Relay X Channels without Channel State Information at the Transmit Sides: Degrees of Freedom

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***Abstract*—This paper focuses on the two-user relay-assisted X channel with no channel state information (CSI) available at the transmitter side. Two relaying modes, namely half-duplex decode-and-forward (DF) and cognitive relays, are considered and the degrees of freedom (DoF) are characterized. It is shown that assisted by a half-duplex DF relay which is equipped with $2M$-antennas, the X channel with two M-antenna users has $\frac{4M}{3}$ DoF, which is achievable through interference alignment (IA). Furthermore, it is shown that in this channel, M-antenna cognitive relay (with non-causal access to information streams) provides $2M$ DoF using interference cancellation (IC) technique. In this setting, IC outperforms interference alignment in the cognitive relay mode, since the latter achieves $\frac{4M}{3}$ DoF.**

***Index Terms*—degrees of freedom; relay X channel; decode-and-forward relay; cognitive relay.**

I. Introduction

Driven by the need for higher spectral efficiency, multiuser communication in which multiple users wish to communicate with multiple receivers in a shared wireless medium has a pivotal role in wireless communication. In such networks, the communication from each transmitter to its respective receiver causes disruptive interference on the unintended receivers. In traditional interference management approaches, interference is either treated as noise (weak interference regimes), decoded and removed (strong interference regimes), or partially decoded and removed (e.g., in the Han-Kobayashi approach [1]).

Motivated by operating wireless network as close as possible to their optimal performance, recently advanced interference management approaches are proposed that can achieve higher spectral efficiencies (rate) over wireless networks. Specifically, interference alignment (IA) is a strong approach that divides the signaling space into two spaces, one being reserved for communicating the desired signal and the other being allocated to the interfering signals [2], [3].

Performing IA, however, requires global and perfect knowledge of the CSI at the transmit side, which is a condition not always possible to meet in practice. Lack of perfect channels state information at the transmit side (CSIT) can potentially make interference alignment unfeasible. Certain conditions under which interference alignment is still feasible without perfect CSIT are studied in [4], [5].

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As remedy to cope with lack of CSIT, relaying techniques can be leveraged to facilitate interference management. For instance, while for X channel with *perfect* CSIT including a relay does *not* improve the achievable degrees of freedom (DoF) [6], [7], for the same X channel *without* CSIT including relays provides DoF gains. More specifically, the work in [8] has studied the gains of including a half-duplex amplify-and-forward (AF) relay in the two-user X channel without CSIT, where it is shown that for single-antenna X channel without CSIT, the DoF is $\frac{4}{3}$. This is in contrast to the same X channel *without* relay, for which the DoF is 1 [9], and establishes the DoF gain of deploying relays in X channel without CSIT. Cognitive message sharing is also found to improve the DoF in MIMO X channel [2]. The gains of cognition in transmitters and relays for interference channels have been studied in [9] and [10], where it is shown that such cognition improves the achievable DoF.

Motivated by the premise that adding relays can recover the lost DoF in the X channel due to lack of perfect CSIT, in this paper, we investigate the impacts of two relaying modes in the X channel with no CSIT. In the first scenario, we consider a $2M$-antenna half-duplex relay and show that the decode-and-forward (DF) strategy can provide $\frac{4M}{3}$ DoF in M-antenna setting in three channel extensions in time. In the second scenario, we consider a cognitive relay for this setting with non-causal access to the information streams and show that interference cancellation (IC) can provide $2M$ DoF through asymmetric complex signaling, while via interference alignment the achievable DoF is $\frac{4M}{3}$. The advantage of using asymmetric complex signaling is that DoF of $2M$ is achieved without any channel extension in time and it takes only one time slot.

The rest of the paper is organized as follows: In section II, the system model of X channel with both DF and cognitive relays have been presented. A DoF analysis of theses channels has been presented in section III and IV, respectively. Finally we conclude with section V.

Notations: We use lower case bold face letters to denote vectors and upper bold case letters to denote matrices. Non bold capital letters denote scalars and $(.)^T$ denotes transpose.

