Improved Orbital Predictions using Pseudo Observations - Maximizing the Utility of SGP4-XP

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ABSTRACT

The new SGP4-XP algorithm was released by USSF near the end of 2020 and is claimed to offer accuracy that is “statistically equivalent” to the high-precision Special Perturbations algorithm. There are several issues that will likely limit the adoption of SPG4-XP: it still relies on a two-line structure without covariance information, new TLEs are not publicly available, and the software is distributed in binary format only. We present an open source software implementation that addresses each of these issues. By using the well-established practice of generating pseudo observations from lower-precision theories and then performing orbit determination with a higher-precision theory, TLEs for SGP4-XP can be used with any force model. The OD process provides missing covariance data. This process allows organizations that do not wish to run third-party binaries on their operational computers to generate pseudo observations elsewhere and only import the necessary output. For organizations willing to trust these binaries, pseudo observations from SGP4 or other ephemeris can be used to generate SGP4-XP TLEs. This allows users of the new algorithm to generate compatible TLEs from existing sources.

1. INTRODUCTION

The Simplified General Perturbations model has been in operational use since the early 1970s [1]. The orbital propagation model known as SGP4 has been available publicly since the release of Spacetrack Report Number 3 [2]. That document also provided the source code for the model which is used extensively in the space community today. In the four decades since the release of SGP4 source code the most substantial improvement in the publicly available version came from [1] when they identified several potential bugs in the software leading to discontinuities. Many current implementations of SGP4 trace back to the 2006 version of the software released with [1]. In order to use the SGP4 model, users require orbit data in the two line element set (TLE) format. For decades the US Government has provided access to a catalog of TLEs for space objects through various websites. The current website is known as Space-Track.org or Space-Track. Users are able to download TLEs after registration. The data is updated multiple times per day by the contractor running the site using a data feed from the 18th Space Control Squadron (18SPCS) which is now part of the United States Space Force (USSF). The Space-Track website has included a link for downloading compiled binaries for the SGP4 algorithm but not the source code for the model for several years.

In the last few months of 2020, the Space-Track website announced the release of the first new algorithm for TLEs in almost 40 years. The algorithm is called SGP4-XP. In the release notes [3] the authors claim that the accuracy of the new algorithm is “statistically equivalent” to numeric integration such as the Special Perturbations (SP) propagator. SGP4-XP uses the same TLE structure as SGP4 with a different ephemeris type value and has a run time that is claimed to be only 1.5 to 2 times longer than the legacy model. The release documents [3] list several important improvements of SGP4-XP over SGP4:

- Improved Lunar perturbation modeling.
- Improved resonance modeling for 8-, 12-, 16-, and 24-hour orbits.
- Solar Radiation Pressure modeling with the AGOM model parameter replacing second derivative of the mean motion value in the first line of the TLE.

- Addition of J5 zonal gravity for consistency with PPT3 theory.
- Deep-space perturbations are always used with no more 225-minute period threshold.
- A region is added for Very Deep Space (VDS) / cislunar multi-day orbits.
- EGM-96 is used instead of WGS-72.
- The Jacchia-70 atmospheric density model is used for drag calculations along with a generic flux model that predicts values for F10 and Ap indices.
- The B* term is replaced with a B term for drag calculations.

More accurate position calculations would be of great benefit to the space community. However, many important applications of orbit predictions require the use of uncertainty information, usually in the form of state error covariance matrix (covariance). For example, the calculation of the probability of collision between two space objects makes use of covariance. The new model still relies on TLEs which do not contain covariance. The software binaries have been publicly available to registered Space-Track users since 2020 with an updated release published in May 2021. However, no TLEs for the new SGP4-XP model are available to regular users on the Space-Track website. Another possible deterrent to widespread use of the new model is that the software is only available in binary format. There is no source code available for users to inspect and compile themselves. This paper seeks to address each of these issues, to validate some of the claims in the release documents, and to help encourage the adoption of the SGP4-XP model.

2. ORBIT DETERMINATION

At the time this paper was prepared, the authors were able to identify only a single TLE released publicly for use with the SGP4-XP algorithm. No TLEs with ephemeris type 4 were available on Space-Track when we queried the database. The single TLE was found in the documentation accompanying the software release [3].

```
1 41085U GPDC     19001.16111410 +.00000000  64817-1  25579 0 4 0001
2 41085  98.8405  84.6159 0030231 128.2631 288.8143 14.1732257715999
```

Object 41085 is a piece of of debris associated with the NOAA 16 satellite launched in 2000. Before proceeding with describing an orbit determination process for generating SGP4-XP TLEs we wish to first demonstrate the accuracy of this single TLE when propagated forward using SGP4-XP.

