Scientific epistemology: How scientists know what they know

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Scientific inquiry is only one epistemological approach to knowledge. The author addresses several ways of knowing in science and contrasts them with other approaches to knowledge in order to better understand how scientists in general, and physicists in particular, come to know things. Attention in this article is focused on the processes of induction and deduction, observation and experimentation, and the development and testing of hypotheses and theories. This chapter takes a physicist's practical approach to epistemology and avoids such statements as "the transcendental deduction of the synthetic a priori" more typical of philosophers. Implications for teaching high school physics are included. This article is one of several chapters produced for the book Teaching High School Physics*, and is intended for use in high school physics teacher education programs at the university level.*

Epistemology

Epistemology concerns itself with ways of knowing and how we know. The word is derived from the Greek words *epistéme* and *logos* – the former term meaning "knowledge" and that latter term meaning "study of". Hence, the word parsed into English implies the nature, source, and limitations of knowledge. As such, the study of epistemology historically has dealt with the following fundamental questions:

- What is knowledge, and what do we mean when we say that we *know* something?
- What is the source of knowledge, and how do we know if it is *reliable?*
- What is the scope of knowledge, and what are its *limitations?*

Providing answers to these questions has been the focus of attention for a very long time. More than 2,000 years ago Socrates (c. 469 BC–399 BC), Plato (428/427 BC – 348/347 BC), and Aristotle (384-322 BC) wrestled with various answers to these questions, but were never able to resolve them. At best they were able only to provide "partial" answers that were attacked time and again by later philosophers the likes of Descartes (1596 – 1650), Hume (1711 –1776), and Kant (1724 – 1804). Not even these giants of philosophy were able to provide lasting answers to these questions, and, indeed, the discussion continues down to the present day. Even a more recently proposed solution to the definition of knowledge – defining knowledge as justified true belief (see Chisholm, 1982) – has failed in the light of arguments proposed earlier by Gettier (1962).

Philosophy and Science

Philosophy often interacts with science – especially physics – at many points and in countless ways. Scientists are often confronted with the question, "How do you know?" Providing an answer to that question frequently is not easy and often moves such a discussion into the field of scientific epistemology. Addressing this subject matter in a brief chapter is a task of great delicacy because, in order avoid being entirely superficial, one must strongly limit the subject matter that one touches upon and the depth of which it is addressed. Authors such as Galileo, Newton, Bacon, Locke, Hume, Kant, Mach, Hertz, Poincaré, Born, Einstein, Plank, Popper, Kuhn, and many, many others have written tomes in this area of the philosophy of science. The present author has been selective in choosing from among the many topics addressed by these authors on the basis of that which will be most suitable for physics teaching majors, and addressing these topics at a level consistent with their need for understanding. Science teachers need to understand the types of arguments that scientists use in actual practice to sustain the subject matter that they claim as knowledge.

Science is more than a conglomeration of facts, and teaching consists of more than just relating the facts of science. Science is a way of knowing that requires a strong philosophical underpinning (whether consciously sought of unconsciously learned). One cannot assume that students who understand the facts, principles, laws, and theories of science necessarily know its processes and their philosophical underpinning. They cannot be assumed to learn the philosophy of science by osmosis; it should be directly taught. It is hoped that the prospective physics teacher will, as a result of reading this chapter, more fully understand the nature and dilemmas of science. It is expected that this understanding will impact his or her teaching for the better. The author also hopes that this chapter sparks the interest in readers to the extent that they will find their way to reading more broadly in this critically important area.

Knowledge versus Faith

When historians say that they know something, is their type of knowledge the same as that of scientists when they say that they know something? Do sociologists speak with the same surety as scientists? When a theologian makes a proclamation, is the degree of certitude the same as that of a scientist? Frankly, the answer to all these questions is in the negative. Science, sociology, history, and religion each have their own ways of knowing and different types of certitude.

One fundamental question with which all scientists ultimately must reckon is how they actually *know* anything. Consider for instance the following statements:

- The Earth is a spheroid.
- The Earth spins daily on its axis.
- The Earth orbits the Sun annually.