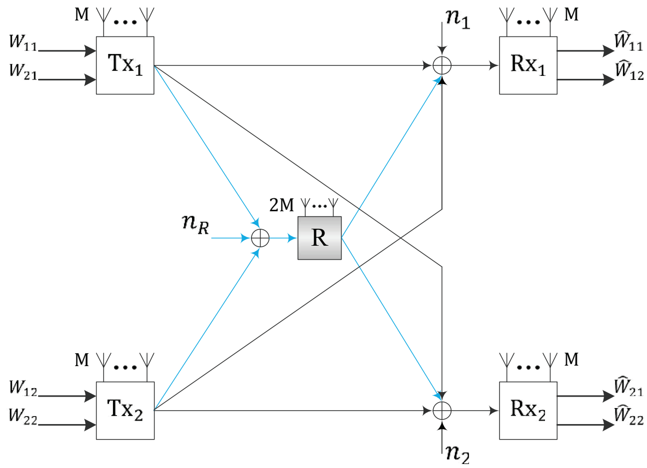

Fig. 1. X Channel with a DF Relay

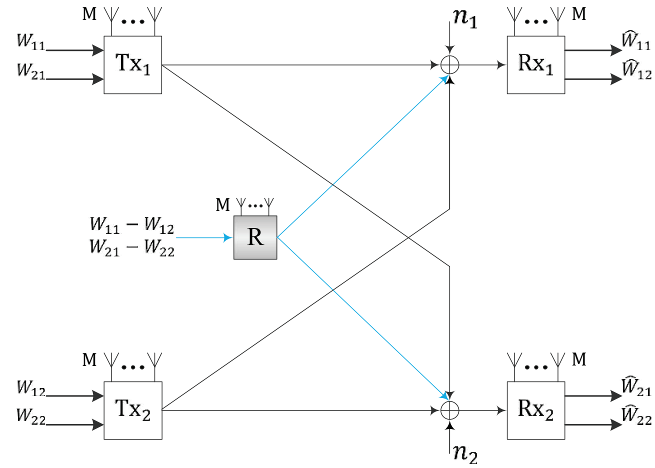

Fig. 2. X channel with a cognitive relay

II. SYSTEM MODEL

Consider a two user Gaussian X channel, consisting of two transmitters and two receivers, where each transmitter sends one independent message to each receiver. The communication between the transmitters and receivers is assisted by a relay in two different modes, namely DF and cognitive, as depicted in Figures 1 and 2, respectively. We assume that the transmitters have no CSI while the relay and receivers have full CSI. Throughout this paper, it is assumed that channel matrices are generated from a continuous distribution, so that the channels are non-degenerate with probability one.

A. DF Relay

In the first scenario, we assume that the relay is operating in the DF mode. In this setting, we assume that the transmitters and receivers are equipped with M antennas and the relay has $2M$ antennas. The relay operates in the half-duplex DF mode. We define $\boldsymbol{x}_i(t) \in \mathbb{C}^M$ and $\boldsymbol{x}_R(t) \in \mathbb{C}^{2M}$ as the signals transmitted by transmitter TX_i for $i = 1, 2$, and the relay, respectively, at time t, and accordingly define $\boldsymbol{y}_i(t) \in \mathbb{C}^M$ and $\boldsymbol{y}_R(t) \in \mathbb{C}^{2M}$ as the signals received by receiver RX_i for $i = 1, 2$, and the relay, respectively, at time t.

We define $\boldsymbol{H}_{ij} \in \mathbb{C}^{M\times M}$ as the channel between transmitter j and receiver i, and defining $\boldsymbol{H}_{iR} \in \mathbb{C}^{M\times 2M}$ and $\boldsymbol{H}_{Rj} \in \mathbb{C}^{2M\times M}$ as the channel between the relay to receiver i, and the channel between transmitter j to the relay, respectively. In the half-duplex relaying mode, the transmissions are accomplished in two phases. At time t, if the relay is in the receiving mode (i.e., silent and listening to the transmitters), then the signals received by the relay and the receivers are given by

$$\boldsymbol{y}_1(t) = \boldsymbol{H}_{11}\boldsymbol{x}_1(t) + \boldsymbol{H}_{12}\boldsymbol{x}_2(t) + \boldsymbol{n}_1(t), \quad (1)$$

$$\boldsymbol{y}_2(t) = \boldsymbol{H}_{21}\boldsymbol{x}_1(t) + \boldsymbol{H}_{22}\boldsymbol{x}_2(t) + \boldsymbol{n}_2(t), \quad (2)$$