The accuracy of this TLE can be demonstrated using inter-TLE comparisons. We take a set of TLEs produced using SGP4 and predict their positions at their respective epochs. We can then compute the distance between these positions and those predicted by the SGP4-XP TLE at the same epochs. For comparison we also perform the calculation using several other SGP4 TLEs from near the same epoch, January 1, 2019. In Fig. 1 it is clear that the XP TLE maintains its accuracy for a much longer time that the SGP4 TLEs. The regular SGP4 TLEs exceed 2 km difference by 4 days after epoch. The XP TLE does not reach that level until almost 10 days after epoch.

The results from this single TLE are encouraging, but more SGP4-XP TLEs are needed in order to make use of the model.

To produce XP TLEs we use an orbit determination process modeled after [4]. For our observation data we use high-precision ephemeris for Satellite Laser Ranging (SLR) targets tracked by the International Laser Ranging Service (ILRS) [5]. There are SLR targets in Low Earth Orbit (LEO), Medium Earth Orbit (MEO), and Geostationary Orbit (GEO) which allow us to test the new model in each orbit regime. The specific ephemeris format we use is the Consolidated Prediction Format Version 2 (CPFv2). The CPF file format actually contains *predictions* of SLR satellite positions. The CPF positions agree to within several meters of the positions determined through very precise fits of SLR observations [6].

We identified several satellites with easily available CPF data to study. The approximate orbit and physical characteristics[1] for each satellite are given in Table 1.

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[1] `https://ilrs.gsfc.nasa.gov/missions/satellite_missions/current_missions/`

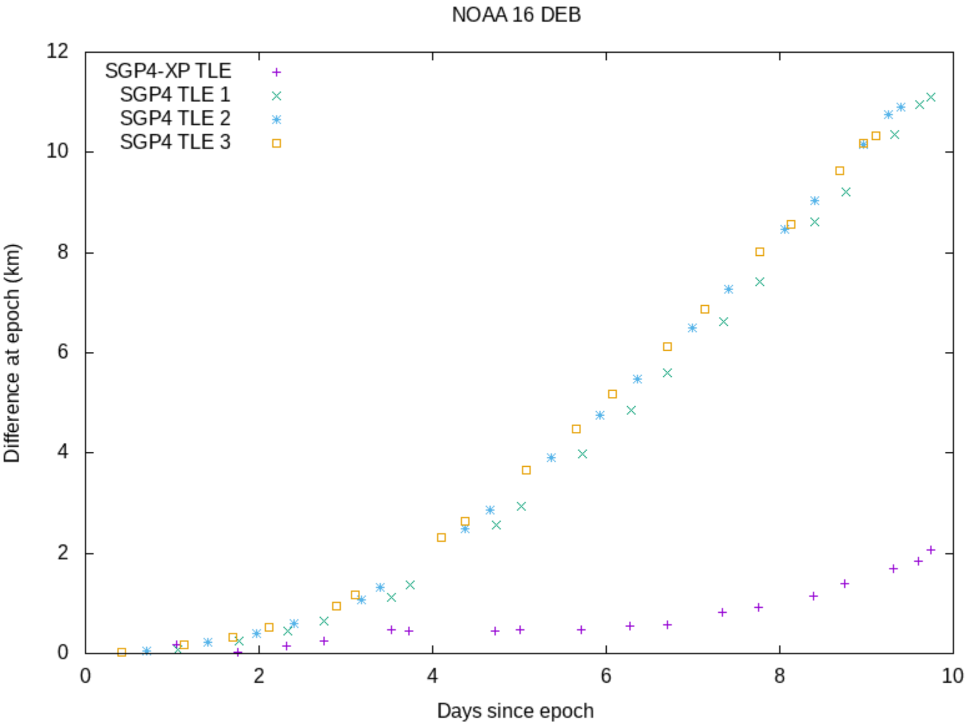

Fig. 1: Comparison of SGP4-XP position accuracy with SGP4.

Using the CPF position vectors we can determine an initial orbit in a given fit span by using three different vectors as inputs to a Gibbs initial orbit determination algorithm. We set the epoch at the end of the seven day fit spin. We then perform a differential correction process to fit each of the elements in the SGP4-XP TLE to the observation data. The CPF vectors are rotated from an earth-centered frame to the TEME frame used by SGP4 and SGP4-XP. The differential correction process uses equinoctal elements internally as recommended by [4] to avoid singularities. A least squares optimization is solved at each iteration using the Levenberg-Marquardt algorithm. The Jacobian Matrix is computed using finite differences.

