Discovery of an 11th-Century Geometrical Compilation: The Istikmāl of Yūsuf al-Mu'taman ibn Hūd, King of Saragossa

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> *It is strange that a work believed to be so important and written by a king should be lost* (G. Sarton [1927 I, 759] in his discussion of the *Istikmāl*)

Two members of the family of the Banū Hūd, kings of Saragossa from 1039 to the Christian conquests of the 12th century, were well known for their mathematical talents: Aḥmad al-Muqtadir bi-llāh, who reigned from 1047 to 1081, and his son Yūsuf al-Mu'taman [1] ibn Hūd, who reigned from 1081 to 1085 [Lane-Poole 1925, 26]. The fame of Aḥmad is evident from the following passage in a letter [2] written by a certain Abu l-Wālid al-Shaqunṭī (from Sagunta, Spain) to Abū Yaḥyā ibn al-Mu'allim al-Tanjī (from Tangier, Morocco) in order to show that Spain is more perfect than Morocco (the letter is quoted by the historian Al-Maqqarī (died 1632; see [EI(1) III, 173–174]):

> Do you have (in Morocco) in astronomy, geometry or philosophy a king such as Al-Muqtadir ibn Hūd, ruler of Saragossa? He was a miracle (*āya*) in that. [Al-Maqqarī 1884–1885 2, 141, lines 9–10]

The son Yūsuf seems to have been equally brilliant; al-Maqqarī mentions him in [1884–1885 1, 206, lines 15–16] and says that among his mathematical works is the book *Istikmāl* (which can be translated as *Perfection* or *Comprehensive Treatise*). This work is also mentioned by other authors. The encyclopedist Muḥammad ibn Ibrāhīm al-Akfānī (died 1348; see [Brockelmann 1943–1949 2, 171]) says that if the *Istikmāl* of al-Mu'taman ibn Hūd had been completed, it would have made the existing geometrical literature superfluous (Arabic text in [Sprenger 1843, 80, lines 11–12], paraphrased in [Wiedemann 1905, 395; Wiedemann 1970 1, 112]). From this we gather that the *Istikmāl* was an unfinished work. The biographer Ibn al-Qifṭī (died A.D. 1248; see [EI(2) III, 840]) says that the *Istikmāl* of al-Mu'taman ibn Hūd is a beautiful comprehensive work, which was edited by Maimonides (ca. 1135–1204; see [DSB IX, 27–32]), and was studied under his guidance [Ibn al-Qifṭī 1903, 319, lines 13–15]. The *Istikmāl* is mentioned by Ibn Aknīn (ca. 1160–1226; see [Steinschneider 1855] = [Steinschneider 1980, 35–73]) in the *Ṭibb al-Nufūs* (Healing of the Souls) in a survey of the works that should be read by students of geometry. Ibn Aknīn discusses the *Elements* of Euclid, the *Spherics* of Theodosius, the *Spherics* of Menelaos, *On the Sphere and Cylinder* and unspecified minor works by Archimedes, and the *Conics* of Apollonios; he then concludes the survey as follows:

> But we refer you to a book which contains all the benefits of geometry, with abbreviation of long-windedness and with maximum conciseness in its proofs; the proofs of its propositions exhibit the knowledge that is subsumed under each of these proofs. This is the book *Istikmāl* by al-Mu'taman ibn Hūd, king of Saragossa. Nothing matches it. It is concisely phrased, and its proofs are splendid.

(Arabic text in note 4, transcribed from the Arabic text in Hebrew characters in [Güdemann 1968, 28, lines 17–21]; German translation in [Güdemann 1968, 87, lines 17–24].)

Ibn Aknīn then provides a summary of the *Istikmāl* (to be discussed below).

Further biographical data of Yūsuf al-Mu'taman ibn Hūd and references to the *Istikmāl* in 12th- and 13th-century works can be found in a recent study by Djebbar [1984], who also points to the importance of the *Istikmāl*.