$$\boldsymbol{y}_R(t) = \boldsymbol{H}_{R1}\boldsymbol{x}_1(t) + \boldsymbol{H}_{R2}\boldsymbol{x}_2(t) + \boldsymbol{n}_R(t), \quad (3)$$

where $\boldsymbol{n}_i(t)$ for $i = 1, 2$ at the ith user and $\boldsymbol{n}_R(t)$ at the relay account for the additive white Gaussian noise (AWGN) distributed as $\mathcal{N}(0, \sigma^2)$[1]. Similarly, if relay is in the transmission mode at time t, the received signals at the receivers are given by

$$\boldsymbol{y}_1(t) = \boldsymbol{H}_{11}\boldsymbol{x}_1(t) + \boldsymbol{H}_{12}\boldsymbol{x}_2(t) + \boldsymbol{H}_{1R}\boldsymbol{x}_R(t) + \boldsymbol{n}_1(t), \quad (4)$$

$$\boldsymbol{y}_2(t) = \boldsymbol{H}_{21}\boldsymbol{x}_1(t) + \boldsymbol{H}_{22}\boldsymbol{x}_2(t) + \boldsymbol{H}_{2R}\boldsymbol{x}_R(t) + \boldsymbol{n}_2(t). \quad (5)$$

B. Cognitive Relay

In the second setting, we assume that the relay is cognitive, i.e, it has non-causal access to the information streams of both users, and it is constantly in the transmission mode. In this setting we assume that all nodes are equipped with M antennas. The received signal at the receiver $k = 1, 2$ is given by

$$\boldsymbol{y}_k(t) = \boldsymbol{H}_{k1}(t)\boldsymbol{x}_1(t) + \boldsymbol{H}_{k2}(t)\boldsymbol{x}_2(t) + \boldsymbol{H}_{kR}(t)\boldsymbol{x}_R(t) + \boldsymbol{n}_k(t). \quad (6)$$

The channels follow a slow-fading model, in which within one time slot through transmission, the channels remain unchanged and then change to independent states in the next slot. We denote the average transmission power of all transmitters by P, i.e., $\mathbb{E}(\|\boldsymbol{x}_k\|^2) \leq P$. We define the rate tuple

$$\boldsymbol{R}(P) \triangleq [R_{11}(P), R_{12}(P), R_{21}(P), R_{22}(P)] \quad (7)$$

as a vector of simultaneously achievable rates for the given power constraint P, where R_{ij} is the rate of message that is transmitted from transmitter j to receiver i. Accordingly, $\mathcal{C}(P)$ is defined as the set of all such achievable rate vectors. Hence, the sum-rate capacity, i.e., $R_{\text{sum}}(P)$ and DoF in the corresponding channel are given by

[1] $\mathcal{N}(a, b)$ denotes a complex Gaussian distribution with mean a and variance b.

$$R_{\text{sum}}(P) = \max_{\boldsymbol{R}(P)\in\mathcal{C}(P)} (\sum_{i=1}^{2}\sum_{j=1}^{2} R_{ij}(P)), \quad (8)$$

$$\text{and} \qquad \text{DoF} = \lim_{P\to\infty} \frac{R_{\text{sum}}(P)}{\log(P)}, \quad (9)$$

that is further examined.

III. DF RELAY

In this section, the relay-aided X channel where the relay operates under the DF mode is considered. It is shown that when the transmitters do not have access to the CSI, IA can be facilitated by the relay via designing beamforming at the transmitters and the relay. We focus on a setting in which every transmitter and receiver have M antennas and the relay is equipped with $2M$ antennas. The main result of this setting is stated in the following theorem.

Theorem 1. *In a two-user relay X channel with M antennas at the transmitters and receivers and $2M$ antennas at the half-duplex* DF *relay, with no CSIT and full CSI at the relay and receivers, the achieved* DoF *is* $\frac{4M}{3}$.