The AGOM parameter for SRP and the B term for drag require special handling. After the initial differential process converges for the rest of the TLE parameters, the SRP and drag terms are fit in alternating iterations. For each iteration trial values from a list of values spanning multiple orders of magnitude are used. The parameter is held fixed and the other TLE parameters are adjusted during the differential correction process. After each iteration, the range of fixed values is reduced. All values for AGOM and B term are required to be greater than or equal to zero in our process.

For all satellites we obtained the CPF files for the first six months of 2021. According to [6] the CPF positions are most accurate during the first 24 hours of each prediction run. We filter each file to include only the first day's worth of predictions. We set the first epoch to be midnight of January 8, 2021. We use the first seven days of CPF values to fit an SGP4-XP TLE[2]. We then advance the epoch by one day and the fit span by one day and fit another TLE. The TLEs will have overlapping fit spans. This is similar to TLEs produced by 18SPCS. We can then use the subsequent CPF position values as "truth" data to test the prediction accuracy of each SGP4-XP TLE.

The results of our orbit determination process are presented in the next section. It is possible to generate covariance information as an output of this differential correction process, but that is not done in our implementation. The SGP4-XP TLEs produced here are intended to approximate the information to eventually be provided on the Space-Track website, which does not include covariance information.

In order to obtain covariance information, we present a second orbit determination process. This orbit determination

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[2] By examining discontinuities between CPF files on subsequent days we were able to identify maneuvers performed by the GEO satellites. We did not use time spans that included maneuvers for either TLE-fitting or predictions.

Table 1: Satellite orbit and physical characteristics.

LEO Satellites

Name	Mean Motion (revs/day)	Semi-major Axis (km)	Inc. (deg)	Eccentricity	Mass (kg)	Surface Area (m^2)
AJISAI	12.4	7870	50	0.001	685	0.036
Starlette	13.8	7340	50	0.02	47.5	0.045
Stella	14.3	7170	98.9	0.001	48	0.045
Swarm A	15.5	6810	87.4	0.001	578	1

MEO Satellites

Name	Mean Motion (revs/day)	Semi-major Axis (km)	Inc. (deg)	Eccentricity	Mass (kg)	Surface Area (m^2)
ETALON 2	2.13	25500	65.5	0.002	1415	1.3
GALILEO 201	1.86	28000	50.5	0.165	700	20.9
GLONASS 140	2.13	25500	64.5	0.001	750	23.3

GEO Satellites

Name	Mean Motion (revs/day)	Semi-major Axis (km)	Inc. (deg)	Eccentricity	Mass (kg)	Surface Area (m^2)
IRNSS 1A	1	42166	31.3	0.002	1425	16.3
IRNSS 1C	1	42166	2.05	0.002	1425	16.3
QZS1	1	42168	41.8	0.08	2281	38.3

process makes use of the SGP4-XP TLEs generated in the previous step. Following a process similar to [7] we generated predicted position vectors using a set of SGP4-XP TLEs. Again we use a 7 day fit span and generate ephemeris from the TLEs during this fit span. The TLEs are propagated backwards one to three full revolutions. LEO orbits have ephemeris generated every 30 seconds. MEO orbits every 300 seconds. GEO orbits every 900 seconds. These pseudo observations are then used as input data for another orbit determination process with a high-precision propagator. For our experiments we used the Orekit software package[8]. We setup a force model using 60x60 gravity, solar and lunar perturbations with solid tides, solar radiation pressure, and drag. The results of these ODs include the state vector, solar radiation pressure coefficient, and the drag coefficient for LEO orbits. This process is designed after [7]. Other organizations use a similar process to determine covariance information for use with conjunction assessments [9],[10]. Several alternative fit spans and pseudo-observation generation procedures are described in [9].

3. SGP4-XP ACCURACY

For each satellite we produce around 170 SGP4-XP TLEs. For each TLE we predict the position of the satellite and compare to the next 45 days worth of CPF position values at the times specified in the CPF files. The prediction error between each position is accumulated in bins. Each bin represents a single day since epoch. In this way, the results of all TLE predictions can be combined into a single plot. For each day we construct a box-and-whisker plot. The lower and upper lines represent the minimum and maximum error. The top of the box is the 75th percentile, and the bottom is the 25th percentile. The horizontal line in the middle is the median error. Fig. 2 below shows how the error grows for predictions from all of the AJISAI XP TLEs for the 45 day span of CPF positions.

We produce the same plot using traditional SGP4 TLEs from Space-Track in Fig. 3.

To avoid a crowded plot the boxes are removed and the median values for both types of TLEs can be combined into a single chart and extended over a longer period of time. The plot of median values is shown in Fig. 4.

SGP4-XP does not seem to be an advantage over the previous SGP4 model until almost 30 days after epoch.

The median plots for STELLA and SWARMA are below in Fig. 5 and Fig. 6.

