Thermodynamics – Basic Concepts

Contents

Laws about gases

Charles' Law

Volume is directly proportional to temperature: $V = cT$, where $c > 0$ is constant.

Scientist Jacque Charles noticed that if air in a balloon is heated, the balloon expands. For an ideal gas, this relationship between V and T should be linear (as long as pressure is constant).

Boyle's Law

Pressure is inversely proportional to volume: $p = \frac{a}{V}$, where a > 0 is a constant.

Robert Boyle noticed that when the volume of a container holding an amount of gas is increased, pressure decreases, and vice versa (while the temperature is held constant). Note that this is not a linear relationship between p and V.

Charles' and Boyle's Laws combined

Mathematically, you can combine the two laws above: $\frac{pV}{T} = k$, where k is a constant.

Ideal Gas Law

This law combines the relationships between p, V, T and mass, and gives a number to the constant! The ideal gas law is: **pV = nRT**, where n is the number of moles, and R is universal gas constant. The value of R depends on the units involved, but is usually stated with S.I. units as: **R = 8.314 J/mol·K**.

For air, one mole is 28.97 g (=0.02897 kg), so we can do a unit conversion from moles to kilograms.

$$R = \frac{8.314\,J}{mol \cdot K} \times \left| \frac{1\,mol}{0.02897\,kg} \right| = \frac{287\,J}{kg \cdot K}$$

This means that for air, you can use the value

R = 287 J/kg·K.

If you use this value of R, then technically the formula should be written as

pV = mRT,

where m represents the *mass* of air in kg (and we avoid having to do any calculations with moles.)

Relevant concepts and definitions for gases

Pressure

Pressure is defined as a force applied over the surface area of an object. The formula defining pressure is: $p = \frac{F}{A}$

For example, picture a person standing on a box that sits on the floor. The person weighs 800 N pounds and the box, viewed from above, is the shape of a square with a side length of 1 meter. The pressure of the box on the floor would be:

$$\begin{aligned} p &= \frac{F}{A} \\ &= \frac{800\ N}{1\ m^2} \\ &= 800\frac{N}{m^2} \\ &= 800\ Pa \end{aligned}$$

Pressure is measured in Pascals (Pa), and 1 Pascal is defined as 1 N/m^2. Therefore, the pressure on the floor is **800 Pa**.

Another common unit for pressure is pounds per square inch (psi).

Barometric Pressure

Also known as atmospheric pressure, this is pressure applied to any object that is in an atmosphere, e.g. on earth. Standard atmospheric pressure, at sea level at a temperature of 25 °C is 101.325 kPa. Barometric pressure is measured with a barometer.

Manometric Pressure

Also known as the **gauge pressure**, this is the *internal* pressure of the system, and does not include the barometric pressure. Manometric pressure is measured with a manometer. Note that this value can actually be negative (see below).

Absolute Pressure

This is the total amount of pressure including both the manometric and barometric pressures. You can use the formula:

Absolute Pressure = Barometric Pressure + Manometric Pressure

With this formula, it is important to note that absolute pressure cannot be negative, but manometric pressure can and will be negative whenever the absolute pressure of the system is lower than the barometric pressure. This happens when you create a vacuum (or partial vacuum) inside something.

Specific Volume

The specific volume of an object is defined as its volume (i.e. space occupied) per unit mass. The symbol is a lower case v (which is sometimes written in italics), and the formula can be written as:

$$v = \frac{V}{m}$$ where V is the total volume.

The normal unit of measurement for specific volume is m^3/kg. Specific volume is also the reciprocal of density: $v = \frac{1}{\rho}$, where ρ (Greek letter rho) is the density of an object (or substance).

Temperature

Temperature is a less straightforward concept than pressure. Temperature is a measure of the amount of thermal energy that exists in a physical space, relative to a zero point of energy, which we define as absolute zero. In science, temperature is usually measured on the Kelvin scale, where 0 K is absolute zero, and 273.15 K is the freezing point of water (273.15 K = 0 °C).

