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AN ALLPASS APPROACH TO DIGITAL PHASING AND FLANGING

by

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An Allpass Approach to Digital Phasing and Flanging

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Abstract

A convenient structure is proposed for implementing digital "phasers," "flangers," "comb filters," and the like. These sound modifiers all work by sweeping "notches" through the spectrum of a sound. The main feature of the proposed structure is that a fixed number of notches is provided which can be controlled independently. Each notch-section is made using a second-order allpass filter.

Phasing and flanging are techniques which have been available in the recording studio for a long time. In both cases, the effect created by moving "notches." (The term "notch" refers to the elimination of sound energy at a single frequency or over a narrow frequency interval.) For our purposes, a *flanger* is defined as a filter which modulates the frequencies of a set of *uniformly spaced notches,* and a *phaser* is defined as a filter which modulates the frequencies of a set of *non-uniformly spaced notches.* Figure 1 shows how the notches appear generically in these two cases.

A model for flanging is a simple delay line with a feed-around, as shown in Fig. 2. The notches are spaced at intervals of $1/\tau$Hz with the first notch occurring at frequency $1/2\tau$. There are two potential problems with the flanger. First, to avoid audible noise when the delay length $\tau(n)$ varies over time, digital signal interpolation is required. In some cases (such as when the signal sampling rate is very high relative to the signal bandwidth) linear interpolation is adequate, but in other cases, more expensive interpolation is required. Second, if the input signal is exactly harmonic, and if the first notch frequency happens to land on half the fundamental frequency of the source, then every harmonic will vanish. In such a case, the output signal fails to exist. (This is analogous to the old adage stating that "if your parents didn't have children, chances are you won't either.") In practice, the signal loudness can be severely modulated as the notches move through alignment with the signal spectrum. This is one reason why flangers are more often used with noise-like or inharmonic sounds. The allpass structure proposed in this paper circumvents both the interpolation problem and the uniform spacing constraint inherent in time-varying delay lines.

The architecture of the allpass-based notch filter is shown in Fig. 2b. It consists of a series connection of second-order allpass filters with a feed-around. Thus delay line of the flanger is replaced by a string of allp filters to obtain a phaser. The phaser will have notc wherever the phase of the allpass chain is at π (180 degre It will be shown that these frequencies occur very close the resonant frequencies of the individual allpass sectio Thus to move just one of the notches, the tuning of pole-pair in the corresponding section is all that needs be changed (which affects only one coefficient in the str ture). The depth of the notches can be varied together changing the gain of the feed-around. The width of ea notch is controlled by the distance of the associated pc pair from the unit circle. So to widen the notch associat with a particular allpass section, one simply increases "damping" of that section. Finally, since the gain of allpass string is unity (by definition of allpass filters), gain of the entire structure is strictly bounded betweer and 2 (given stability of each individual allpass, which easy to ensure). This property allows arbitrary notch cc trols to be applied without fear of the overall gain becomi ill-behaved.

As mentioned above, it is desirable to avoid exact h monic spacing of the notches. One possibility is to spa the notches according to the *critical bands of hearing,* sin this gives *uniform notch density* with respect to "plac along the basilar membrane in the ear. There is no need follow closely the critical-band structure, and many simp functional relationships can be utilized to tune the notch Due to the immediacy of the relation between notch chara teristics and the filter coefficients, the notches can easily placed under musically meaningful control.

Presentation viewgraphs

The last five pages contain the viewgraphs (VG) the presentation at the ICMC-84 at IRCAM. Here we gi a few auxiliary notes.

VG1

According to lore, the term "flanging" arose from t way the effect was originally achieved: a disk-jockey add (mixed equally) the outputs of two turntables playing t same record in unison. He would place is thumb gently on the "flange" of one turntable to slow it behind the other. At this point the records are slightly "out of phase." However, the delay is kept below the threshold of echo perception. Next he would place his thumb on the flange of the other turntable, slowing it until it re-synchronized, and then further slowing it behind the other. The process is repeated as desired, pressing the flange of each turntable in alternation. When this is done, uniformly-space notches are swept through the spectrum, and the effect has been described as a "whoosh" passing subtly through the sound. If the flanging is done rapidly, a Doppler shift is introduced which approaches the "Leslie" effect commonly used for organs.

