Lecture 28: Disjunct Matrices

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1 Overview

We have seen in Group Testing we have to find d objects out of n objects with t tests where $t \leq n$. We compute an unknown vector x with as few tests as possible. We looked at the Adaptive Tests and saw some tight bounds.

$\Omega(dlogn) \leq t^a(d,n) \leq O(dlogn)$

In this lecture we will look at some Non Adaptive Group Testing methods in which all the tests needs to be fixed apriori and represent tets as matrix with one row per test. This matrix has various names like Disjunct Matrix, Dedisjunct Matrix or Coverfree Matrix.

Diagram to be added

Where,

ith test $S_i \subseteq [n]$

$\chi_{S_i}(j) = 1 iff j \in S_i$

we will attack this and get

$\Omega(dlogn) \leq t(d,n) \leq n$, Where the running time we will get is $O(d^2 log^2 n)$.

Now decoding can be defined as $r = Mx$ where M is a matrix given and x is the ouput.

2 Definition1- M is d-Separable.

iff $\forall S_1 \neq S_2 \leq [n]$

$\bigcup_{j \in S_i} S_{M_j} \neq \bigcup_{i \in S_2} S_{M_j}$

Where $|S_1|, |S_2| \leq d$

3 Decoding Algorithm.

We know that, $r = Mx$ and $|x| \leq d$.

Now, check all the possible subsets.

$\forall$ $T \leq [n]$ such that $|T| \leq d$ and check if

$S_r = \bigcup_{j \in T} S_{M_j}$

We will get the running time of $n^{O(d)}$.

4 Definition2- M is d-Disjunct.

Diagram to be added

Pick a subset S (not necessarily together). Then pick a column that is not present in that subset. There will always be i such that 1 in column j and all zero in subset exists.

$\forall s \leq [n], |S| \leq d, \forall j \notin S$

There exist an i such that $M_{ij} = 1$.

But $\forall k \in S$, $M_{ik} = 0$.

$S_{M_j} \notin \bigcup_{k \in S} S_{M_k}$

5 Claim

Any matrix iwhich is d-disjunct is also d-separable.

Proof: By contradiction, Let M be disjunct but not d-separable. (***Diagrams to be included later***)

If M is not d-seperable, then union of the two is the same.

$\therefore \bigcup_{K \in S_1} S_{m_k} = \bigcup_{K \in S_2} S_{m_k}$ - by contradiction.

This implies that for all $j \in S_2$ and $j \notin S_1$

This is a contradiction because one column that is outside is also contained in the union, and hence the matrix is disjunct.

6 Lemma

There exists a $O(nt)$ time decoding algorithm for any disjunct matrix.

Observation - (** Diagram to be added later ***)

So if we have 1 in R, then there exists a j in matrix that made it possible.

If $r_i = j$, then there exists a j such that $M_{ij} = 1$ and $x = 1$

7 Observation 2

d - disjunct matrix - (** Diagram to be added later)

*D*ecoding algorithm:

*i*nput: $(r \in \{0,1\}^t, (M))$ $\forall j \in [n]$ and $X_j = 1$

For all tests, if $r_i = 0, \forall j \in n$, if $M_{ij} = 1$, set $X_j = 0$

*E*nd
We will get a runtime of $O(nt)$