Hitherto the *Istikmāl* was known by references only, and the work itself was considered to be lost ([Suter 1900, 108; Sarton 1927 I, 759], quoted above; [Matvievskaya & Rozenfeld 1983 2, 305; Djebbar 1984, 2, 7] [7]). The purpose of this note is to show that large parts of the *Istikmāl* survive in four anonymous manuscripts. Before presenting the evidence for the identification I will provide a description of the work, based on the extant manuscript material. Pending the identification, I indicate the *Istikmāl* and Yūsuf al-Mu'taman ibn Hūd by the terms "work" and "author."

The following Arabic manuscripts contain fragments of the work:

K: Copenhagen, Or. 82 (see [COBRH, 64–67]), an undated manuscript which originally must have contained the complete work. Many leaves of the manuscript are lost; the 128 extant leaves are bound (and numbered) in a very incorrect order (see Fig. 1).

L: Leiden, Universiteitsbibliotheek, Or. 123/1 (see [Voorhoeve 1957, 432; De Goeje 1873; No. 2646, pp. 238–239]), undated, a continuous fragment of 80 leaves, but containing only one-fifth of the amount of text in K.

C: Cairo, Dār al-Kutub Muṣṭafā Fāḍil Riyāḍa, 41m, f. 1b–37b, written in A.D. 1746 [King 1981, 439; King 1985, 489, No. B 110], the first part of the work [7].

D: Damascus, Ẓāhiriyya, ʿāmm 5648, f. 223b–277a, written in A.D. 1888 [Āʿidī 1973, 106], containing exactly the same part as C. This is not surprising, because the entire ms. Damascus, Ẓāhiriyya, ʿāmm 5648 is very closely related to ms. Cairo, Dār al-Kutub Muṣṭafā Fāḍil Riyāḍa 41m (see [King 1979, 459]).

The overlaps of K with L, C, and D (see Fig. 1) show that the text in the four manuscripts belongs to the same work.

No preface or introduction to the work is extant. The work is divided into five "species" (*nauʿ*), which I will abbreviate as N1, . . . , N5.

The species N3, N4 are subdivided into three (sub)species N31, N32, N33 and N41, N42, N43, respectively; N5 is subdivided into two (sub)species N51, N52. The work uses the same term (*nauʿ*) for the species N3, N4, N5 and their (sub)species N31, N32, etc.

The species N1, N2 and the subspecies N31, . . . , N52 are divided into up to four sections (*fuṣūl*); each section is divided into on average 15 to 20 proposi-

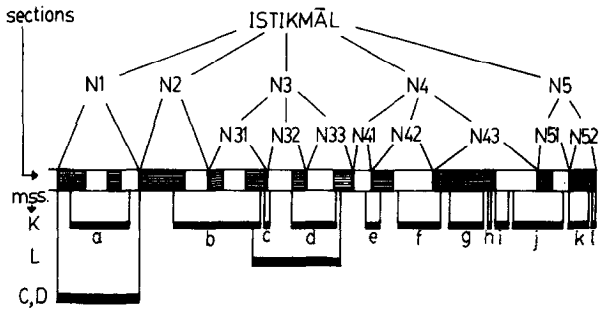

FIGURE 1

tions. Figure 1 shows in descending order the title of the work, the species, the subspecies, the sections (indicated by white and shaded rectangles), and the extant parts of the work (indicated in black) in the manuscripts K, L, and C–D, respectively. The different sections are represented as shaded rectangles in such a way that the length of each rectangle is proportional to the (estimated) length of the relevant section.

The figure shows the badly organized state of the Copenhagen manuscript (K). The text in this manuscript can be divided into twelve continuous fragments, indicated as a, . . . , l in the figure. These fragments consist of the following leaves in the following order:

a = 1–20; b = 21–50; c = 51 only; d = 58–71; e = 74–78; f = 79–92; g = 93–95 + 113–120; h = 53 only; i = 54 + 72 + 73 + 55; j = 56 + 52 + 57 + 96–112; k = 122–127 + 121; l = 128 only.