General Physics Stuff

Force

In physics, force is defined using Newton's 2nd Law:

$$F_{net} = ma,$$

where m is the mass of an object, a is the objects acceleration and F_{net} is the amount of unbalanced force being applied to the object. The S.I. unit for force is the Newton (N), where

1 N = 1 kg·m/s^2.

Conceptually, 1 N is the amount of force required to move a mass of 1 kg at a rate of acceleration equal to 1 m/s^2.

Any unit for force should always be made up of a unit for mass (in this case kg) multiplied by a unit for acceleration (m/s^2).

In fact, the formal definition of pounds force (lbf, the Imperial unit of force) is

$$1 \text{ lbf} = 1 \text{ slug}\cdot\text{ft/s}^2$$

or the force required to accelerate a mass of 1 slug at a rate of acceleration of 1 ft/s^2. Since one slug equals 32.174 pounds mass (lb), we get the useful conversion factor:

$$1 \text{ lbf} = 32.174 \text{ lb}\cdot\text{ft/s}^2,$$

which is very useful for changing between weights in lbf to masses in lbm, and vice versa.

Energy and Work

There are many different types of energy, and different formulas for calculating it, but they should all be measured with the same unit, the Joule.

The following are a few different mathematical definitions of energy.

Kinetic Energy = ½ mv^2

where m is mass and v is speed (or velocity without direction). Kinetic Energy is the energy of motion – anything that is moving has kinetic energy. The higher the speed, the more kinetic energy an object has.

Gravitational Potential Energy = m g h

where m is mass, g is gravitational acceleration, and h is the height of the object relative to a reference point (e.g. the ground). The higher something is above the ground, the more potential it has to fall, increase speed, and thus gain kinetic energy.

In general, all forms of energy have something to do with movement, even including thermal and electrical energy, while potential energy is the potential for movement to happen at some point. For example, electric potential is the potential for an object with charge (e.g. an electron) to move through an electric field. Chemical potential energy can be thought of as the energy held in some substances that can be released through a chemical reaction, such as burning a fuel for heat.

Work is defined as the change in the level of energy of a system, so it is also measured in Joules. Mechanical work can be measured by this formula:

$$W = F \cdot s,$$

where F is the force exerted and, and s is the distance over which the force was exerted.

So, if I apply a steady force of 100 N on an object, and the object moves 5 m, then the amount of work I did on the object is:

$$\begin{aligned} W &= 100\,N \times 5\,m \\ &= 500\,N \cdot m \\ &= 500\,J \end{aligned}$$

The definition of a Joule can be understood easiest based on the above definition of work:

$$W = F \cdot s$$

Therefore,

$$\begin{aligned} 1\,J &= 1\,N \cdot m \\ &= 1\,kg \cdot \frac{m}{s^2} \cdot m \\ &= 1\,kg \cdot \frac{m^2}{s^2} \end{aligned}$$

As an exercise, use the above definitions of kinetic and gravitational potential energy to show that they are measured in the same units.

In fact another formula for mechanical work is:

$$W = p\,\Delta V$$

Where p is pressure and ΔV is the change in volume. The units still work out to be in Joules as long as pressure is measured in Pascals, and volume in m^3.

Power

The concept of power follows directly from work. Power is simply the rate at which work is done and is measured in Watts (W).

$$Power = \frac{Work}{time}$$
$$1\,W = \frac{1\,J}{1\,s}$$
$$= \frac{1\,kg \cdot \frac{m^2}{s^2}}{1\,s}$$
$$= 1\,kg \cdot \frac{m^2}{s^3}$$

So, if a generator supplies 500 J of electricity every second, we can call it a 500 W generator. If a light bulb uses up 60 J of energy every second to brighten up our lives, it is a 60 W bulb.

And Back to Energy!!

Another common unit for energy worth mentioning is the **kilowatt-hour** (kWh). This is a unit people often see on their electricity bills. The kilowatt-hour may seem like a unit for power, but it is actually energy.

Think of it this way: 1 kW is 1000W, which is an amount of power. Power is work (or energy change) divided by time. So now, we are multiplying that unit of power by a different unit of time, the hour. Thus, the kWh is energy divided by time, then multiplied by time, bringing us back to energy!