A model for flanging is a simple delay line with a feed-around, as shown in the lower half of VG1. This structure can be written as

$$y(t) = x(t) + x(t - \tau(t)), \qquad (1)$$

where $x(t)$ is the input signal amplitude at (discrete) time t ($t = 0, 1, 2, \ldots$), $y(t)$ is the output at time t, and $\tau(t)$ is the length of the delay-line at time t. Equation (1) is the difference equation for a so-called *feed-forward comb-filter.*

VG2

The magnitude frequency response of (1) is shown in the lower half of VG3. (The notches are the "teeth" of the "comb.") The notches are spaced at intervals of $1/\tau$ Hz with the first notch occurring at frequency $1/2\tau$ Hz.* Since the delay $\tau(t)$ is a function of time, so are the notches.

VG3

This is the analytical derivation of the graph in VG2. The magnitude frequency response is derived from the difference equation.

VG4

The notch-spacing is inversely proportional to delay length.

Hopefully the remaining viewgraphs can be followed as self explanatory.

Acknowledgment

The author thanks Robert Poor and Jim McConkey for fruitful discussions on the subject of this talk.

References

[1] B. Friedlander and J. O. Smith, "Analysis and Performance Evaluation of an Adaptive Notch Filter," *IEEE Trans. on Info. Theory*, vol. IT–30, pp. 283–295, March 1984.

* The horizontal axis in the plot is in units of radian frequency $\omega = 2\pi f$ where f is frequency in Hertz.

[2] L. R. Rabiner and B. Gold, *Theory and Applicati of Digital Signal Processing,* Prentice-Hall Inc., Engl wood Cliffs, NJ, 1975.

[3] J. O. Smith, "Introduction to Digital Filter Theory In J. Strawn, ed., *Digital Audio Signal Processin An Anthology.* William Kaufmann, Inc., Los Alto California, 1985.

[4] K. Steiglitz, *An Introduction to Discrete System* John Wiley and Sons, Inc., New York, 1974.

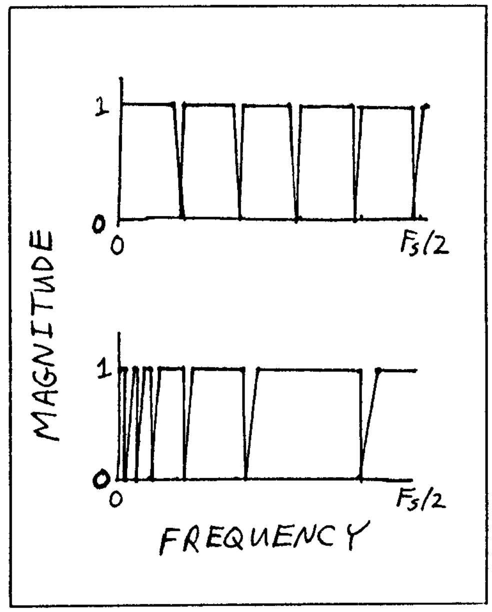


Figure 1. Magnitude frequency response for the a) flanger and b) phase shifter. *(Idealized)*

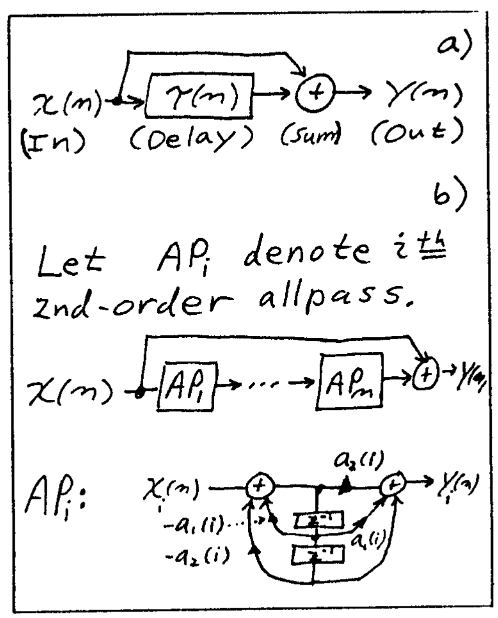
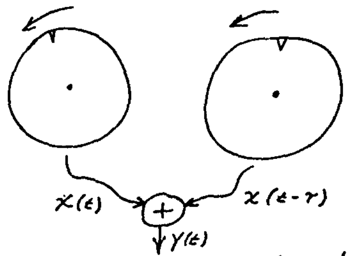


Figure 2. System diagram for the a) flanger and b) phase shifter.