Proof: We consider a transmission scheme that takes place over three phases (time slots). Let $\boldsymbol{d}_{ij}$ denotes the $M\times 1$ signal representing the message $\boldsymbol{w}_{ij}$, which is the message of transmitter j intended to be transmitted to receiver i. In the first phase, transmitters 1 and 2 send messages $\boldsymbol{w}_{11}$ and $\boldsymbol{w}_{12}$, respectively

$$\boldsymbol{x}_1(1) = \boldsymbol{d}_{11} \quad \text{and} \quad \boldsymbol{x}_2(1) = \boldsymbol{d}_{12}. \quad (10)$$

In the second phase, the signals transmitted by transmitters 1 and 2 are

$$\boldsymbol{x}_1(2) = \boldsymbol{d}_{21} \quad \text{and} \quad \boldsymbol{x}_2(2) = \boldsymbol{d}_{22}. \quad (11)$$

By invoking the signal models in (1)-(3) and noting that the transmitters and receivers are equipped with M antennas, the channel outputs in the ith phase for $i = 1, 2$ are given by

$$\boldsymbol{y}_1(i) = \boldsymbol{H}_{11}(i)\boldsymbol{d}_{i1} + \boldsymbol{H}_{12}(i)\boldsymbol{d}_{i2}, \quad (12)$$

$$\boldsymbol{y}_2(i) = \boldsymbol{H}_{21}(i)\boldsymbol{d}_{i1} + \boldsymbol{H}_{22}(i)\boldsymbol{d}_{i2}, \quad (13)$$

$$\boldsymbol{y}_R(i) = \boldsymbol{H}_{R1}(i)\boldsymbol{d}_{i1} + \boldsymbol{H}_{R2}(i)\boldsymbol{d}_{i2}, \quad (14)$$

where the noise terms can be ignored as the focus is on the high power regime (i.e., $P\to\infty$). Based on expressions stated above, receiver 1 gets a linearly combination of its desired messages (which are messages $\boldsymbol{w}_{11}$ and $\boldsymbol{w}_{12}$) in the first phase, while the signal received in receiver 2 is all interference. Transmission in the second phase is vice versa, receiver 2 gets a linearly combination of its desired messages (which are messages $\boldsymbol{w}_{21}$ and $\boldsymbol{w}_{22}$), while receiver 1 gets interference.

It is noteworthy that since the relay has $2M$ antennas, it can decode all messages that it receives during each phase. Finally in the third phase, the transmitters are silent and the relay constructs and sends the combination of all messages as follows

$$\boldsymbol{x}_R(3) = \begin{bmatrix} \boldsymbol{U}_1\boldsymbol{d}_{11} + \boldsymbol{U}_2\boldsymbol{d}_{22} \\ \boldsymbol{U}_3\boldsymbol{d}_{21} + \boldsymbol{U}_4\boldsymbol{d}_{12} \end{bmatrix}, \quad (15)$$

where $\boldsymbol{U}_i$ is $M\times M$ beamforming matrix. The beamforming matrices at the relay are determined, such that the received signal at each receiver provides excessive freedom that enables it decoding the desired messages.

Denoting the channel between the first M antennas of the relay and receiver 1 by $\boldsymbol{H}_{1R,1}$ and the channel between the second M antennas of the relay and receiver 1 by $\boldsymbol{H}_{1R,2}$, we can show the signals received at the receiver 1 in three phases as (16) that is shown at the top of the next page. The whole signal space is $3M$-dimensional. We need to make sure interference is confined in a M-dimensional subspace, so that the signal can be recovered. To this end, we need to have following condition

$$\big[\boldsymbol{H}_{1R,2}(3)\boldsymbol{U}_3, \boldsymbol{H}_{1R,1}(3)\boldsymbol{U}_2\big] \in \mathcal{R}(\big[\boldsymbol{H}_{11}(2), \boldsymbol{H}_{12}(2)\big]), \quad (17)$$

where $\mathcal{R}(.)$ denotes the row space of its argument matrix. Determining $\boldsymbol{U}_2$ and $\boldsymbol{U}_3$ as

$$\boldsymbol{U}_2 = \boldsymbol{H}_{1R,1}^{-1}(3)\boldsymbol{H}_{12}(2), \quad (18)$$

$$\boldsymbol{U}_3 = \boldsymbol{H}_{1R,2}^{-1}(3)\boldsymbol{H}_{11}(2), \quad (19)$$

satisfies the condition. Similarly $\boldsymbol{U}_1$ and $\boldsymbol{U}_4$ can be determined as

$$\boldsymbol{U}_1 = \boldsymbol{H}_{2R,1}^{-1}(3)\boldsymbol{H}_{21}(1), \quad (20)$$

$$\boldsymbol{U}_4 = \boldsymbol{H}_{2R,2}^{-1}(3)\boldsymbol{H}_{22}(1). \quad (21)$$