Most readers will agree with these assertions, but how many of them actually *know* that the Earth is a spheroid, spins daily upon its axis, and orbits the Sun annually? Do they *know* these statements to be correct, or do they merely have *faith* that they are correct? The fact of the matter is that the vast majority of even physics majors will not know the basis for these statements that took scientists many years to develop. The facts underlying these understandings are by no means clear. Indeed, the philosopher-scientist Aristotle argued so eloquently against the motion of the Earth that his reasoning held sway for nearly two millennia. He argued that if the Earth were spinning we should feel the motion, encounter prevailing easterly winds, see the oceans cast off at the equator, and find that projectiles are left behind when thrown into the air – yet we see none of these! So, on what *basis* do current scientists make the above three claims? How do they *know* the answers; how do they *justify* their beliefs?

If a person claims to *know* something rather than merely have *faith* in something, then that person should be able to provide evidence to support the claim. If there is no support for the claim, then one has mere faith and not knowledge. Anyone who claims to know something should always be ready, willing, and able to answer the question, "How do you know?" Scientists – as should all science teachers – must always be watchful of embracing unjustified beliefs for in doing so they are merely embracing opinion. According to Blaise Pascal, "Opinion is the mistress of error; she cannot make us wise, only content."

The Nature of Knowledge

What then is knowledge? It appears that knowledge is to some extent a *justified belief.* In the not too distant past efforts were made to expand upon this definition by including an additional qualifier as in *justified true belief* Chisholm, 1982). Such a definition stated that we know X if, and only if,

X is true;
We believe X; and
We are justified in believing X.

Let's look at an example by considering the following argument:

- When someone jumps out of an open window, the person falls to the ground.
- We believe that when someone jumps out of an open window, the person falls to the ground.
- We are justified in believing that when someone jumps out of an open window, the person falls to the ground.

The first statement clearly has been the case since windows were invented or one can legitimately make that argument. However, might one not be equally justified in saying that someone who jumps out of an open window will fall to the ground *until next Tuesday at noon after which time people will then fall into the sky?* The inferential process based on experience could support both claims unless one makes a *presumption* about the nature of the world: the laws of nature are forever constant and apply the same way to all matter across both time and space.

This view is known as the Uniformity of Nature Principle, and is one upon which all science and scientists rely. It is based on a long human record of experiences with nature, and is supported even in our observations of outer space that show the same physical principles in operation over the entire universe and throughout the distant past.

How We Know in General

There are several ways of knowing things in general, but not all ways would be considered "scientific." Sociologists, historians, and theologians know things in ways quite different from that of scientists. Sociologist might refer to surveys and draw conclusions from demographic data. Historians might refer to primary sources such as written documents, photographs, and eyewitnesses; theologians might rely on scripture considered inspired or the word of God or on the work of a highly distinguished theologian. Scientists, however, would not make these sorts of claims as no scientist or scientific writing is considered the ultimate authority. All paths to knowledge, however, do apply human *reason* to a greater or lesser extent as a generic way of knowing.

Rationalism

Adherents of *rationalism* believe that logic is the source of knowledge. *Syllogisms,* one form of logic, can be used to derive knowledge if applied properly. Here we use a form of syllogism known to logicians as "modus ponens" reasoning. (There is an opposite form logical construct not dissimilar to this known as the "modus tollens" that denies a particular conclusion, but it will not be dealt with here.) The modus ponens syllogism takes the following form.

If A, then B;
A;
Therefore, B.

The first step of this logical argument is called the major premise; the second step is the minor premise; the third step is the conclusion. Consider the following argument that illustrates the modus ponens type of logical argument. If humans are cut, they will bleed. I am human. Therefore, when I am cut I will bleed. Sounds reasonable. But what is the problem with the following argument?

- If I can locate the North Star, I can use it to find north at night.
- I can locate the North Star because it is the brightest star in the night sky.
- Therefore, the brightest star in the night sky shows the direction north.