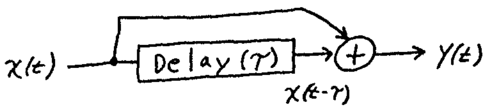
1

The FLANGER



(Two-turntables Method)

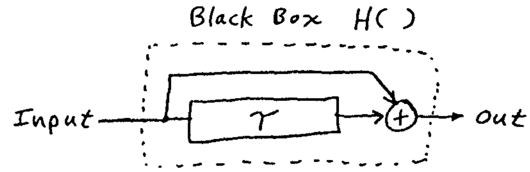
Mathematical Model



The delay τ varies with time (e.g.)

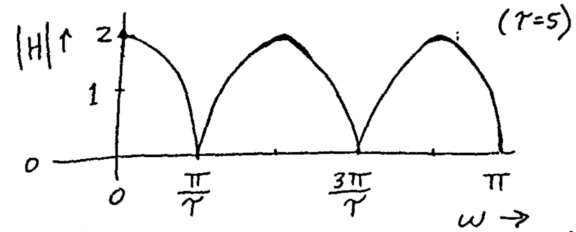
2

SYSTEM DIAGRAM



FREQUENCY RESPONSE

$$|H(e^{j\omega})| = 2\,|\cos(\omega\tau/2)| \quad \left(\begin{array}{l}=\text{gain at}\\ \text{freq. } \omega\end{array}\right)$$



(Frequency $\omega=\pi$ is half the sampling rate)
$\omega = 2\pi f/f_s$, f in Hz
f_s = sampling rate (Hz)

3

Derivation of Flanger Frequency Response

$$Y(t) = x(t) + x(t-\tau)$$

$$Y(z) = X(z) + z^{-\tau} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + z^{-\tau}$$

$$H(e^{j\omega}) = 1 + z^{-\tau}\Big|_{z=e^{j\omega}}$$

$$= 1 + e^{-j\omega\tau}$$

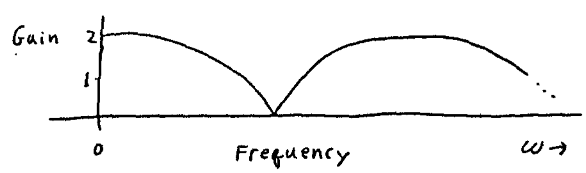
$$= e^{-j\omega\tau/2}\left(e^{j\omega\tau/2} + e^{-j\omega\tau/2}\right)$$

$$= 2e^{-j\omega\tau/2}\cos(\omega\tau/2)$$

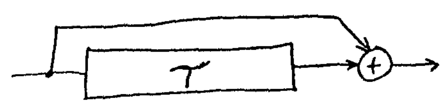
$$|H(e^{j\omega})| = 2\,|\cos(\omega\tau/2)|, \quad \omega = 2\pi\frac{f}{f_s}$$

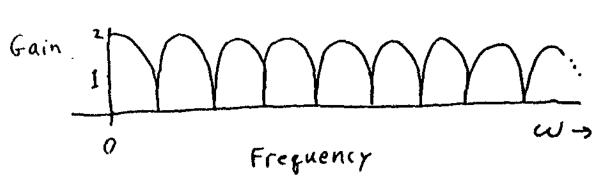
4

Small Delay



Large Delay





(5)

Essential Feature:

$\boxed{\text{Moving notches}}$

Problems with Flanger

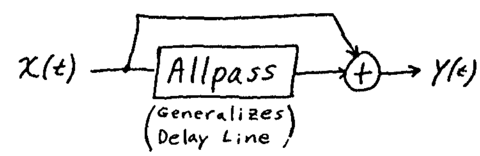
- Digital Delay-Line Interpolation
- Harmonic notch spacing (Linear in ω) (periodic signal can vanish!)

(6)

Solution

New structure which directly implements sweeping notch filters.

Structure



Property: Allpass gain is unity. A notch is obtained at each frequency for which the allpass phase is 180°.

(7)

General Case

$$H(z) = 1 + \underbrace{z^{-\nu}\frac{\tilde{C}(z)}{C(z)}}_{\text{ALLPASS}}$$

where

$$C(z) = 1 + c_1 z^{-1} + \cdots + c_m z^{-m}$$

$$\tilde{C}(z) = c_m + c_{m-1} z^{-1} + \cdots + z^{-m}$$

$$= z^{-m} C(z^{-1})$$

$$\nu \geq 0 \text{ (integer)}$$

$\boxed{\text{Note: } c_i = 0 \text{ gives pure delay line}}$

Theorem

If $z^m C(z)$ has k roots inside the unit circle, then $H(z)$ has $|m-2k-\nu|$ zeros on the unit circle.

