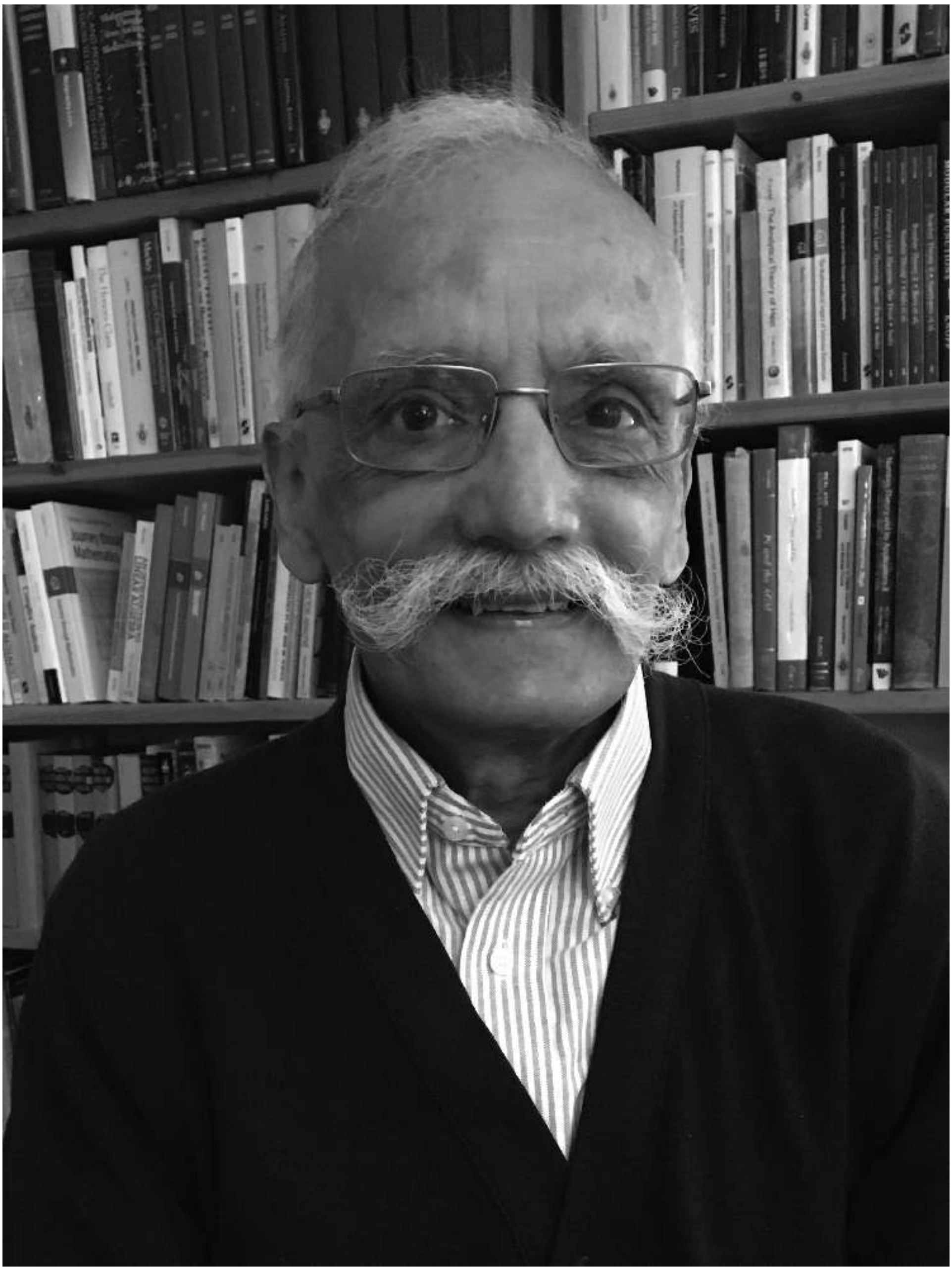
Series and Products in the Development of Mathematics

Volume 1

This is the first volume of a two-volume work that traces the development of series and products from 1380 to 2000 by presenting and explaining the interconnected concepts and results of hundreds of unsung as well as celebrated mathematicians. Some chapters deal with the work of primarily one mathematician on a pivotal topic, and other chapters chronicle the progress over time of a given topic. This updated second edition of *Sources in the Development of Mathematics* adds extensive context, detail, and primary source material, with many sections rewritten to more clearly reveal the significance of key developments and arguments. Volume 1, accessible even to advanced undergraduate students, discusses the development of the methods in series and products that do not employ complex analytic methods or sophisticated machinery. Volume 2 treats more recent work, including de Branges's solution of Bieberbach's conjecture, and requires more advanced mathematical knowledge.