Fig. 5 shows the SPG4-XP performs much worse for STELLA than regular SGP4. For SWARMA both models have median errors that grow at a similar rate.

The error with LEO orbits could be due to the implementation of drag in SGP4-XP requiring special fitting procedures

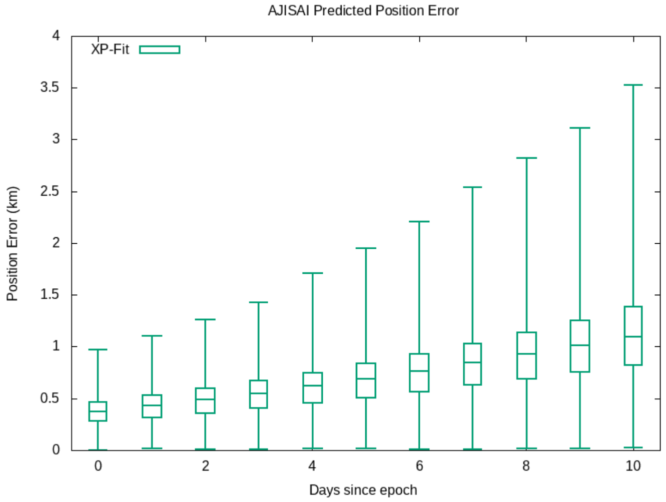

Fig. 2: AJISAI prediction errors for SGP4-XP TLEs displayed as box and whiskers.

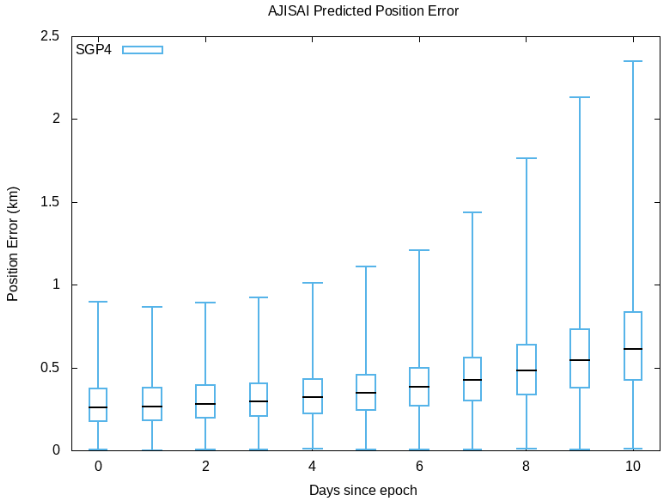

Fig. 3: AJISAI prediction errors for SGP4 TLEs displayed as box and whiskers.

to determine the correct B term value. For LEO orbits, it's not clear that SGP4-XP provides a benefit over SGP4.

The situation is very different in MEO and GEO. The median prediction errors for ETALON 2 are shown in Fig. 7.

For ETALON 2 we see that the size and growth rate of median prediction errors for SGP4-XP is much less than that for SGP4. After 45 days the median prediction error for SGP4-XP is slightly larger than 1 km. For the first 10 days SGP4-XP median error is under 0.33 km.

Fig. 8 shows that for the first 10 days of predictions using SGP4-XP TLEs for QZS1 the median error is under 0.66 km. SGP4-XP has enhanced accuracy for MEO and GEO orbits compared to SGP4.

4. HIGH-PRECISION OD FROM SGP4-XP TLES

The accuracy of SGP4-XP TLEs for MEO and GEO orbits is encouraging. To make full use of the enhanced accuracy it is still necessary to have some estimate of the uncertainties involved. For orbit predictions, the covariance information from the orbit determination process is used. To generate covariance information we used the high-precision

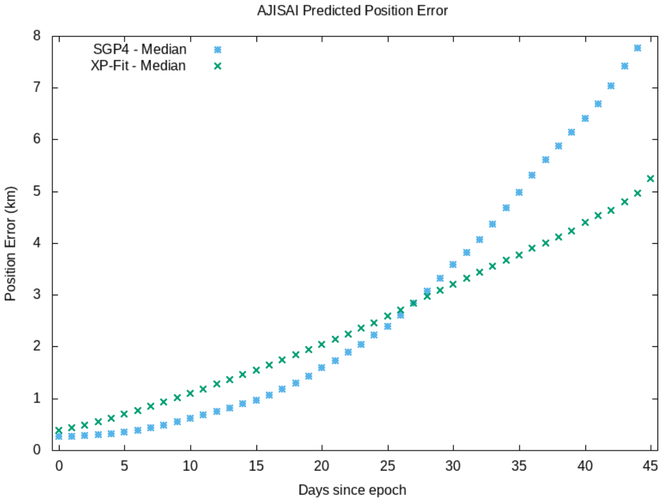

Fig. 4: AJISAI median prediction errors for XP and SGP4 TLEs.

