Mathematics Level D: Lesson 2 – Representations of a Line

Targeted Student Outcomes

- Students graph a line specified by a linear function.
- Students graph a line specified by an initial value and rate of change of a function and construct the linear function by interpreting the graph.
- Students graph a line specified by two points of a linear relationship and provide the linear function.

Targeted Content Standards:

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*,*y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Lesson Notes

Linear functions are defined by the equations of a line. This lesson reviews students' work with the representation of a line, and in particular, the determination of the rate of change and the initial value of a linear function from two points on the graph or from the equation of the line defined by the function in the form $y = mx + b$, or an equivalent form. Students interpret the rate of change and the initial value based on the graph of the equation of the line, and also in the context of the variables.

Classwork

Example 1 (10 minutes): Rate of Change and Initial Value given in the Context of the Problem

Here, verbal information giving an initial value and a rate of change is translated into a function and its graph. Work through the example as a class.

In part (b), explain why the value 0.5 given in the question is the rate of change.

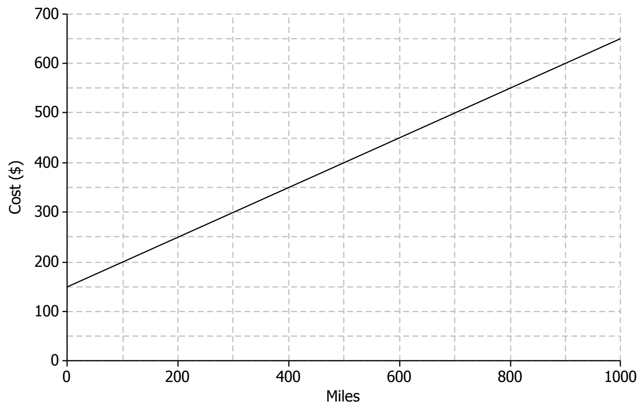
- It would be a good idea to show them this on the graph, demonstrating that each increase of 1 unit for m (miles) results in an increase for 0.5 in the C (cost). An increase of $1{,}000$ for m will result in an increase of 500 units for C.
- Point out that if the question stated that each mile driven *reduced* the cost by $\$0.50$ then the line would have negative slope.

It is important for students to understand that when the scales on the two axes are different, the rate of change cannot be used to plot points by simply counting the squares. Encourage the students to use the rate of change by holding on to the idea of increasing the variable shown on the horizontal axis and showing the resulting increase in the variable shown on the vertical axis (as explained in the previous paragraph).

Given the rate of change and initial value, the linear function can be written in slope-intercept form ($y = mx + b$) or an equivalent form, such as $y = a + bx$. Students should pay careful attention to variables presented in the problem; m and C are used in place of x and y.

Example 1: Rate of Change and Initial Value given in the Context of the Problem

A truck rental company charges a $\$150$ rental fee, in addition to a charge of $\$0.50$ per mile driven. In this problem, you will graph the linear function relating the total cost of the rental in dollars, C, to the number of miles driven, m, on the axes below.



a. **If the truck is driven zero miles, what will be the cost to the customer? How will this be shown on the graph?**

$\$150$, shown as the point $(0, 150)$. This is the initial value. Some students might say, "b." Help them to use the language, "initial value".

b. **What is the rate of change that relates cost to number of miles driven? Explain what it means within the context of the problem.**

The rate of change is 0.5. It means that the cost will increase by $\$0.50$ for every mile driven.

c. **On the axes given, graph the line that relates C to m.**

Students can plot the initial value $(0, 150)$, and then use the rate of change to identify additional points as needed. A $1,000$ unit increase in m results in a 500 unit increase for C, so another point on the line is $(1,000, 650)$.

d. **Write the linear function that models the relationship between number of miles driven and total rental cost?**

$C = 0.5m + 150$

Exercises 1–5 (10 minutes)

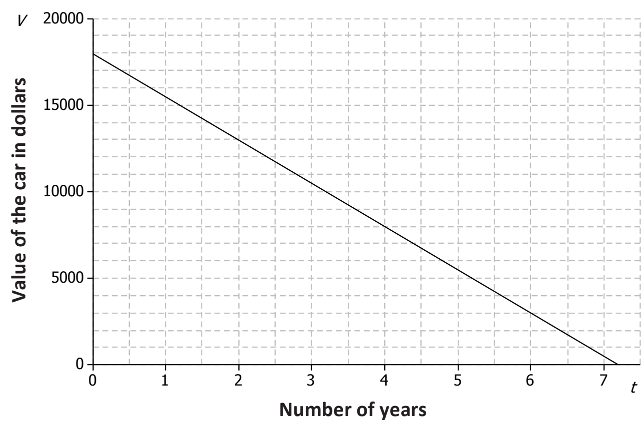
Here, students have an opportunity to practice the ideas to which they have just been introduced. Let students work independently on these exercises. Then discuss and confirm answers as a class.

Exercise 3, part (c) provides an excellent opportunity for discussion about the model and whether or not it continues to make sense over time.

- In Exercise 3, you found that the price of the car in the 7th year was less than $600. Does this make sense in general?
  - *Not really.*
- Under what conditions might the car be worth less than $600 after seven years?
  - *The car may have been in an accident.*

Exercises 1–5

Jenna bought a used car for $\$18,000$. She has been told that the value of the car is likely to decrease by $\$2,500$ for each year that she owns the car. Let the value of the car in dollars be V and the number of years Jenna has owned the car be t.