(8)

Special Case

If $C(z)$ has all roots inside the unit circle, then $H(z)$ has all $(m+\nu)$ zeros on the unit circle.

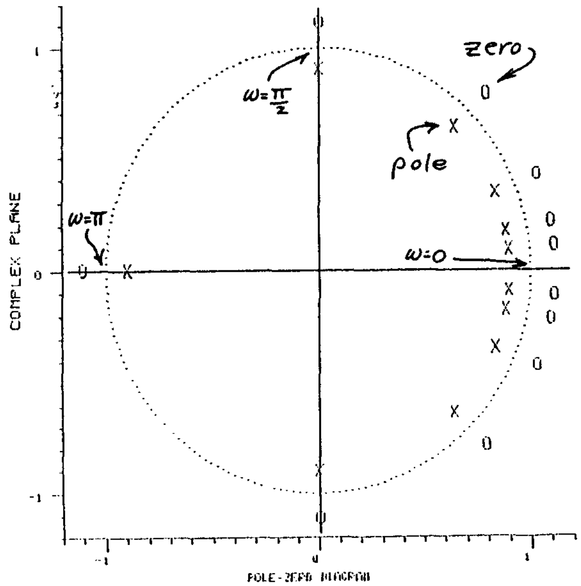
In other words

If the allpass is stable and order m, then there are m notches "somewhere" along the frequency axis.

9

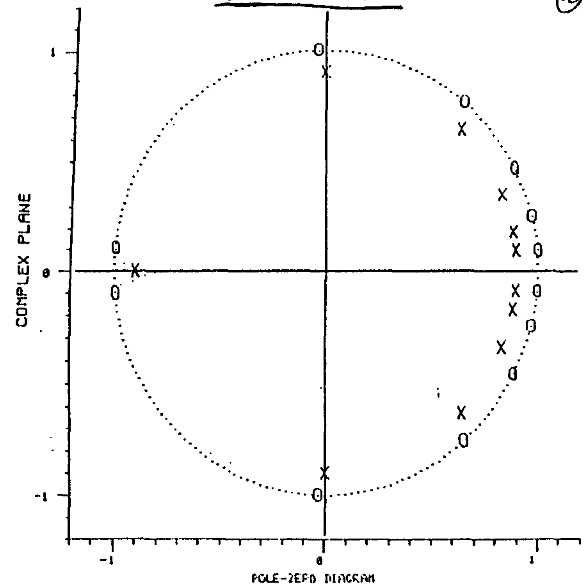
ALLPASS POLE-ZERO DIAGRAM

(order = 12)



10

1 + AllPass



Important Observation:

Poles close to unit circle ($R \approx 1$) $\Rightarrow$ Notches close to pole angles.

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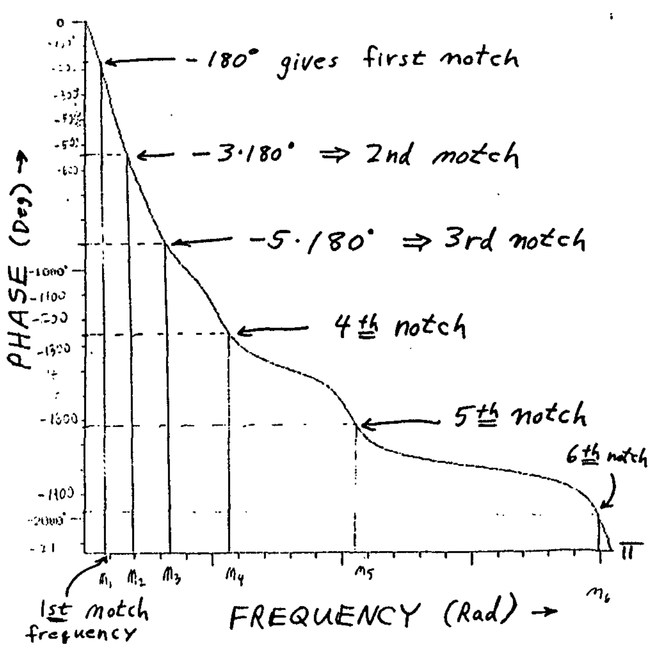
Allpass Phase Response