Therefore, interference terms at the receivers occupy only M dimensions. Since the channels at different time slots are independent and every channel matrix has full rank almost surely, one can simply show that the desired signal space has full rank of $2M$ and is linearly indepent of the interference space. Therefore the intended messages can be decoded correctly and DoF $= \frac{4M}{3}$ can be achieved ■

Remark 1. *We have shown that achievable* DoF *in the* MIMO *X channel without* CSIT *assisted by 2M-antenna* DF *relay is* $\frac{4M}{3}$, *which is the maximum* DoF *for X channel with a relay, while its all nodes have full* CSI *that is investigated in [7].*

IV. COGNITIVE RELAY

In this section we focus on the X channel enabled with a cognitive relay. We study the achievable DoF under two different interference management schemes, namely, IC and IA. Similar to the previous setting (DF relay), in this section we demonstrate that when the transmitters do not have access to the CSI, including a cognitive relay can still facilitate interference management effectively when the transmitters have full CSI.

A. Interference Cancellation

In this section, the IC strategy in the relay X channel of interest is investigated which relies on the notation of asymmetric complex signaling. We analyze the one-antenna setting for the first and show that it achieves 2 DoF enabled by a one-antenna cognitive relay. Then we extend the results for general case with M-antenna users and relay.

Theorem 2. *In the two-user X channel with one antenna at the transmitters, receivers and cognitive relay, with no CSIT*

$$\begin{bmatrix} \boldsymbol{y}_1(1) \\ \boldsymbol{y}_1(2) \\ \boldsymbol{y}_1(3) \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{11}(1) & \boldsymbol{H}_{12}(1) & 0 & 0 \\ 0 & 0 & \boldsymbol{H}_{11}(2) & \boldsymbol{H}_{12}(2) \\ \boldsymbol{H}_{1R,1}(3)\boldsymbol{U}_1 & \boldsymbol{H}_{1R,2}(3)\boldsymbol{U}_4 & \boldsymbol{H}_{1R,2}(3)\boldsymbol{U}_3 & \boldsymbol{H}_{1R,1}(3)\boldsymbol{U}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{d}_{11} \\ \boldsymbol{d}_{12} \\ \boldsymbol{d}_{21} \\ \boldsymbol{d}_{22} \end{bmatrix} \tag{16}$$

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and full CSI at the relay and receivers, the DoF *achieved by* IC *is* 2*.*

Proof: To show the achievability of 2 DoF, we consider sending four messages w_{11}, w_{21}, w_{12}, and w_{22} using two dimensions. Specifically, each transmitter sends its signal in the complex form where one message is embedded in the real part and the other one in the imaginary part. Hence, the transmitted signals are $x_1 = d_{11} + jd_{21}$ and $x_2 = d_{12} + jd_{22}$ and the relay sends a linear combination of four messages in the complex form by using the weighting coefficients u_1, u_2, u_3, and u_4, i.e., $x_R = u_1 d_{11} + u_2 d_{22} + j(u_3 d_{21} + u_4 d_{12})$.

For formalizing the effects of the complex channels on these complex signals, we denote the real and imaginary parts of the channel coefficient, i.e., h_{kl} by h_{kl}^r and h_{kl}^i, respectively, i.e., $h_{kl} = h_{kl}^r + jh_{kl}^i$ for $k, l = \{1, 2\}$. The signals at the receivers are

$$\begin{aligned} y_1 &= (h_{11}^r + jh_{11}^i)(d_{11} + jd_{21}) + (h_{12}^r + jh_{12}^i)(d_{12} + jd_{22}) \\ &\quad + (h_{1R}^r + jh_{1R}^i)(u_1 d_{11} + u_2 d_{22} + j(u_3 d_{21} + u_4 d_{12})), \end{aligned} \tag{22}$$

and
$$\begin{aligned} y_2 &= (h_{21}^r + jh_{21}^i)(d_{11} + jd_{21}) + (h_{22}^r + jh_{22}^i)(d_{12} + jd_{22}) \\ &\quad + (h_{2R}^r + jh_{2R}^i)(u_1 d_{11} + u_2 d_{22} + j(u_3 d_{21} + u_4 d_{12})). \end{aligned} \tag{23}$$