1. **What is the value of the car when $t = 0$? Show this point on the graph.**

   $\$18,000$. *Shown by the point* $(0, 18,000)$.

2. **What is the rate of change that relates V to t? (Hint: Is it positive or negative? How can you tell?)**

   $-2,500$. *The rate of change is negative because the value of the car is decreasing.*

3. **Find the value of the car when**

   a. $t = 1$: $18,000 - 2,500 = \$15,500$

   b. $t = 2$: $18,000 - 2(2,500) = \$13,000$

   c. $t = 7$: $18,000 - 7(2,500) = \$500$

4. **Plot the points for the values you found in Exercise 3, and draw the line (using a straight-edge) that passes through those points.**

   See graph above.

5. **Write the linear function that models the relationship between the number of years Jenna has owned the car and the value of the car.**

   $V = 18,000 - 2,500t$ or $V = -2,500t + 18,000$

Exercises 6–11 (10 minutes)

Here, in the context of the pricing of a book, students are given two points on the graph and are expected to draw the graph, determine the rate of change, and answer questions by referring to the graph.

Point out that the horizontal axis does not start at 0. Ask students:

- Why do you think the first value is at 15?
  - *The online bookseller may not sell the book for less than* $15.

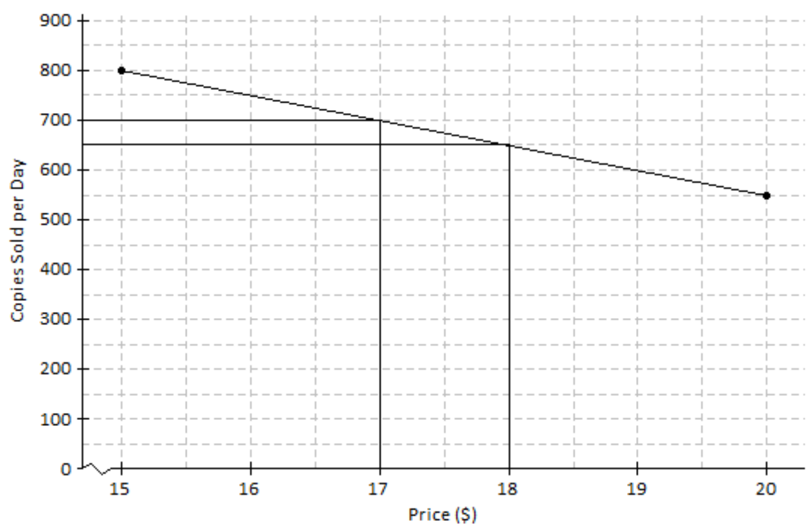
In Exercise 8, students are asked to find the rate of change; it might be worthwhile to check that they are using the scales on the axes, not purely counting squares.

For Exercises 9 and 10, encourage the students to show their work by drawing vertical and horizontal lines on the graph, as shown in the sample student answers below.

Let students work with a partner. Then discuss and confirm answers as a class.

Exercises 6–10

An online bookseller has a new book in print. The company estimates that if the book is priced at $\$15$ then 800 copies of the book will be sold per day, and if the book is priced at $\$20$ then 550 copies of the book will be sold per day.



6. **Identify the ordered pairs given in the problem. Then plot both on the graph.**

   The ordered pairs are $(15, 800)$ *and* $(20, 550)$. *See graph above.*

7. **Assume that the relationship between the number of books sold and the price is linear. (In other words, assume that the graph is a straight line.) Using a straight-edge, draw the line that passes through the two points.**

8. **What is the rate of change relating number of copies sold to price?**

   Between the points $(15, 800)$ *and* $(20, 550)$ *the run is* 5 *and the rise is* $-(800 - 550) = -250$. *So, the rate of change is* $\frac{-250}{50} = -50$.

9. **Based on the graph, if the company prices the book at $\$18$, about how many copies of the book can they expect to sell per day?**

   650

10. **Based on the graph, approximately what price should the company charge in order to sell 700 copies of the book per day?**

    $\$17$

Closing (5 minutes)

If time allows, consider posing the following questions:

- How would you interpret the meaning of the rate of change (-50) from Exercise 8?
  - *Answers will vary, pay careful attention to wording. The number of copies sold would decrease by* 50 *units as the price increased by* $\$1$ *or for every dollar increase in the price, the number of copies sold would decrease by* 50 *units.*
- Does it seem reasonable that the number of copies sold would decrease with respect to an increase in price?
  - *Yes, if the book was really expensive someone may not want to buy it. If the cost remained low, it seems reasonable that more people would want to purchase it.*
- How is the information given in the truck rental problem different than the information given in the book pricing problem?
  - *In the book pricing problem, the information was given as ordered pairs. In the truck rental problem, the information was given in the form of a slope and initial value.*

Lesson Summary

When the rate of change, b, and an initial value, a, are given in the context of a problem, the linear function that models the situation is given by the equation $y = a + bx$.

The rate of change and initial value can also be used to graph the linear function that models the situation.

When two or more ordered pairs are given in the context of a problem that involves a linear relationship, the graph of the linear function is the line that passes through those points. The linear function can be represented by the equation of that line.