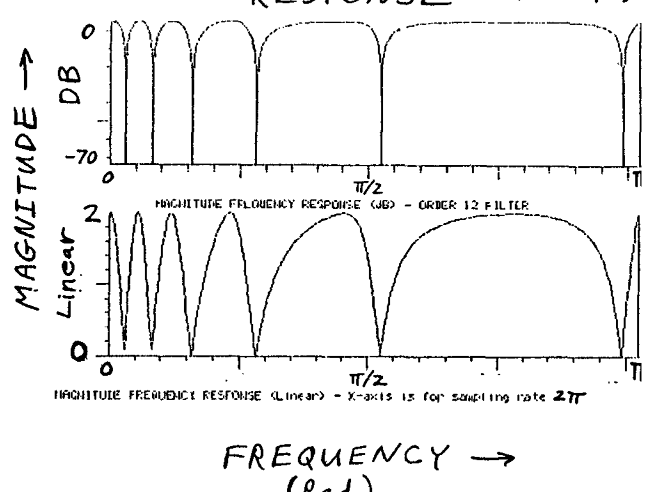
(order 12 example)



12

FREQUENCY RESPONSE

(order 12 example)



13

Problem

How to directly control notch frequencies?

Clue

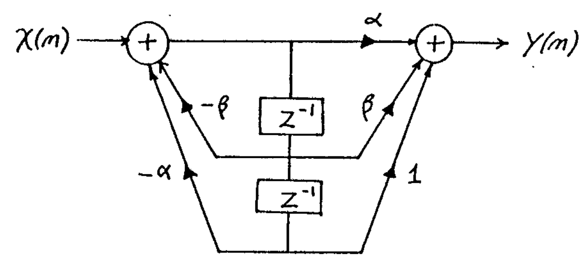
Notch frequencies close to Allpass pole/zero frequencies

Solution

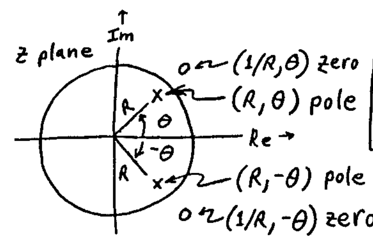
Use cascade 2nd-order allpass filters. The pole/zero frequencies are simply related to the coefficients of each 2nd-order section.

SECOND ORDER ALLPASS

14



$$H(z) = \frac{\alpha + \beta z^{-1} + z^{-2}}{1 + \beta z^{-1} + \alpha z^{-2}}$$

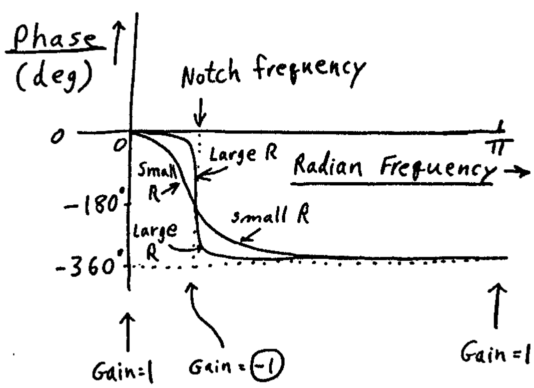


$$\boxed{\begin{aligned}\alpha &= R^2 \\ \beta &= -2R\cos\theta\end{aligned}}$$

$$\boxed{\begin{aligned}R &= \text{pole radius} \\ \theta &= \text{pole angle}\end{aligned}}$$

15

Phase of 2nd-order Allpass



R controls notch width

θ controls notch tuning

16

2nd-Order Allpass

$$H(z) = \frac{\alpha + \beta z^{-1} + z^{-2}}{1 + \beta z^{-1} + \alpha z^{-2}}$$

$$\boxed{\begin{aligned}\alpha &= R^2 \\ \beta &= -2R\cos\theta\end{aligned}}$$

$$\boxed{\begin{aligned}R &= e^{-\pi B T} \\ \theta &= 2\pi f T\end{aligned}}$$

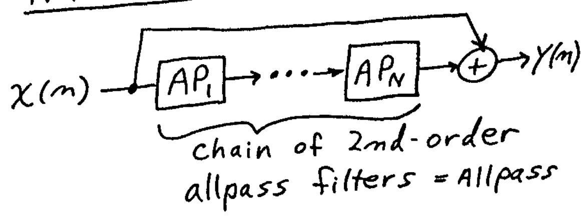
$f \approx$ Notch frequency (Hz)