Listed next are the titles of all the (sub)species given in the extant manuscript material, with references to page and line numbers in the manuscripts; these references also indicate the beginning of each (sub)species. A discussion of the "sections" (*fuṣūl*) is omitted for reasons of brevity. Remarks on the subject matter and other additions of my own are in parentheses. The Arabic text of the titles can be found in note 5. The following list includes some references of my own to works such as Euclid's *Elements*, but the reader should bear in mind that the extant parts of the work do not contain any references at all to other geometrical works or to other authors.

N1: "The first species of the first genus of the two genera of the mathematical sciences, on the knowledge of the properties of numbers, separately and in relation to one another" (C 1b:1–2, D 223b:2–3). (Contains number theory in the style of the arithmetical Books of Euclid's *Elements*. N1 consists of four sections. Propositions 5–13 of section 4 contain the practically unchanged text of Thābit ibn Qurra's treatise on amicable numbers [3].)

N2: (The title does not occur in the extant fragments. Section 2 is called "on circular figures" (K 23b:2). Judging from the extant part of N2, the subject of the species was elementary plane geometry of single lines, triangles, circles, etc.)

N3: "The third species of the first genus of the two genera of the mathematical sciences, on the properties of lines, angles and plane figures in relation to one another" (K 32b:2–3). (Consists of N31, N32, and N33.)

N31: "The first (sub)species on the properties of lines, angles and plane figures in relation to one another, concerning ratio" (K 32b:5–6). (Theorems on ratio and proportion, in the style of Euclid's *Elements*, Books V and VI.)

N32: "The second (sub)species of the third species of the first genus of the two genera of the mathematical sciences on the properties of lines, angles and plane figures in relation to one another, concerning commensurability and incommensurability" (K 51b:2–4, L 13a:2–5). (Theorems mostly from Book X of the *Elements*.)

N33: "The third (sub)species of the third species of the first genus of the two genera of the mathematical sciences, on the properties of lines, angles and plane figures in relation to one another, concerning assumption and position" (K 62a:2–4, L 49b:2–6). (Theorems involving the concept "given," in the style of Euclid's *Data*, for use in geometrical analysis.)

N4: (Title not found in the extant fragments. Consists of N41, N42, and N43.)

N41: (Title not found in the extant fragments. Consisting of only one section, of which the latter half is extant. N41 deals with elementary theorems on planes in space and solid angles from Book XI of the *Elements*.)

N42: "The second (sub)species of the fourth species on the properties of spheres and the sections produced in them (the spheres being) not in relation to one another" (K 76b:2–3). (Includes theorems concerning great and small circles on a single sphere from the *Spherics* of Theodosius and the *Spherics* of Menelaos.)

N43: "The third (sub)species of the fourth species on sections of cylinders and circular cones" (K 90b:14–15). (Conic sections; mainly a summary of the *Conics* of Apollonius.)

N5: "The fifth species of the two genera of the mathematical sciences, on the relation of solids and their surfaces to one another" (K 104a:2–3). (Consists of N51 and N52.)

N51: "The first (sub)species on the relation of solids with plane surfaces and their surfaces to one another" (K 104a:3–4). (The first section contains among other things preliminaries for N52; the second section treats paralellepipeda, pyramids, and regular polyhedra, i.e., the topic indicated in the title.)

N52: "The second (sub)species (the ms. reads: section) of the fifth species on the relation of solids and the surfaces of round solids to one another" (K 122a:2–3). (Deals mostly with material from Archimedes' *On the Sphere and Cylinder*. The confusion between "section" and "species" in the ms. was probably caused by the fact that N52 consists of only one section.)

At the end of N52, the text in K states that "the book has ended" (K 128a). Thus N5 was the last species of the work.