Many people will agree with the conclusion of this statement. If you are skeptical, go out and try this line of reasoning on a number of people. You will be amazed with how many will find the argument and conclusion perfectly acceptable. The problem with this statement, as you may well know, is that the conclusion is completely wrong. The major premise is correct; the minor premise is a broadly held misconception that leads to an incorrect conclusion. The North Star, Polaris, is the 49th brightest star in the night sky. Sirius, the Dog Star, is the brightest star in the night sky. Sirius rises roughly in the southeast and sets in roughly the southwest for observers in the mid northern latitudes where the North Star is plainly visible about half way up in the northern sky. Sirius is likely to "point" southeast or southwest near its rising and setting respectively, and south only when it is highest in the sky. Scientists tend to avoid the syllogistic approach to knowledge, as it is "empty". The conclusion cannot state more than what has been noted in the premises, and thus only makes explicit what has been stated previously.

Reason alone, without the support of evidence, is quite limited and subject to error. For example, consider the claim by Aristotle that heavier objects fall faster than lighter objects. This makes perfect sense in light of natural human reason. If a larger force is applied to an object, it accelerates at a higher rate. Now, if the earth is pulling on one object more than another, doesn't it make logical sense that the heavier object should fall faster? But despite human reason, experimental evidence shows that this is wrong. Barring friction, all objects accelerate at the same rate independent of their weight. If Aristotle had only known about Newton's second law, he would have understood that greater mass requires greater force to accelerate it thus canceling the "advantage" of weight over mass. Another example of the failure of reason can be exhibited in responding to the question, "What is the weight of smoke?" One might weigh an object before burning it and then measure the weight of the ashes. The difference between the two is the weight of the smoke. The process fails because it does not take into account the addition of oxygen from the air when it enters into the burning process.

We must keep in mind that one's outlook as well as lack of understanding can sway reason. As anyone who has examined the religious and political arenas will be aware, we tend to believe what we want to believe, and take facts as opinions if we do not agree, and opinions as facts if we do agree. We sometimes gain false impressions when we pre-judge someone or something on the basis of prior impressions. With all these critiques of pure reason, how can anyone actually ever know anything using the approach of rationalism alone?

Reliabilism

Adherents of *reliabilism* say that they are justified in knowing something only if that something is arrived at using a reliable cognitive process that extends beyond mere human reason. Less subjective than human reason and not subject to self-deception or human bias is *artificial inference* such as the rules of mathematics or Boolean logic. These are ideal approaches for deriving knowledge. Structured logic is the *sine qua non* of reliabilists. Consider for instance, the following knowledge derived from the axiomatic proofs of mathematics. From the relationship $4x + 2 = 10$ one can follow the rules of algebra to reliably conclude that $x = 2$. No question about it. But what can we conclude from the following manipulation where x is a variable and c a constant?

$$x = c$$

Now, multiply each side by x.

$$x^2 = cx$$

Next, subtract c^2 from each side.

$$x^2 - c^2 = cx - c^2$$

Factor.

$$(x + c)(x - c) = c(x - c)$$

Cancel the common term $(x - c)$.

$$x + c = c$$

Substitute c for x and combine.

$$2c = c$$

Cancel the common term c.

$$2 = 1$$

Now, does 2 really equal 1? Of course not. But why not? Clearly, we have arrived at a false conclusion because we have violated one of the rules of algebra. Can you tell which one? The point is that if a person is using artificial inference to derive knowledge, one must be exceedingly careful not to broach any of the rules of mathematics and logic – assuming that all are actually known.

Coherentism

Adherents of *coherentism* believe that knowledge is secure when its ideas support one another to form a logical construct, much like bricks and mortar of a building supporting one another to form an edifice. Knowledge is certain only when it coheres with similar information. To this means of knowing, *universal consent* can prove to be fruitful. According to the coherentist viewpoint, because "everyone" believes something that it must be so.

No one in their right mind would dispute the statements that Indiana is located between Ohio and Illinois, and that the Eiffel Tower is located in Paris. Many there are who have traveled to Indiana and Paris and know from personal experience the locations of the state and the tower. Besides, there are books and maps and internet references that all say the same thing. Everyone and everything, it seems, agrees with these statements. But be careful. Just because "everyone" believes something, doesn't necessarily make it so. It was once believed by nearly everyone that diseases resulted from humans having displeasured the gods, that the Earth was flat, and that the Earth stood unmoving at the center of the universe.