$B \approx$ Notch width (Hz)

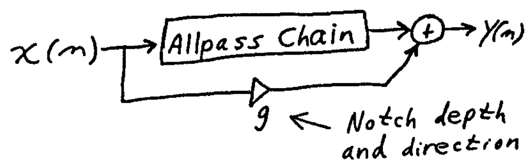
T = sampling period (sec)

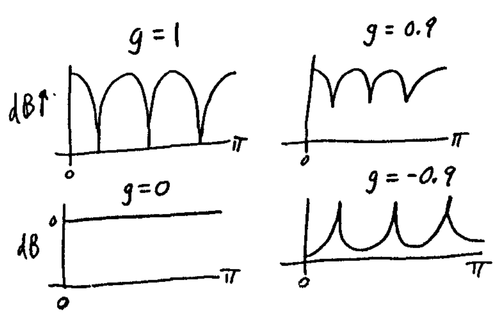
Easy to control notch location and width in this form.

N notches

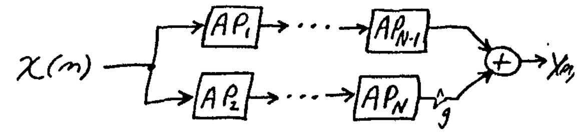


Depth Control



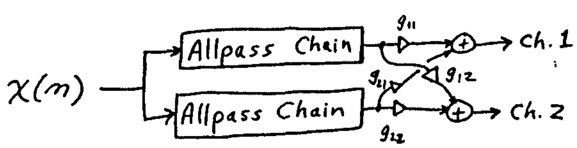


Balanced Structure (Rob Poor):



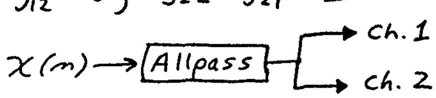
- Better numerical conditioning
- Eliminates delay compensation
- Notches can be turned off

Stereo Version (Rob Poor)



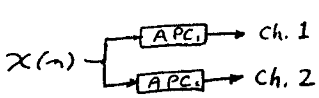
Special Cases

- $g_{11} = g_{12} = 0, \quad g_{22} = g_{21} = 1$



(no notches)

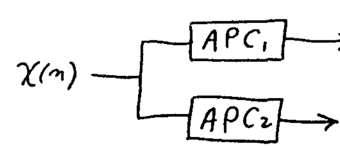
- $g_{11} = g_{22} = 1, \quad g_{12} = g_{21} = 0$



Notches created in the air only!

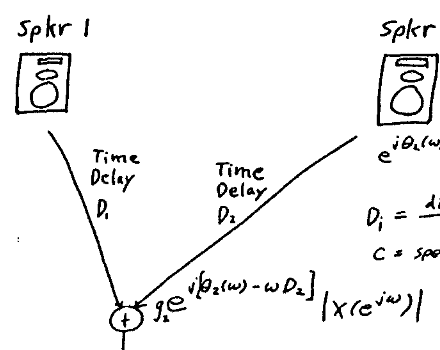
Notch Pattern Moves with observer angle

Stereo Spatial Phaser



$e^{j\theta_1(\omega)}|X(e^{j\omega})|$

$e^{j\theta_2(\omega)}|X(e^{j\omega})|$



$e^{j\theta_2(\omega)}|X(e^{j\omega})|$

$D_i = \dfrac{\text{distance}}{c}$

c = speed of sound

$g_2 e^{j[\theta_2(\omega) - \omega D_2]} |X(e^{j\omega})|$

$$Y(e^{j\omega}) = \left\{ g_1 e^{j[\theta_1(\omega) - \omega D_1]} + g_2 e^{j[\theta_2(\omega) - \omega D_2]} \right\} |X(e^{j\omega})|$$

$$|Y(e^{j\omega})|^2 = \left(g_1^2 + g_2^2 + 2 g_1 g_2 \cos\{[\theta_1(\omega) - \theta_2(\omega)] - \omega[D_1 - D_2]\} \right) |X|^2$$

$$\approx [2 + 2\cos(\Delta\theta - \omega \Delta D)] |X|^2$$