Just for fun, and to practice unit conversions, let's convert 1 kWh into Joules.

(Remember that $1\,J = 1\,kg \cdot \frac{m^2}{s^2}$)

$1\,kW \cdot h \times \left|\frac{1000W}{1kW}\right| = 1000\,W \cdot h = 1000\,kg \cdot \frac{m^2}{s^3} \cdot h$, from the definition of the Watt above.

$$1000\,kg \cdot \frac{m^2}{s^3} \cdot h \times \left|\frac{3600s}{1h}\right| = 3600000\,kg \cdot \frac{m^2}{s^3} \cdot s$$
$$= 3600000\,kg \cdot \frac{m^2}{s^2}$$
$$= 3600000\,J$$

Therefore 1 kWh is the same as 3.6×10^6 J

This may also give you an idea of why we often use kWh to measure our energy consumption. If we measured it using Joules, the numbers would be huge! Also, when you know the "wattage" of all your appliances, it makes it easier to do a quick calculation in kWh for an hour's use of those appliances, and then multiply by the electric company's rate per kWh to find out how much it costs to run your air conditioner for 6 hours, for example.

Problems

Piston pressure problem

1) In a piston system, the piston diameter is 25.0 mm, and the mass of the piston and platform is 55.0 g. If a 0.584 kg weight is placed on top of the platform to equilibrate the pressure, then what is the manometric pressure inside? Assume the barometric pressure is 101.300 kPa and $g = 9.807\ m/s^2$.

Gas law problems

2) The pressure-volume relationship for a given gas is assumed to be $Pv^{1.2}$ = Constant. At a pressure of 1.5 bars, the volume is of the gas is 55 mL. The mass of the gas inside is 80 mg. If temperature is kept constant, what should the new pressure be (in bars) when the volume is decreased to 35 mL?

3) Calculate the volume of 1.2 kg an ideal gas with a molar mass of 55 g/mol at a pressure of 120 kPa, and a temperature of 80°C. Also, what is are specific volume and the density of the gas?

4) Calculate the mass in grams of air contained in a spherical balloon with a diameter of 5 m. The temperature of the air inside the balloon is 55.0°C and the manometric pressure is measured at 1.5 bars.

Weight, mass, density, etc.

5) What is the specific volume of a gas that has a weight of 1.5 kg and is contained in a cylindrical container with a volume of 2.0 m^3? If this is the answer on earth, where $g = 9.807\ m/s^2$, what is the gas's new specific volume if I move the cylinder to Phobos, where $g = 0.0015\ m/s^2$?

6) You measure the weight of a platinum bar with a mass of exactly 10 kg to be 98.12 N. What is the acceleration due to gravity at your location? If you move to a place where $g = 9.807\ m/s^2$, what will be the weight of the bar?

7) While on the Jovian moon Ganymede, you use a spring scale to measure the weight of a brick as 1.5 pounds. If you then travel to Io and measure it again with the same scale, what will the weight of the brick be (in pounds)? (Assume the gravitational acceleration is 1.8 m/s^2 on Io, and 1.4 m/s^2 on Ganymede.) Also, given that the brick is a cube with a length of 50 cm on each side, what is its density in kg/m^3?

Answers

Question 1 (Piston Pressure Problem)

There are three forces pushing the piston downward:

F_{bar} = The force of the barometric pressure

$F_{applied}$ = The force of the applied weight

F_{cyl} = The force of the weight of the piston and platform.

There is only one force pushing upward:

F_{mano} = The force of the manometric pressure inside the piston

Since the system is in equilibrium (i.e. it isn't moving), the sum of the forces acting downward must be equal to the forces acting upward.

$$F_{bar} + F_{applied} + F_{cyl} = F_{mano}$$
$$p_{bar} \cdot A + m_{applied} \cdot g + m_{cyl} \cdot g = p_{mano} \cdot A$$
$$\frac{p_{bar} \cdot A + m_{applied} \cdot g + m_{cyl} \cdot g}{A} = p_{mano}$$

Since Force of weight = mass × gravitational acceleration

And Force of pressure = pressure × area

All of the quantities on the left side of this equation can be determined from the given information, but we have to be careful to use the correct units: kg, m, and Pa.

$$A = \pi r^2$$
$$= \pi(0.0125\text{m})^2$$
$$= 4.91 \times 10^{-4}\text{m}^2$$

By converting the given diameter of 25 mm to 0.025 m and dividing by 2 to get the radius.