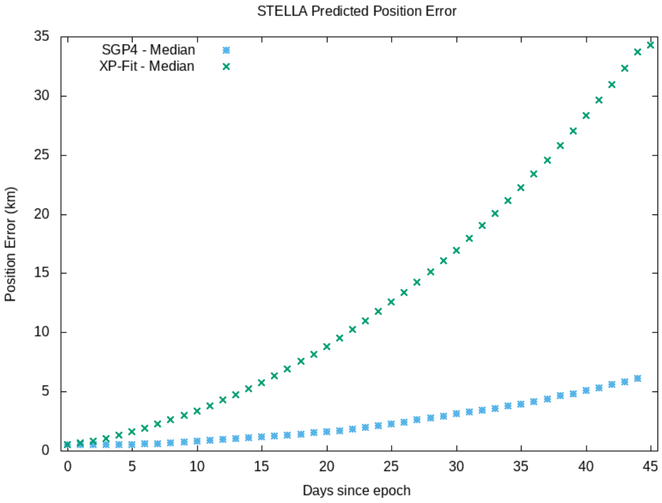

Fig. 5: STELLA median prediction errors for XP and SGP4 TLEs.

propagator in Orekit along with the orbit determination capabilities of the software. The force model included 60x60 gravity, solar and lunar perturbations with solid tides, solar radiation pressure, and drag.

The OD process as described above included a 7-day fit span followed by a 45 day prediction span. The fit span and prediction span do not overlap. In addition to fitting the XP-derived ephemeris, we also performed the OD for each satellite using a 7-day fit span of the original CPF position vectors.

The prediction errors for AJISAI are shown in Fig. 9 below.

For comparison the prediction errors when fitting the CPF position vectors directly are in Fig. 10.

The median errors can be compared to the direct predictions from SGP4-XP in Fig. 11.

The median errors for the HP-fitted XP ephemeris (HPXP) remain close to the values of the direct HP fit of the CPF vectors. After 30 days the median errors cross. So some additional accuracy can be gained by fitting SGP4-XP TLEs with a high-precision propagator.

For ETALON 2 in MEO, Fig. 12 shows the median error. The HP-fitted XP ephemeris is slightly less accurate than

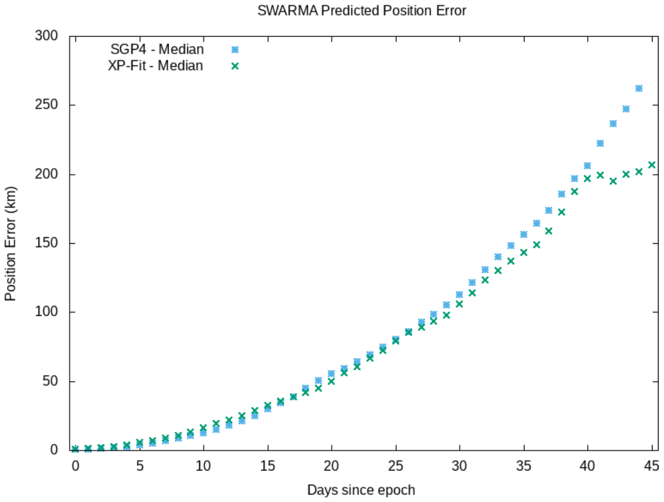

Fig. 6: SWARM A median prediction errors for XP and SGP4 TLEs.

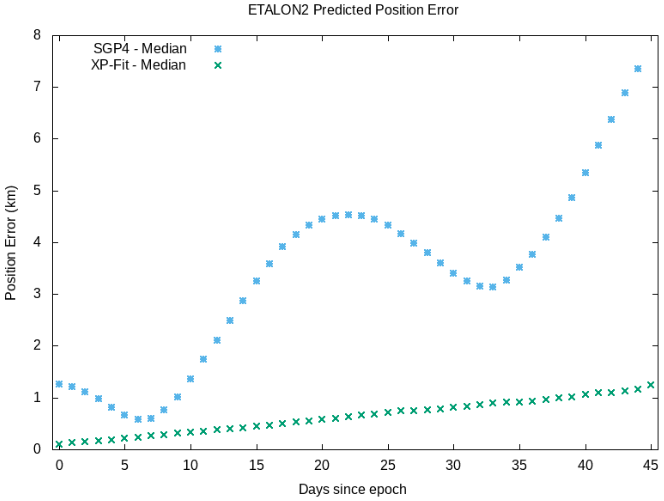

Fig. 7: ETALON2 median prediction errors for XP and SGP4 TLEs.

just the SGP4-XP TLEs themselves. The direct fit of the CPF vectors for ETALON 2 shows a very small prediction error.