Coherentism lends itself to yet another way of knowing that can be similarly flawed, that of *perfect credibility.* To the medieval mind it was only reasonable that the Earth was at the center of the universe, the lowest point possible under the heavens. To medieval thinkers humanity was at the center of the universe not because of our noble status as the pinnacle of creation, but because we were so very despicable with our fallen nature. Closer to the center of the universe still was that place at the very center of the Earth that was reserved for the most despicable of all – hell. Those not so terribly bad were relegated to the underworld or Hades upon death, but not hell. This is the reason why the medieval viewpoint envisioned heaven as "up" and hell as "down." Man's position near or at the center of the universe was not pride of place; rather, it was a matter of making perfect sense in man's relationship with the deities. This belief was perfectly credible. Interpreting things in any other way would have made no sense given the then prevailing theological understanding. Still, such conclusions were flawed. Remember, all Aristotle's evidence and argumentation at one time pointed to the fact that the Earth was stationary, but today we know that it spins daily upon it axis and revolves annually around the Sun which is just one of billions of stars located in a typical galaxy, one of billions seemingly scattered almost entirely at random around a universe that has no evident center.

Credible authority is another way of knowing based on coherentism, and it is the way that almost everyone has come to "know" what they claim know about the universe. It is this approach that is often used in schools to teach children. The teacher is the authority figure; the children are empty vessels to be filled with "knowledge". While this viewpoint is quite wrong, it does have its uses – and also its limitations. Let's look at the following questions. What is your name? How do you know? Is Labor Day a legal holiday in the USA? How do you know? You know your name because those entitled to name you at birth, your parents, did so. They are credible authorities as only parents have a right to name their children. We know that Labor Day is a national holiday because the United States Congress declared by law that it should be so in 1894. By their legal authority, parents and Congress have performed an act by the very power vested in them. Relying entirely on this approach to knowing can be problematic in many situations as not all authorities are credible. For instance, many religious sects claiming to possess the "truth" preach contradictory beliefs; they can't all be correct. Psychics might intentionally make false claims in order to influence the direction of lives. Financial consultants might seek to mislead clients in an effort to achieve financial gain.

There are several unresolved problems associated with coherentism. When ideas or beliefs conflict, it is not possible to tell which one is to be accepted. How do we distinguish a correct idea from an incorrect idea when incorrect ideas sometimes are consistent with what we already know, or a new idea conflicts with what we "know" to be correct? How do we distinguish a better or more important idea from one less so? What role does bias play a role in our ability to distinguish correctly? Coherentism, it appears, is unable to provide meaningful answers to these questions.

<u>Empiricism</u>

Adherents of classical *empiricism* (a type of empiricism perhaps best suited to teaching high school physics) believe that logic, connected to verification though observation or experimentation, leads to knowledge. The empirical approach to knowledge consists of reason constrained by physical evidence. For example, reason in conjunction with observation helps scientists know that the Earth is spheroidal. Careful observers will note that the North Star descends below the northern horizon for travelers crossing from north to south of the equator at any longitude, that the masts of ships disappear long after the hull when ships travel over the horizon in any direction, circumnavigation of the globe being possible in any direction, and the shadow of the Earth on the moon during a lunar eclipse at any time of night are all pieces of evidence that one can logically use to conclude that the Earth is roughly spherical. Observation in conjunction with reason will lead to no other conclusion.

In its simplest form, one might know something through *personal experience.* If one's hand is burned by a hot piece of metal, one knows it and has the evidence to prove it. One's hand might be red and painful as with a first degree burn, or there might be blisters with excruciating pain as with a second degree burn, or there might even be charred flesh with an acrid smell as in a third degree burn. One's belief is substantiated with evidence; hence, one can support a belief with evidence. One's belief in a burned hand is not merely a matter of faith; one actually possesses knowledge based on reason sustained by ample evidence. One must be careful, however, of assuming that personal experience is the final arbiter of whether or not an experience provides incontrovertible evidence. Some concrete experiences can be interpreted or viewed in different ways. The failure of eyewitnesses to provide identical interpretations is a good example of this. In the case of a robbery, the person who has a gun shoved into his or her face might remember things about the perpetrator of the crime quite differently from someone who witnessed the act from a hidden location. One's perspective can, indeed, influence what one sees or remembers, or how one interprets evidence. People don't always draw the same conclusion based on the same evidence either. In the case of the traditional "boy who called wolf" story, two conclusions can be drawn – either don't lie, or don't tell the same lie more than once!