We now proceed to the identification of the work. Our first witness is Ibn Aknīn, who provides the following survey of the *Istikmāl*, immediately after the quoted passage in which he recommends the work:

> He (the author) divided it into five species. The first species is about number; he mentioned in it what Euclid mentioned in the seventh, eighth and ninth (Book) of his book (the *Elements*), and also what Thābit ibn Qurra mentioned in his treatise on amicable numbers. The second species is on the properties of lines, angles and plane figures, not in relation to one another. In it he mentioned what Euclid mentioned in the first, second, third and fourth (Books of the *Elements*), and he added problems to this. In the third species he mentioned the properties of lines, angles, plane figures, and much (more) knowledge. He mentioned in it what Euclid mentioned in what is known as the *Book of Data* and also as the *Book of Assumptions*. In the fourth species he mentioned what Euclid mentioned in the eleventh book (of the *Elements*). In the fifth species he mentioned the relation of solids [with] plane [surfaces] to one another.
>
> We have set out here the contents of the book, in order to show that its name *Istikmāl* (Perfection) is appropriate for what is denoted by it.

(Arabic text in note 4, based on the Arabic text in Hebrew characters in [Güdemann 1968, 28:24–29:1]; imperfect German translation in [Güdemann 1968, 87:24–88:17].)

This survey proves that the text in the manuscripts K, L, C, and D is the *Istikmāl*. The inaccuracies in Ibn Aknīn's account are easy to explain: the subjects of N4 and N5 in his survey are in fact the subjects of N41 and N51, respectively. It appears from Ibn Aknīn's survey that the *Istikmāl* never contained more than the five species N1, . . . , N5. These must have belonged to the "first genus of the two genera of the mathematical sciences" (though this is not stated clearly in the title of N5). We can assume that the first genus was geometry, but we do not know for sure what al-Mu'taman ibn Hūd regarded as the second of the two genera of the mathematical sciences; this second genus may have included astronomy or optics or both. Al-Mu'taman ibn Hūd probably intended to write a comprehensive treatise on the two genera, but he finished only the part dealing with the first genus. This is consistent with Ibn al-Akfānī's statement that the *Istikmāl* was an incomplete work.

Our second witness for the identification is the author of the marginal remarks in the mathematical and astronomical manuscript Istanbul, Aya Sofya 4830, the otherwise unknown geometer Muḥammad ibn Jawbār ibn Sartāq al-Marāghī. According to f. 102a and f. 165 of the manuscript he wrote the marginal notes in 727 and 728 H./A.D. 1327, at the Niẓāmiyya madrasa in Baghdad. The Aya Sofya manuscript contains Thābit ibn Qurra's treatise on amicable numbers, at the end of which Muḥammad ibn Sartāq wrote the following remark:

> Exactly this treatise is written in the book *Istikmāl* by al-Mu'taman ibn Hūd, khalīf of Spain (Andalus). I have edited it, and I called it *Ikmāl* (*Perfection* or *Complete Treatise*). It is the fourth section of the first species of it on numerical (propositions). Muhammad ibn Sartāq al-Marāghī, who is in need of God the Sublime, edited it.

(Ms. Aya Sofya 4830, f. 121b; Arabic text in note 6.)

As stated before, the fourth section of N1 does indeed contain the text of Thābit's treatise, with only very minor editorial changes. We note that Muḥammad ibn Sartāq had a somewhat too optimistic view of the power of al-Mu'taman ibn Hūd, who was a petty king of a small part of Spain.

Muḥammad ibn Sartāq also "edited" a number of treatises in the Aya Sofya manuscript, but his "editions" did not entail any changes in the order of the propositions. It is reasonable to assume that he followed the same procedure in editing the *Istikmāl*. His marginal remarks contain references to his recension of the *Istikmāl*, which confirm our identification of the manuscripts K, L, C, and D. We cite two examples:

1. On f. 177b, he wrote in the margin of Al-Kūhī's additions to the *Data* of Euclid (GAS V, 319, No. 17):

"These two rules are proved in proposition 5 of section 1 of class (*ṣinf*) 3 of species 3 of genus 1 of my book *Ikmāl fi l-riyāḍī*" (the *Complete Treatise in the Mathematical (Science)*; obviously the title of his recension of the *Istikmāl*).

In modern notation these rules are as follows: If x and y are magnitudes in a known ratio and if p, q are two known magnitudes, then:

(i) if $p < x$ and $q < y$, either the ratio of $x - p$ to $y - q$ is known, or there is a known ratio r and a known magnitude s such that $x - p = r(y - q) + s$.