For the GEO object QZS1, the situation is different still. Fig. 13 shows the median errors. The HP-fitted XP ephemeris and the HP-fitted CPF vectors show a faster growth in median error that the SGP4-XP TLEs themselves.

Table 2 summarizes the results for all the satellites we studies. The first column presides the residual when performing the SGP4-XP orbit determination process. For high-orbit satellites it is on the order of 10s of meters. This means the typical error for "predictions" over the fit span (i.e. times before epoch) using SGP4-XP TLEs will be very accurate. For LEO satellites the residuals rise to 100s of meters. Next we provide the median prediction error after day 1, day 10, and day 30. Each cell summarizes the results of approximately 170 individual orbit determinations.

The low residual over the fit span interval and the accuracy of predictions in the MEO and GEO regions provides some support for the statement that the accuracy of SGP4-XP is "statistically equivalent" to numeric integration. This paper does not offer support for that statement in the LEO region.

While the accuracy of refitting SGP4-XP TLEs may not be an improvement over using the TLEs directly for predic-

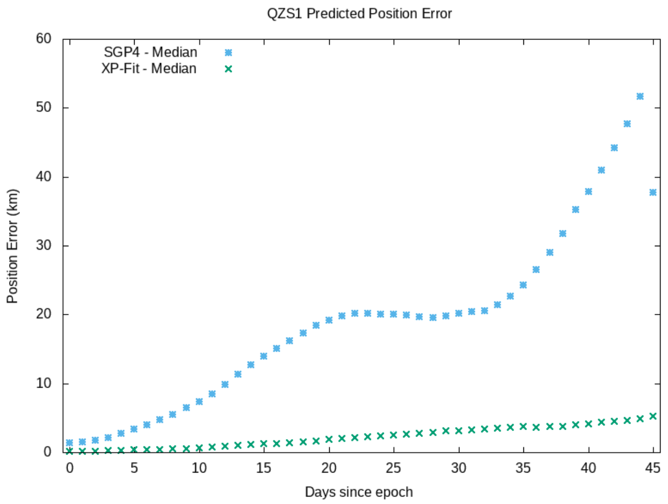

Fig. 8: QZS1 median prediction errors for XP and SGP4 TLEs.

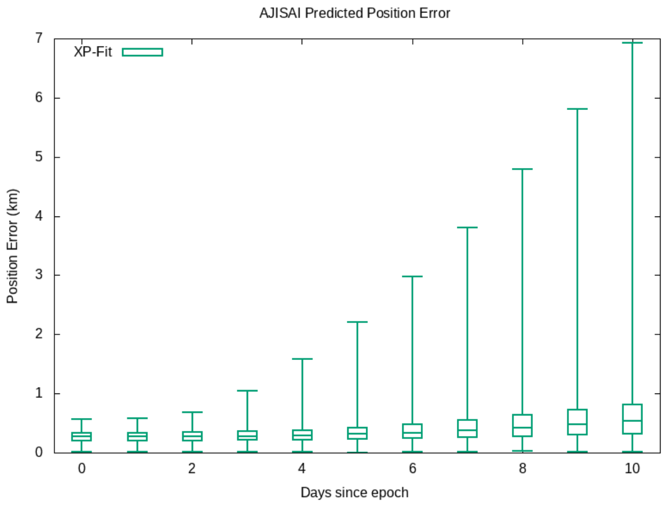

Fig. 9: AJISAI prediction errors for SGP4-XP TLEs refit to a high-precision model (HPXP) displayed as box and whiskers.

tions, the OD process does generate a covariance matrix. The diagonals of the covariance matrix produced by Orekit for a typical OD are in Table 3.

The covariance values for AJISAI are similar between the two types of high-precision fits. They are within a factor of 1.5 to 4 for each value. This means that fitting to XP ephemeris gives a similar covariance to fitting directly to the high-precision ephemeris.

This approach, similar to [7], [9], [10], is a reliable way to obtain covariance. The same approach can be adapted to other high-precision propagators. Those tools must be able to process the psuedo-observations consisting of position (or position and velocity) vectors generated from XP TLEs.