Improvements in technology can lead to increased precision in observations. Refined observations can then lead to overturning knowledge based on reason and new observations. The history of science is littered with evidence-based models now discarded that were once thought to constitute knowledge. A review of the history of scientific models – the solar system, evolution, the atom, the nature and origin of the universe, the nature and cause of gravitation, predator-prey relationships, genetics, heat and energy – all point to the fact that scientists spend a great deal of time building, testing, comparing and revising models in light of new evidence.

As history shows, even scientific knowledge is tentative. This is so for more than one reason: (1) scientists presume the Uniformity of Nature principle and to the extent that this presumption is wrong, our conclusions based upon it are similarly wrong; and (2) what is accepted at any one point in time by the converged opinion of institutional science is what constitutes established scientific knowledge. Borrowing a page from the book of coherentism, when all the indicators suggest that something is correct, it is assumed to be so until new empirical evidence overrules it. Scientists therefore do not claim to possess "truth" as such because this would constitute something that is known now and forever to be correct, and totally consistent with reality. To make a claim of possessing "truth" would be worse than presumptuous.

This is not to say that scientific knowledge is "weak". The vast majority of what we teach in high school science – especially physics – is not likely to change. Quite the contrary. Our understanding of momentum, energy, optics, electricity, magnetism, and such, is extremely well supported and there is no reason to believe that it ever should change. It is for this reason that scientists say they their knowledge is tentative, while at the same time durable.

Induction, Deduction, and Abduction

Induction and *deduction* are at the heart of empiricism. In the process of induction, one generalizes from a set of specific cases; in the process of deduction, one generates specifics from a general rule. Induction can be thought of as a search for generality; deduction can be thought of as a search for specificity. A very simple example will suffice to explain the concepts of induction and deduction.

Suppose a person goes to a roadside fruit stand wanting to buy sweet apples. The fruit stand owner offers up some slices of apples as samples. Taking a bite of one sample our shopper finds that it is sour. He examines the apple and sees that it is hard and green. He then takes another sample and finds that it too is hard, green, and sour. Before picking a third sample our shopper observes that all the apples are hard and green. He departs having decided not to buy any apples from this fruit stand concluding they are all sour.

Granted, two samples is a very minimal basis for performing induction, but it suffices for this example. If one were to examine the thought process that was used by our would-be buyer, one would determine that this is how he reasoned:

All hard and green apples are sour;
these apples are all hard and green;
therefore, these apples are all sour.

We have seen this form of reasoning before and recognize it as a modus ponens form of syllogism. Our shopper has performed an inductive process that relied on specific cases of evidence to generate a general rule. Note then the next lines of the shopper's reasoning:

Because all of the apples are sour,
I do not want to purchase any of these apples.

When the shopper decides to depart the fruit stand without purchasing any apples he does so on the basis of deduction. Using the conclusion established via induction, he made a decision via deduction to leave without purchasing any apples.

Scientists rarely use the syllogistic process when they deal with the subject matter of science because they are not interested in drawing "empty conclusions" about material objects. For instance, "All light travels in straight lines; we have light; therefore, what we have is traveling in straight lines" contributes nothing to scientific knowledge or understanding. To justify the claim that light travels in straight lines we must make observations that lead observers to this conclusion. Data related to the phenomenon must be accounted for in terms of this principle.

Abduction is at the heart of generating explanations in science. It is the process of creating hypotheses. The formulation of hypotheses – constructs designed to provide predictions and explanations – begins with examination of available evidence and devising an explanation for it. Abduction sometimes relies upon analogies with other situations. In the previous example, one might conclude from knowledge that sugar gives the taste of sweetness to those things that contain it, that natural sugars are absent in hard green apples. This would explain the lack of sweetness in the apples sampled at the fruit stand. The statement that hard green apples are sour because they lack natural sugars present in sweet apples is a hypothesis derived by abduction. They hypothesis serves to explain why the samples of hard green apples all tasted sour.