Hence, the real and imaginary parts of the received signal at the receiver k for $k = 1, 2$ are

$$\begin{aligned} y_k^r &= (h_{k1}^r + h_{kR}^r u_1)d_{11} + (h_{k2}^r - h_{kR}^i u_4)d_{12} \\ &\quad + (-h_{k1}^i - h_{kR}^i u_3)d_{21} + (-h_{k2}^i + h_{kR}^r u_2)d_{22}, \end{aligned} \tag{24}$$

and
$$\begin{aligned} y_k^i &= (h_{k1}^i + h_{kR}^i u_1)d_{11} + (h_{k2}^i + h_{kR}^r u_4)d_{12} \\ &\quad + (h_{k1}^r + h_{kR}^r u_3)d_{21} + (h_{k2}^r + h_{kR}^i u_2)d_{22}. \end{aligned} \tag{25}$$

In order to cancel interference in both dimensions at each receiver, the following equations should hold at the first receiver

$$(-h_{11}^i - h_{1R}^i u_3)d_{21} + (-h_{12}^i + h_{1R}^r u_2)d_{22} = 0, \tag{26}$$

and
$$(h_{11}^r + h_{1R}^r u_3)d_{21} + (h_{12}^r + h_{1R}^i u_2)d_{22} = 0. \tag{27}$$

Similarly, the following equations should hold at the second receiver

$$(h_{21}^r + h_{2R}^r u_1)d_{11} + (h_{22}^r - h_{2R}^i u_4)d_{12} = 0, \tag{28}$$

and
$$(h_{21}^i + h_{2R}^i u_1)d_{11} + (h_{22}^i + h_{2R}^r u_4)d_{12} = 0. \tag{29}$$

Also it should be noted that the relay can satisfy these conditions with the information of channel coefficients and messages of the transmitters, d_{ij} for $i, j = \{1, 2\}$. The equations (26)-(29) can be written as,

$$\boldsymbol{A} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \boldsymbol{b}, \quad \text{and} \quad \boldsymbol{C} \begin{bmatrix} u_1 \\ u_4 \end{bmatrix} = \boldsymbol{d}. \tag{30}$$

where

$$\boldsymbol{A} = \begin{bmatrix} d_{22}h_{1R}^r & -d_{21}h_{1R}^i \\ d_{22}h_{1R}^i & d_{21}h_{1R}^r \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} d_{22}h_{12}^i + d_{21}h_{11}^i \\ -d_{22}h_{12}^r - d_{21}h_{11}^r \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} d_{11}h_{2R}^r & -d_{12}h_{2R}^i \\ d_{11}h_{2R}^i & d_{12}h_{2R}^r \end{bmatrix}, \quad \boldsymbol{d} = \begin{bmatrix} -d_{11}h_{21}^r - d_{12}h_{22}^r \\ -d_{11}h_{21}^i - d_{12}h_{22}^i \end{bmatrix}.$$

The optimum coefficients u_p for $p = 1, 2, 3, 4$ at the relay are determined uniquely if matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ have full rank. The channel matrices are generated from a continuous probability distribution as it is assumed, so that almost surely, any matrix composed of the channel coefficients will have full rank. The matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ can be written as,

$$\boldsymbol{A} = \begin{bmatrix} h_{1R}^r & -h_{1R}^i \\ h_{1R}^i & h_{1R}^r \end{bmatrix} \begin{bmatrix} d_{22} & 0 \\ 0 & d_{21} \end{bmatrix}, \tag{31}$$

$$\boldsymbol{C} = \begin{bmatrix} h_{2R}^r & -h_{2R}^i \\ h_{2R}^i & h_{2R}^r \end{bmatrix} \begin{bmatrix} d_{11} & 0 \\ 0 & d_{12} \end{bmatrix}. \tag{32}$$

The first matrix in (31) and (32) has full rank and multiplication with a nonsingular matrix (second matrix in (31) and (32) respectively) does not affect the rank of a matrix. Therefore, the matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ have full rank.