5. RUNNING THIRD-PARTY BINARIES

The final shortcoming of the current situation with SGP4-XP is the dependence on third-party binaries. Members of the space community that wish to use the SGP4-XP algorithm must use the compiled code as distributed by USSF

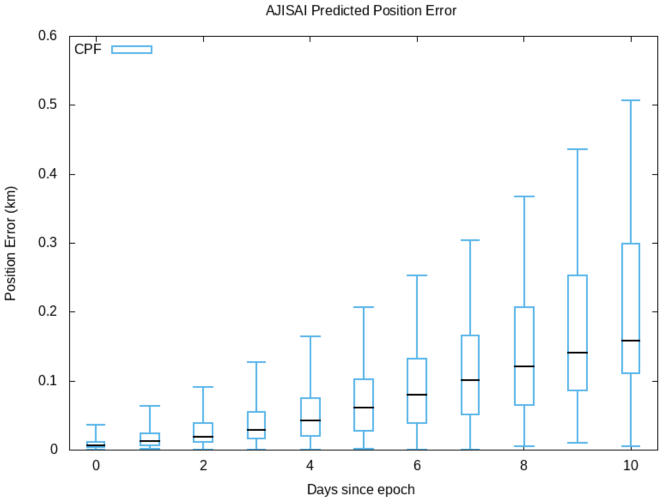

Fig. 10: AJISAI prediction errors for CPF displayed as box and whiskers.

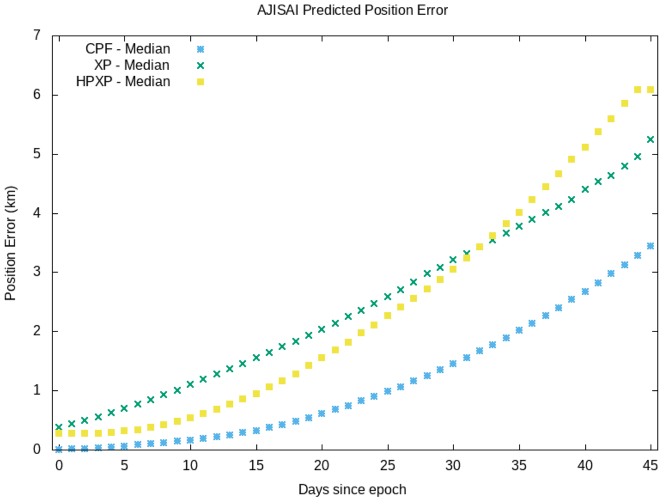

Fig. 11: AJISAI median prediction errors for SGP4-XP, HPXP, and CPF.

through the Space-Track website. We outline a simple architecture for incorporating the software into workflows. A basic implementation of these features as a set of webservices is available with an open source license[3].

Individual researchers that do not want to grant full trust to the binaries can use them from within a virtual machine or container (such as Docker) to prevent the code, while executing, affecting other systems or making unwanted outgoing network connections. No such misbehavior was observed by the authors when using the code. The authors make no claim either way whether other researchers should trust the code.

For groups on a network that wish to include SGP4-XP in a more complicated workflow, they can establish an isolated node in the network. The node can be fully air-gapped (less convenient) or merely restricted from making outbound network connections (more convenient).

Whether in a container or on an isolated node a web server will run hosting a few basic webservices. The first is an SGP4-XP ephemeris generation service. The source TLE, start date, end date, and time stamp are specified. The TEME position and velocity vectors are computed and returned. A similar service can take a series for SGP4-XP

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[3] `https://www.odutils.space/`

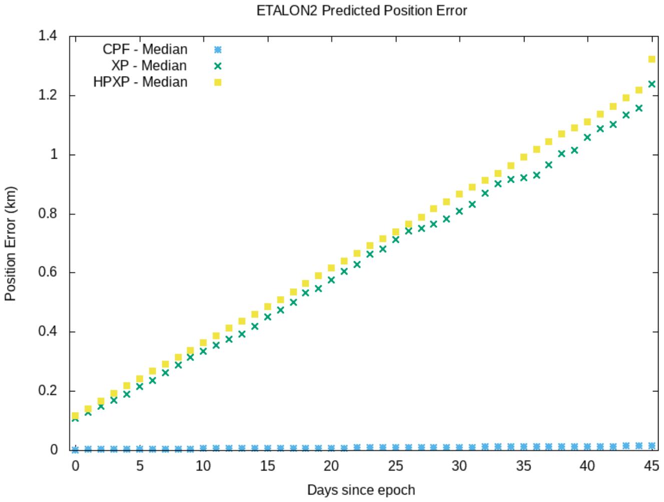

Fig. 12: ETALON2 median prediction errors for SGP4-XP, HPXP, and CPF. The Orekit model fit directly to the CPF data is a very close match even after 45 days.