$m_{applied}$ = 0.584 kg This is already in the correct units ☺

$m_{cylinder}$ = 55.0 g = 0.055 kg

p_{bar} = 101300 Pa

g = 9.807 m/s^2

Substituting all of this into the equation for p_{mano} above, we get:

$$\frac{101300\text{Pa} \cdot 4.91 \times 10^{-4}\text{m}^2 + 0.584\text{kg} \cdot 9.807\text{m/s}^2 + 0.055\text{kg} \cdot 9.807\text{m/s}^2}{4.91 \times 10^{-4}\text{m}^2} = p_{mano}$$

All three of the multiplications on the type should result in N as the units, since they are the three forces acting downward from the original equation. The result is:

$$\frac{49.74\text{N} + 5.727\text{N} + 0.5394\text{N}}{4.91 \times 10^{-4}\text{m}^2} = p_{mano}$$
$$\frac{56.0\text{N}}{4.91 \times 10^{-4}\text{m}^2} = p_{mano}$$

$p_{mano} = 114053\text{N/m}^2 \cong 114000\text{Pa}$ A Pascal is a Newton per square metre.

Therefore, the manometric pressure is about 114000 Pa or 114 kPa.

Question 2 (Gas laws)

Given:

$p_1 = 1.5\ bar$ $V_1 = 55\ mL$

$p_2 = ?$ $V_2 = 35\ mL$

$= 80\ mg$

Equation:

$$pV^{1.2} = constant$$

$$p_1V_1^{1.2} = p_2V_2^{1.2}$$

$$p_2 = \frac{(p_1)(V_1)^{1.2}}{(V_2)^{1.2}}$$

$$p_2 = \frac{(1.5\ bar)(55\ mL)^{1.2}}{(35\ mL)^{1.2}}$$

$$= 2.58\ bar$$

Note that we didn't need to know that the mass was to solve this problem – i.e. the mass (80 mg) was extraneous information. We also didn't need to convert from mL to m^3 because the mL cancelled out in the equation on the right (because it is a proportion).

Therefore, the new pressure, after compression, is 2.58 bar.

Question 3 (Gas laws)

Given:

$$P = 120\ kPa$$
$$= 120000\ Pa$$

$$V = ?$$

$$mass = 1.2\ kg, molar\ mass = 55\ g/mol$$

$$n = \frac{mass}{molar\ mass} = \frac{1200\ g}{55 \frac{g}{mol}} = 21.82\ moles$$

$$T = 80°\ C = 353.15\ K$$

$$R = 8.314\ \frac{N \cdot m}{mol \cdot K}$$

Equations:

$$PV = nRT$$

$$V = \frac{nRT}{P}$$

$$V = \frac{(21.82\ mol)\left(8.314\ \frac{N \cdot m}{mol \cdot K}\right)(353.15\ K)}{120000\ \frac{N}{m^2}}$$

$$V = 0.534\ m^3$$

Specific Volume

$$v = \frac{V}{m} = \frac{0.534\ m^3}{1.2\ kg} = 0.455\ {m^3}/{kg}$$

Density

$$\rho = \frac{1}{v} = \frac{1}{0.455\ \frac{m^3}{kg}} = 2.25\ {kg}/{m^3}$$

Therefore the volume of the gas is 0.534 m^3, the specific volume is 0.455 m^3/kg and the density is 2.25 kg/m^3.