RANJAN ROY (1947–2020) was the Ralph C. Huffer Professor of Mathematics and Astronomy at Beloit College, where he was a faculty member for 38 years. Roy published papers and reviews on Riemann surfaces, differential equations, fluid mechanics, Kleinian groups, and the development of mathematics. He was an award-winning educator, having received the Allendoerfer Prize, the Wisconsin MAA teaching award, and the MAA Haimo Award for Distinguished Mathematics Teaching and was twice named Teacher of the Year at Beloit College. He coauthored *Special Functions* (2001) with George Andrews and Richard Askey and coauthored chapters in the *NIST Handbook of Mathematical Functions* (2010); he also authored *Elliptic and Modular Functions from Gauss to Dedekind to Hecke* (2017) and the first edition of this book, *Sources in the Development of Mathematics* (2011).



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Preface

Sources in the Development of Mathematics: Series and Products from the Fifteenth to the Twenty-first Century, my book of 2011, was intended for an audience of graduate students or beyond. However, since much of its mathematics lies at the foundations of the undergraduate mathematics curriculum, I decided to use portions of my book as the text for an advanced undergraduate course. I was very pleased to find that my curious and diligent students, of varied levels of mathematical talent, could understand a good bit of the material and get insight into mathematics they had already studied as well as topics with which they were unfamiliar. Of course, the students could profitably study such topics from good textbooks. But I observed that when they read original proofs, perhaps with gaps or with slightly opaque arguments, students gained very valuable insight into the process of mathematical thinking and intuition. Moreover, the study of the steps, often over long periods of time, by which earlier mathematicians refined and clarified their arguments revealed to my students the essential points at the crux of those results, points that may be more difficult to discern in later streamlined presentations. As they worked to understand the material, my students witnessed the difficulty and beauty of original mathematical work, and this was a source of great enjoyment to many of them. I have now thrice taught this course, with extremely positive student response.

In order for my students to follow the foundational mathematical arguments in *Sources*, I was often required to provide additional material, material actually contained in the original works of the mathematicians being studied. I therefore decided to expand my book, as a second edition in two volumes, to make it more accessible to readers, from novices to accomplished mathematicians. This second edition contains about 250 pages of new material, including more details within the original proofs, elaborations and further developments of results, and additional results that may give the reader a better perspective. Furthermore, to give the material greater focus, I have limited this second edition to the topics of series and products, areas that today permeate both applied and pure mathematics; the second edition is thus entitled *Series and Products in the Development of Mathematics*.

This first volume of my work discusses the development of the fundamental though powerful and essential methods in series and products that do not employ complex analytic methods or sophisticated machinery such as Fourier transforms. Much of this material would be accessible, perhaps with guidance, to advanced undergraduate students. The second volume deals with more recent work and requires considerable mathematical background. For example, in volume 2, I discuss Weil's 1949 paper on solutions of equations in finite fields and de Branges's conquest of the Bieberbach conjecture. Each volume contains the same complete bibliography.

The exercises at the end of the chapters present many additional original results and may be studied simply for the supplementary theorems they contain. The exercises are accompanied by references to the original works, as an aid to further research. Readers may attempt to prove the results in the problems and, by use of the references, compare their own solutions with the originals. Moreover, many of the exercises can be tackled by methods similar to those given in the text, so that some exercises can be realistically assigned to a class as homework. I assigned many exercises to my classes, and found that the students enjoyed and benefited from their efforts to find solutions. Thus, the exercises may be useful as problems to be solved, and also for the results they present.

Detailed study of original mathematical works provides a point of entry into the minds of the creators of powerful theories, and thus into the theories themselves. But tracing the discovery and evolution of mathematical ideas and theorems entails the examination of many, many papers, letters, notes, and monographs. For example, in this work I have discussed the work of more than three hundred mathematicians, including arguments and theorems contained in approximately one hundred works and letters of Euler alone. Locating, studying, and grasping the interconnections among such original works and results is a ponderous, complex, and rewarding effort. In this second edition, I have added numerous footnotes and almost five hundred works to the bibliography. My hope is that the detailed footnotes and the expanded bibliography, containing both original works and works of distinguished expositors and historians of mathematics, may encourage and facilitate the efforts of those who wish to search out and study the original sources of our inherited mathematical wealth.

I first wish to thank my wife, who typeset and edited this work, made innumerable corrections and refinements to the text, and devotedly assisted me with translations and locating references. I am also very grateful to NFN Kalyan for his encouragement and for creating the eloquent artwork for the cover of these volumes. I greatly appreciate Maitreyi Lagunas's unflagging support and interest. I thank Bruce Atwood who cheerfully constructed the nice diagrams contained in this work, and Paul Campbell who generously provided expert technical support and advice. I am grateful to my student Shambhavi Upadhyaya, who has an unusual ability to proofread very accurately, for spending so much time giving useful suggestions for improvement. I am indebted to my students whose questions and enthusiasm helped me refine this second edition. I also thank the very capable librarians at Beloit College, especially Chris Nelson and Cindy Cooley. Finally, I wish to acknowledge the inspiration provided me by my friend, the late Dick Askey.