(ii) if $q < y$, there is a known ratio t and a known magnitude u such that $x + p = t(y - q) + u$.

The two rules are proved in proposition 5 of section 1 of N33 in the Leiden ms. (L 53a–54a); in the Copenhagen ms. (K 62b) the number of the same proposition is 6, because the definitions at the beginning of the section, which do not have a number in L, have the number 1 in K.

2. On f. 162b in the margin of Al-Kūhī's treatise on the construction of an equilateral pentagon in a given square (GAS V, 318, No. 3) Muḥammad ibn Sartaq wrote that a certain theorem on conic sections was proved in proposition 4 of section 1 of class 3 of species 4 of genus 1 (see for the details of the reference [Hogendijk 1985a, 255, 273]). N43 deals indeed with conic sections, but a further confirmation of the reference is not possible because the relevant proposition is not extant in our manuscripts.

It is interesting to note that Muḥammad ibn Sartāq in his *Ikmāl* used the term "class" (*ṣinf*) for subspecies, thus removing the ambiguity in the terminology for species and (sub)species in the *Istikmāl*. This difference in terminology shows, incidentally, that the extant manuscripts K, L, C, and D do not contain Muḥammad ibn Sartāq's recension.

A further discussion of the contents of the *Istikmāl* or the (rather strange) classification of its subject matter into "species," etc., would transcend the scope of this paper. I conclude with a brief discussion of the interest of the work.

The *Istikmāl* is essentially an intelligent and efficient abridgment of other sources, with a few original contributions. Al-Mu'taman ibn Hūd did not restrict himself to elementary subjects. N43, for example, contains abstracts of the difficult Books V and VII of the *Conics*. Al-Mu'taman is therefore an exception to the general rule that the level of mathematics in Spain was lower than that in the Eastern part of the Islamic world. We have seen that the *Istikmāl* was studied by Maimonides and his pupils, and that the work was available in Baghdad in the 14th century; thus the *Istikmāl* seems to have been widely studied (compare also [Djebbar 1984, 5–7]).

Among the sources on which the *Istikmāl* is based are the following: Euclid's *Elements* and *Data,* Archimedes' *On the Sphere and Cylinder,* the *Conics* of Apollonius, the *Spherics* of Menelaos, the *Spherics* of Theodosios, Thābit ibn Qurra's treatise on amicable numbers, Eutocius' commentary on Archimedes' *On the Sphere and Cylinder II,* Ptolemy's *Almagest,* Ibn al-Haytham's *Optics,* the treatise by the Banū Mūsā on the measurement of plane and spherical figures, and others. It is interesting to know that such a good collection of mathematical works was available in northeastern Spain in the end of the 11th century. One wonders whether the existence of al-Mu'taman ibn Hūd's library was in any way beneficial to the transmission of mathematical texts from Arabic into Latin in 12th-century Spain. Dr. C. Burnett draws my attention to a statement by the translator Hugo de Sanctalla to the effect that his patron Michel, Bishop of Tarrazona from 1119 to 1151, had obtained Arabic manuscripts in Rota, the stronghold of the Banū Hūd after the fall of Saragossa in 1118 ([Haskins 1960, 71, 73]; for more details see [Burnett 1985]).

I hope to publish a detailed survey of the surviving geometrical parts of the *Istikmāl* (i.e., the species 2–5) with an analysis of its sources and translations of a few historically important parts of the work. The many lacunae in the manuscript material would make an edition of all extant fragments at this time a frustrating task, even though part of the missing material can be retrieved by means of numerous badly legible references in the margin of K to nonextant sections and propositions. However, since four fragments of the *Istikmāl* have turned up in a short time, there may well be more. I would be most grateful for any relevant information, on manuscripts or otherwise, from readers of this article.

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NOTES

1. Djebbar [1984, 17, note 5] points out that the name is al-Mu'taman, not al-Mu'tamin, as earlier writers had assumed.

2. I have not been able to establish the exact date of the letter. The letter was written at the request of Abū Yaḥyā ibn Abī Zakariyya, ruler of Ceuta, who was related by marriage to the Banū (i.e., sons of) ʿAbd al-Muʾmin (the famous Almohad conqueror, who died in 1163) [Al-Maqqarī 1884–1885 2, 138]. I have no further information on Abū Yaḥyā.

