the performance between noise reduction and edge preservation [1].

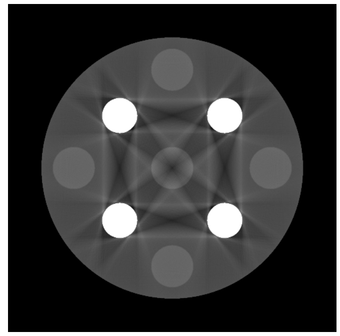
D. Alternating Optimization

Combining our simplified log likelihood model in (8), (11) and (12), and the log prior model in (17), we obtain the final objective function of MBIR-BHC algorithm as

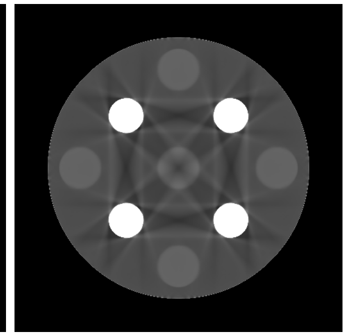
$$\{\hat{x}, \hat{\gamma}\} = \arg \min_{x \ge 0, \gamma} \left\{ \frac{1}{2} \sum_{i=1}^{M} w_i \Big(y_i - \sum_k \sum_l \gamma_{k,l} (p_{L,i})^k (p_{H,i})^l \Big)^2 \right. \\ \left. + \sum_{\{s,r\} \in \mathcal{C}} a_{s,r} \rho(x_s - x_r) \right\} \quad (18)$$

where we also simultaneously estimate the coefficients of the correction polynomial in this optimization. This can be considered as a joint MAP/ML estimation of the image x and the nuisance parameter γ [10]. Notice that one advantage of this method is that since coefficients are estimated during the iterative process, no additional system information, such as the X-ray spectrum or mass attenuation functions, other than the normal sinogram are needed, and the method adapts to the dataset.

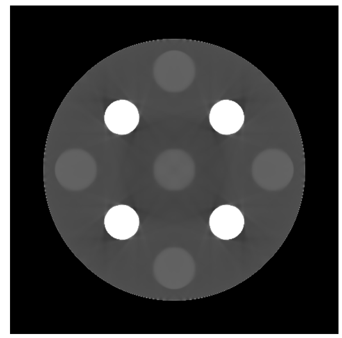
To solve the optimization (18), we alternatively optimize over γ and x until convergence. Fixing x, optimization of γ is reduced to the standard weighted least square problem since in this case $p_{L,i}$ and $p_{H,i}$ can be computed explicitly using (9) and (10). The optimization over x while fixing γ is a high-order non-linear problem. Our approach is to approximate the high-order objective function by a quadratic function using Taylor's expansion and refine the expansion point iteratively. For the approximated objective at each iteration, it is minimized using the iterative coordinate descent (ICD) algorithm [11].



(a) FBP



(b) generic MBIR



(c) MBIR-BHC

Fig. 1: Reconstruction results of the circular water phantom using different methods. The display window is [-200 400] HU.

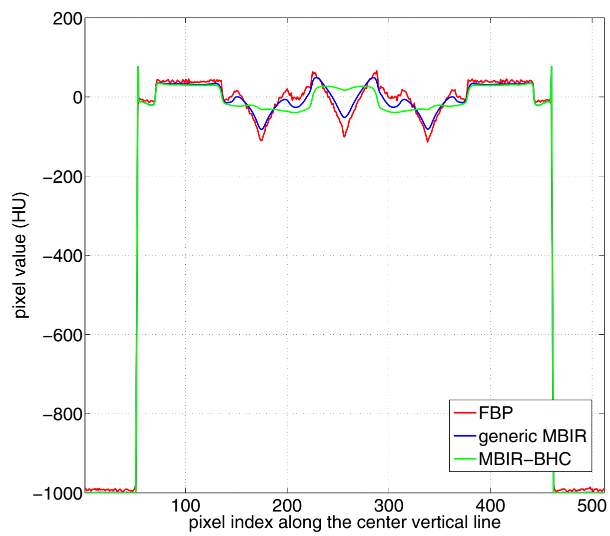


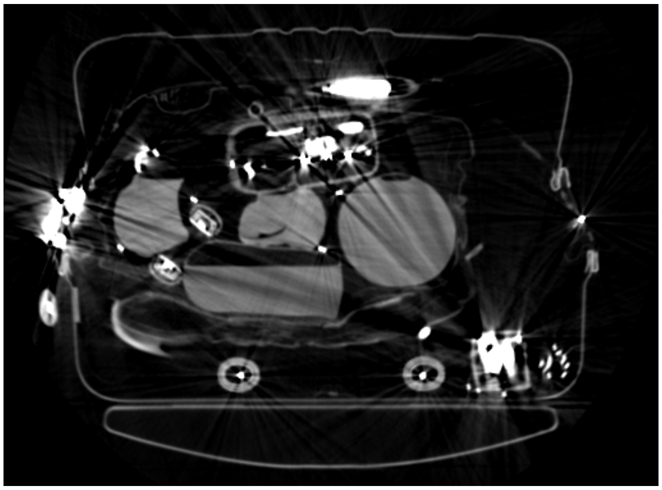
Fig. 2: Pixel profile through the center vertical line of the circular water phantom