Finally, by substituting the optimum values of the relay coefficients in (24) and (25), we have,

$$y_1^r = (h_{11}^r + h_{1R}^r u_1^{opt})d_{11} + (h_{12}^r - h_{1R}^i u_4^{opt})d_{12}, \tag{33}$$

$$y_1^i = (h_{11}^i + h_{1R}^i u_1^{opt})d_{11} + (h_{12}^i - h_{1R}^r u_4^{opt})d_{12}, \tag{34}$$

$$y_2^r = (-h_{21}^i - h_{2R}^i u_3^{opt})d_{21} + (-h_{22}^i + h_{2R}^r u_2^{opt})d_{22}, \tag{35}$$

$$y_2^i = (h_{21}^r + h_{2R}^r u_3^{opt})d_{21} + (h_{22}^r + h_{2R}^i u_2^{opt})d_{22}. \tag{36}$$

where the superscript *opt* denotes the optimum value. As we have seen from the above equations, four messages can be transmitted and detected correctly using two dimensions, so that $\text{DoF} = \frac{4}{2} = 2$ is achievable. ■

Based on the insights gained from the single-antenna setting, next we generalize the results for the setting in which the users and relay have M antennas. For this purpose, we denote the $M \times 1$ signal transmitted by transmitter j and intended to receiver i by $\boldsymbol{d}_{ij}$ for $\{i, j\} = \{1, 2\}$. The transmit signals are similar to the single-antenna setting, i.e., $\boldsymbol{x}_1 = \boldsymbol{d}_{11} + j\boldsymbol{d}_{21}$, $\boldsymbol{x}_2 = \boldsymbol{d}_{12} + j\boldsymbol{d}_{22}$. The relay sends the messages as $\boldsymbol{x}_R = U_1\boldsymbol{d}_{11} + U_2\boldsymbol{d}_{22} + j(U_3\boldsymbol{d}_{21} + U_4\boldsymbol{d}_{12})$ designing the beamforming $M \times M$ diagonal matrices U_i. These beamforming matrices are designed to cancel interference in both real and imaginary parts of received signals. To this end, we need to have the following equalities

$$\begin{bmatrix} \boldsymbol{H}_{1R}^r & -\boldsymbol{H}_{1R}^i \\ \boldsymbol{H}_{1R}^i & \boldsymbol{H}_{1R}^r \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_2\boldsymbol{d}_{22} \\ \boldsymbol{U}_3\boldsymbol{d}_{21} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{12}^i\boldsymbol{d}_{22} + \boldsymbol{H}_{11}^i\boldsymbol{d}_{21} \\ -\boldsymbol{H}_{12}^r\boldsymbol{d}_{22} - \boldsymbol{H}_{11}^r\boldsymbol{d}_{21} \end{bmatrix}, \tag{37}$$

$$\begin{bmatrix} \boldsymbol{H}_{2R}^{r} & -\boldsymbol{H}_{2R}^{i} \\ \boldsymbol{H}_{2R}^{i} & \boldsymbol{H}_{2R}^{r} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_1\boldsymbol{d}_{11} \\ \boldsymbol{U}_4\boldsymbol{d}_{12} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{H}_{21}^{r}\boldsymbol{d}_{11} - \boldsymbol{H}_{22}^{r}\boldsymbol{d}_{12} \\ -\boldsymbol{H}_{21}^{i}\boldsymbol{d}_{11} - \boldsymbol{H}_{22}^{r}\boldsymbol{d}_{12} \end{bmatrix}. \tag{38}$$

Similar to the single antenna case, it can be shown that the above matrices have full rank, so that it is possible to decode $4M$ messages using 2 dimensions and $\mathrm{DoF} = \frac{4\mathrm{M}}{2} = 2\mathrm{M}$ would be achievable.

B. Interference Alignment

We continue with implementing an IA-based interference management scheme. The main result of this section is stated in the following theorem.

Theorem 3. *In the two-user X channel with M antennas at the transmitters, receivers and the cognitive relay, with no* CSIT *and full* CSI *at the relay and receivers, the* DoF *achieved by interference alignment is $\frac{4M}{3}$.*

Proof: To show the achievability of $\frac{4M}{3}$ DoF, we consider sending the signals during three time slots. For this purpose, in the first phase, each transmitter send the $M \times 1$ message is treated as intended at receiver 1 and interference at receiver 2. In the second phase, each transmitter send the $M \times 1$ message is treated as intended at receiver 2 and interference at receiver 1. Accordingly we have

$$\boldsymbol{x}_1(1) = \boldsymbol{d}_{11}, \quad \boldsymbol{x}_2(1) = \boldsymbol{d}_{12}, \quad \boldsymbol{x}_1(2) = \boldsymbol{d}_{21}, \quad \boldsymbol{x}_2(2) = \boldsymbol{d}_{22}. \tag{39}$$