3. [GAS V, 270, No. 13]; see for further references [Hogendijk 1985b]. Thābit's treatise consists of 10 propositions. Propositions 7 and 8 are combined into one proposition (No. 11) in the manuscripts K, C, and D.

4. ونحن نرشدكم إلى كتاب جمع فوائد الهندسة كلها باختصار التطويل وقصر الإيجاز في براهينه يتبين من براهين أشكاله علوم انطوت تحت كل برهان منها

فهو كتاب الاستكمال للمؤتمن بن هود ملك سرقسطة لا يعدله شي وجيز اللفظ نبيل البرهان

فإنه قسمه إلى خمسة أنواع النوع الأول في العدد ذكر فيه ما ذكره أقليدس في السابعة والثامنة والتاسعة من كتابه وما ذكره أيضا ثابت بن قرة في مقالته في الأعداد المتحابة

والنوع الثاني في خواص الخطوط والزوايا والسطوح من غير إضافة بعضها إلى بعض ذكر فيه ما ذكر أقليدس في الأولى والثانية والثالثة والرابعة وزاد عليه مسائل

والنوع الثالث ذكر فيه خواص الخطوط والزوايا والسطوح وعلوم كثيرة وذكر فيه ما ذكره أقليدس في المعروف بكتاب المعطيات ويعرف أيضا بكتاب المفرضات

والنوع الرابع ذكر فيه ما ذكره أقليدس في المقالة الحادية عشر

والنوع الخامس ذكر فيه إضافة المجسمات المستقيمة ⟨السطوح⟩ بعضها إلى بعض وإنما بينا هنا ما حوى عليه الكتاب لنذكر أن تسميته الاستكمال طابق مسماه

5.

النوع الأول من الجنس الأول من جنسي التعاليم الرياضية في علم خواص الأعداد مفردة ومضافة

النوع الثالث من الجنس الأول من جنسي التعاليم الرياضية في خواص الخطوط والزوايا والسطوح بحسب إضافة بعضها إلى بعض

النوع الأول في خواص الخطوط والزوايا والسطوح بحسب إضافة بعضها إلى بعض على جهة النسبة

النوع الثاني من النوع الثالث من الجنس الأول من جنسي التعاليم الرياضية في خواص الخطوط والزوايا والسطوح بحسب إضافة بعضه إلى بعض من جهة الاشتراك والتباين

النوع الثالث من النوع الثالث من الجنس الأول من جنسي التعاليم الرياضية في خواص الخطوط والزوايا والسطوح بحسب إضافة بعضه إلى بعض من جهة الفرض والوضع

النوع الثاني من النوع الرابع في خواص الأكر والقطوع الحادثة فيها من غير إضافة بعضها إلى بعض

النوع الثالث من النوع الرابع في قطوع الأساطين والمخروطات المستديرة

النوع الخامس من جنسي التعاليم الرياضية في إضافة المجسمات وسطوحها بعضها إلى بعض

النوع الأول في إضافة المجسمات المستقيمة السطوح وسطوحها بعضها إلى بعض (من ms.)

النوع (الفصل ms.) الثاني من النوع الخامس في إضافة مجسمات وسطوح المجسمات المستديرة بعضها إلى بعض

6. هذه الرسالة بعينها مسطورة في كتاب الاستكمال للمؤتمن خليفة أندلس وقد حررته وسميته الإكمال وهو الفصل الرابع من النوع الأول في العدديات منه حرره الفقير إلى الله تعالى محمد بن سرتاق المراغي

7. After the present article was submitted, Dr. Djebbar kindly informed me that he independently identified manuscript C as a fragment of the *Istikmāl*, and that he is working on an analysis of the first species on number theory.

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The Three Parts of the Dirichlet *Nachlass*

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The *Nachlass* of Peter Gustav Lejeune Dirichlet (1805–1859) has not yet been intensively studied by historians of mathematics, nor is it well known that its