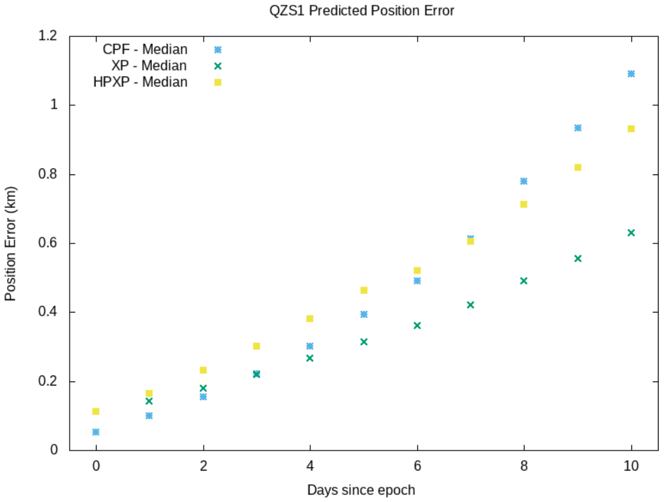

Fig. 13: QZS1 median prediction errors for SGP4-XP, HPXP, and CPF.

TLEs and generate a block of ephemeris that could be used as pseudo-observations on another machine. Finally, if the high-precision propagator that is of ultimate use is available to install in the same container or isolated node, the pseudo-observations can be processed by the orbit determination routine and a high-precision state vector with covariance returned. This allows practitioners to benefit from situations where SGP4-XP provides enhanced accuracy and obtain covariance information without exposing their workstations or networks to a potentially untrusted, third-party software binary.

6. SUMMARY

SGP4-XP is the first new model that uses the TLE format in almost 40 years. We demonstrated that there are situations where the accuracy exceeds that of SGP4, especially in higher orbits. The new drag model for LEO may require special handling when fitting the B term to receive the full benefit of the new model. We demonstrated how to combine multiple TLEs to obtain covariance information using standard pseudo-observation techniques. Finally we described an approach to incorporating the binaries into a software workflow using containers, network isolation, and web

Table 2: Prediction errors for SGP4-XP TLEs, HP fits based on SGP4-XP TLEs, and HP fits using the CPF data.

LEO Satellites										
	Fit Residual (km)	Day 1 Median Error (km)			Day 10 Median Error (km)			Day 30 Median Error (km)		
Name	SGP4-XP	SGP4-XP	HP SGP4-XP	HP CPF	SGP4-XP	HP SGP4-XP	HP CPF	SGP4-XP	HP SGP4-XP	HP CPF
AJISAI	0.34	0.43	0.27	0.01	1.01	0.52	0.20	3.21	2.73	1.51
Starlette	0.45	0.67	0.44	0.05	1.85	1.45	0.79	6.67	9.17	6.04
Stella	0.50	0.64	0.25	0.09	3.38	1.51	1.48	16.95	11.9	11.33
Swarm A	0.66	1.12	2.09	2.04	15.66	37.82	24.87	116.56	263.55	178.46

MEO Satellites										
	Fit Residual (km)	Day 1 Median Error (km)			Day 10 Median Error (km)			Day 30 Median Error (km)		
Name	SGP4-XP	SGP4-XP	HP SGP4-XP	HP CPF	SGP4-XP	HP SGP4-XP	HP CPF	SGP4-XP	HP SGP4-XP	HP CPF
ETALON 2	0.05	0.13	0.14	0.001	0.33	0.34	0.003	0.81	0.80	0.008
GALILEO 201	0.07	0.16	0.24	0.21	1.37	1.86	1.96	8.91	10.23	11.61
GLONASS 140	0.05	0.11	0.07	0.02	0.30	0.20	0.20	1.13	0.92	1.19

GEO Satellites										
	Fit Residual (km)	Day 1 Median Error (km)			Day 10 Median Error (km)			Day 30 Median Error (km)		
Name	SGP4-XP	SGP4-XP	HP SGP4-XP	HP CPF	SGP4-XP	HP SGP4-XP	HP CPF	SGP4-XP	HP SGP4-XP	HP CPF
IRNSS 1A	0.10	0.17	0.21	0.05	0.65	0.92	0.43	3.46	13.39	2.16
IRNSS 1C	0.12	0.24	0.23	0.01	0.59	0.51	0.07	1.26	9.05	0.76
QZS1	0.06	0.14	0.13	0.06	0.63	0.73	0.64	3.09	3.13	3.45

Table 3: The diagonals of the covariance matrix for best-fit Cartesian position and velocity vector fit to either XP TLE ephemeris or CPF ephemeris for AJISAI.

	Fit to XP	Fit to CPF
Rx	8.5	10.5
Ry	9.5	21.8
Rz	13.5	24.7
Vx	4.90 e-6	1.80 e-5
Vy	1.18 e-5	1.54 e-5
Vz	3.29 e-6	7.68e-6

services. Hopefully USSF will soon release SGP4-XP TLEs, the source code for the model, and a detailed description of the theory. The authors wish to acknowledge the use of CPF data provided by ILRS [5]. The authors also wish to thank Leonard Wilson for providing valuable feedback at all stages of this research.

7. REFERENCES

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