III. Results

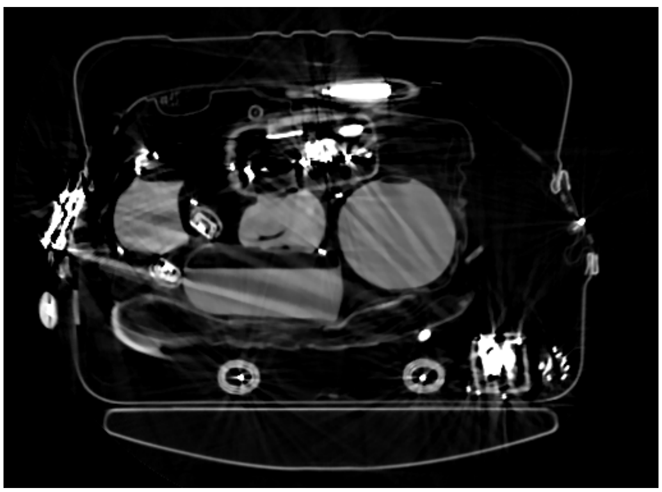
In this section, we applied the proposed MBIR-BHC algorithm on both the simulated and real scan datasets.

In the simulation study, we generated the parallel-beam transmission polychromatic X-ray projection using Monte Carlo method. The phantom is a circular water phantom over a field of view (FOV) of 250 mm $\times$ 250 mm with aluminum and soft tissue inserted and their attenuation coefficients are obtained from the NIST XCOM database [12]. The simulated sinogram has 1024 detector measurements with 0.24 mm spacing and 720 views over 180 degrees. We further did a water pre-correction on the sinogram. Figure 1 shows the comparisons of the reconstructed images using different methods. The resolution of the images are 512 $\times$ 512. From the FBP reconstruction in Figure 1(a), it can be seen that it has severe streaks through the aluminum insertions, and the beam hardening effects due to the metal can not be easily reduced by the simple sinogram water pre-correction. Similar artifacts are also observed in the generic MBIR algorithm using a linear forward projection model in Figure 1(b). As a comparison, the MBIR-BHC algorithm reduces most of the streaking artifacts. Figure 2 further plots the pixel profile through the center vertical line. The MBIR-BHC method reduces the fluctuation significantly compared to the other two methods.

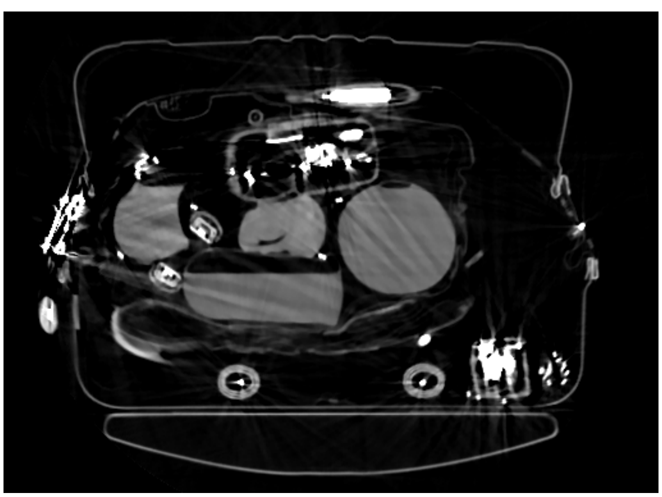
Figure 3 presents the reconstruction results on a real X-



(a) FBP



(b) generic MBIR



(c) MBIR-BHC

Fig. 3: Reconstruction results of the real baggage scan using different methods. The display window is [-1000 1000] HU.

ray CT scan of an actual baggage with low and high density objects, provided by ALERT DHS center, Northeastern University, USA. The parallel-beam sinogram has 1024 detector measurements and 720 projection angles over 180 degrees, rebinned from a fan-beam scan acquired at 130kV X-ray tube voltage. The reconstructed images are of size 512 $\times$ 512 over a field of view of 475 mm $\times$ 475 mm. Due to the beam hardening effects of the high density objects, the FBP reconstruction in Figure 3(a) has streaks and blooming artifacts. While the generic MBIR using a linear forward projection model in Figure 3(b) improves the image quality in general, most of the artifacts still remain. On the other hand, the proposed MBIR-BHC algorithm reduces the streaks in homogeneous regions and the blooming around high density objects significantly and improves the image resolution as well.

IV. Conclusions

In this paper, we have presented a model-based iterative reconstruction method with simultaneous beam hardening correction (MBIR-BHC), which does not require any additional prior knowledge of the system. We developed a simplified poly-energetic X-ray forward model which accounts for the nonlinear behaviors of multiple materials. Based on the assumption that different materials can be separated according to their densities, we proposed the forward projection model as a joint polynomial parametrization of the “low” and “high” densities materials and simultaneously estimated the coefficients of the correction polynomial and the image during the iterative optimization process. The proposed method has better performance than the traditional FBP and generic MBIR in terms of the streaking artifacts reduction. Further investigation will assess the reconstruction accuracy and evaluate the potential benefits in real applications.

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