Question 4 (Gas laws)

Given:

$P = 1.50\ bars = 1.50 \times 10^5\ Pa$

$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(5)^3 = 523.6\ m^3$

$m = ?$

$R = 287\ \frac{J}{kg \cdot K}$ (for air)

$T = 55.0\ °C = 328.15\ K$

Equations:

$PV = mRT$ (using the value of R for air)

$$m = \frac{PV}{RT} = \frac{(1.50 \times 10^5\ Pa)(523.6\ m^3)}{\left(287\ \frac{J}{kg \cdot K}\right)(328.15\ K)} = 833.94\ kg = 834000\ g$$

Therefore, the mass of the air in the balloon is 834000 g.

Question 5 (weight, mass, density, etc.)

Given:

$v = ?$ (specific volume)

$m = 1.5\ kg$

$V = 2.0$ (net volume)

$g = 9.807\ m/s^2$

Equation:

$$\text{Specific volume} = \frac{\text{net volume}}{\text{mass}}$$

$$v = \frac{V}{m} = \frac{2.0\ m^3}{1.5\ kg} = 1.7\ {m^3}/{kg}$$

Therefore the specific volume on earth is 1.7 m^3/kg. The specific volume on Phobos will be exactly the same as on Earth because specific volume only depends on mass and net volume, which both remain the same regardless of change in location or gravity.

Question 6 (weight, mass, density, etc.)

Given:

$m = 10\ kg$

$Weight = F_g = 98.12\ N$

$g = ?$

Equations:

$F = ma$

$F_g = mg$

$g = \frac{F_g}{m}$

$g = \frac{98.12\ N}{10\ kg}$

$g = 9.812\ m/s^2$

Therefore, the acceleration due to gravity at this location is 9.812 m/s^2.

Now, we are told that g=9.807 m/s^2, and need to find the new value for weight (F_g)

Given:

$m = 10\ kg$

$g = 9.807\ m/s^2$

$Weight = F_g = ?$

Equations:

$F_g = mg$

$F_g = (10\ kg)\left(9.807\ \frac{m}{s^2}\right)$

$= 98.07\ N$

Therefore, the weight of the platinum rod is now 98.07 N at this new location.

Question 7 (weight, mass, density, etc.)

On Ganymede:

Given:

$F_g = 1.5\ lbf \times \left|\frac{32.174\ \frac{lbm \cdot ft}{s^2}}{1\ lbf}\right| = 48.261 \frac{lbm \cdot ft}{s^2}$

$g = 1.4\ \frac{m}{s^2} \times \frac{3.28\ ft}{1\ m} = 4.592\ \frac{ft}{s^2}$

$m = ?$

Equations:

$F_g = mg$

$m = \frac{F_g}{g}$

$m = \frac{48.261 \frac{lbm \cdot ft}{s^2}}{4.592\ \frac{ft}{s^2}}$

$m = 10.5\ lbm$

The mass of the brick is 10.5 lbm. We can use this find the weight on Io.

On Io:

Given:

$m = 10.5\ lbm$

$g = 1.4\ \frac{m}{s^2} \times \left|\frac{3.28\ ft}{1\ m}\right| = 4.592\ \frac{ft}{s^2}$

Equations:

$F_g = mg$

$F_g = (10.5\ lbm)\left(4.592\ \frac{ft}{s^2}\right)$

$F_g = 48.2\ \frac{lbm \cdot ft}{s^2} \times \left|\frac{1\ lbf}{32.174\ \frac{lbm \cdot ft}{s}}\right|$

$F_g = 1.50\ lbf$

Therefore, the weight of the brick on Io is 1.50 lbf.

Density of the brick in kg/m^3:

Given:

$m = 10.5\ lbm \times \left|\frac{1\ kg}{2.2\ lbm}\right| = 4.77\ kg$

Length: $l = 0.5\ m$

$V = ?, \rho = ?$

Equations:

$\rho = \frac{m}{V}$ (density = mass/volume)

$$V = l^3 = (0.5\ m)^3 = 0.125\ m^3$$

$$\rho = \frac{m}{V}$$

$$\rho = \frac{4.77\ kg}{0.125 m}$$

$$\rho = 38.2\ {}^{kg}/_{m^3}$$

Therefore the density of the brick is 38.2 kg/m^3.