Therefore the received signals at the receivers in the ith phase for $i = 1, 2$ are given by

$$\boldsymbol{y}_1(i) = \boldsymbol{H}_{11}(i)\boldsymbol{d}_{i1} + \boldsymbol{H}_{12}(i)\boldsymbol{d}_{i2}, \tag{40}$$
$$\boldsymbol{y}_2(i) = \boldsymbol{H}_{21}(i)\boldsymbol{d}_{i1} + \boldsymbol{H}_{22}(i)\boldsymbol{d}_{i2}, \tag{41}$$
$$\boldsymbol{y}_R(i) = \boldsymbol{H}_{R1}(i)\boldsymbol{d}_{i1} + \boldsymbol{H}_{R2}(i)\boldsymbol{d}_{i2}. \tag{42}$$

Finally, in the third time slot, the beamforming is designed at the relay to provide an additional DoF for both receivers to decode their intended messages correctly. To this end, the relay sends a linear combination of all messages as

$$\boldsymbol{x}_R = \boldsymbol{U}_1\boldsymbol{d}_{11} + \boldsymbol{U}_2\boldsymbol{d}_{22} + \boldsymbol{U}_3\boldsymbol{d}_{21} + \boldsymbol{U}_4\boldsymbol{d}_{12}, \tag{43}$$

where $\boldsymbol{U}_i$ is $M \times M$ beamforming matrix. Same as what we have shown in section III, signal received at each receiver in three phases would be recovered if interference is confined in a M-dimensional subspace of $3M$-dimensional signal space, with the difference that relay has M antennas in this setting. To this end, in order to determine $\boldsymbol{U}_2$ and $\boldsymbol{U}_3$ we should have the following condition

$$\left[\boldsymbol{H}_{1R}(3)\boldsymbol{U}_3, \boldsymbol{H}_{1R}(3)\boldsymbol{U}_2\right] \in \mathcal{R}(\left[\boldsymbol{H}_{11}(2), \boldsymbol{H}_{12}(2)\right]). \tag{44}$$

As a result we have

$$\boldsymbol{U}_2 = \boldsymbol{H}_{1R}^{-1}(3)\boldsymbol{H}_{12}(2), \quad \text{and} \quad \boldsymbol{U}_3 = \boldsymbol{H}_{1R}^{-1}(3)\boldsymbol{H}_{11}(2). \tag{45}$$

Similarly $\boldsymbol{U}_1$ and $\boldsymbol{U}_4$ get the following values

$$\boldsymbol{U}_1 = \boldsymbol{H}_{2R}^{-1}(3)\boldsymbol{H}_{21}(1), \quad \text{and} \quad \boldsymbol{U}_4 = \boldsymbol{H}_{2R}^{-1}(3)\boldsymbol{H}_{22}(1). \tag{46}$$

By considering these values for the relay beamforming, it is possible to correctly detect $4M$ messages at the receivers over three time slots, which yields $\frac{4M}{3}$ DoF. ■

As we have seen above, cognitive relay provides $2M$ DoF using IC technique while it provides $\frac{4M}{3}$ DoF applying IA in the M-antenna setting. Therefore, in the X channel with cognitive relay, the performance of IC is better than IA from DoF point of view.

V. Conclusion

DoF for the relay X channel with no transmit side CSI is investigated. We showed that in this channel, DF relay makes it possible to access full DoF. For M-antennas users, it provides $\frac{4M}{3}$ DoF by using the IA technique which is equipped with $2M$ antennas. We have also shown that when the non-causal access to the information streams is possible for the relay, the cognition compensates for the lack of CSIT. In this mode, cognitive M-antenna relay can apply IC strategy, and thus, $2M$ DoF is achievable for the X channel with M-antenna users. As it can be seen, the achievable DoF is higher than the optimum DoF of the X channel with full CSI, i.e., $\frac{4M}{3}$. With the similar situation, $\frac{4M}{3}$ DoF is achieved via IA by cognitive relay, that shows the IC technique outperforms IA from DoF point of view in this particular setting.

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