Some authors have falsely claimed that hypotheses are generated from the processes of induction. This is incorrect. Inductive processes can only provide general statements and, as such, cannot explain anything. The relationships between induction, deduction, and abduction are shown in Table 1.

Intellectual processes and their connections to science
Induction is most closely related to the generation of principles and laws in science. Principles identify general relationships between variables such as "When water is heated in an open container, it evaporates." Laws identify specific relationship between certain observable quantities such as "The period of a pendulum is proportional to the

square root of its length." Principles and laws are descriptive, and almost without exception can be stated in a single formulation, and have no explanatory power. Laws and principles are established on the basis of direct evidence. Principles and laws are resilient because they are based directly on observational evidence and not upon a hypothesis or theory. Even when a hypothesis or theory that explains them is proven false (e.g., Wien's displacement law with the failings of classical electrodynamics, Balmer's spectral law in the light of the failed Bohr model), principles and laws survive the demise of the hypothesis or theory.
Deduction is most closely related to the generation of predictions in science – the process of using principles, laws, hypotheses, or theories to predict some observational quantity under certain specified conditions.
Abduction is most closely related to the generation of hypotheses in science – tentative explanations that almost always consist of system of several conceptual statements. A hypothesis, because it often deals with unobservable elements, often cannot be directly tested via experiment. An example of this would be electron theory that notes that electrons are carriers of an elementary charge, the assumption of which served as the basis of the Millikan oil-drop experiment. Sometimes, the sole basis for accepting hypotheses is their ability to explain laws, make predictions, and provide explanations. For instance, Newton's formulation of gravity was accepted on the basis that it was able to account for Kepler's three laws of planetary motion. So it was with Copernican theory, the corpuscular theory of light, atomic theory of the Periodic Table, and the kinetic theory of gases. Bohr's model for the atom and Einstein's special and general theories were similarly accepted on the basis of their ability to make accurate predictions and provide explanations.

Table 1. *Connections between intellectual processes and scientific nomenclature.*

Induction in Science

Central to the inductive process in science is observation. *Observation* is key to many sciences. Biologists, for instance, learn about the lives and behaviors of animals by making observations. They accumulate a large amount of data about, say, gorillas, and how they interact under certain conditions. Geologists likewise collect data by studying minerals and maps, examining rock formations, and reviewing earthquake data from their seismographs. Meteorologists similarly collect data about the weather such as temperature, barometric pressure, relative humidity, wind speed and direction, and so forth. Scientists do not stop there, however. Raw data per se are of little use, and no scientific journal will publish long lists of data. Scientists are not merely "cameras" expected to record data (Bronowski, 1965). Rather, it is only when they synthesize conclusions based on observations that they are doing the work of scientists. (See sidebar story 1.)

SIDEBAR STORY 1

Induction and the Genius of Isaac Newton

Isaac Newton (1643-1727, Julian calendar) used induction as the basis of what is known today as his theory of gravitation. Now, the story of Newton sitting under an apple tree seeing an apple fall and thinking about the form of gravitation is probably apocryphal. Nonetheless, it could have occurred to Newton that the fall of an apple is not unlike the fall of the Moon as it orbits the Earth. It was the fact that he was able to understand the relationship between the Moon's and the apple's acceleration that constitutes the genius of Isaac Newton. Couched in modern SI terms, and *using the simplifying assumption of circular motion,* this is what Newton did. First, he realized that the acceleration of, say, an apple near the surface of the Earth was

$$a_{\oplus} = 9.8\frac{m}{s^2}$$

He then calculated the centripetal acceleration of the Moon in its orbit around the Earth by using an equation first provided by the Dutch scientists of his day:

$$a_{\supset} = \frac{v^2}{r}$$

The speed of the Moon's motion was easily derived from the relationship into which he put the proper values for the orbital radius of the Moon and its orbital period (both known with a relatively high degree of precision in Newton's day)

$$v = \frac{d}{t} = \frac{\text{circumference}}{\text{period}} = \frac{2\pi r}{P} = \frac{2\pi(3.84x10^8 m)}{2{,}360{,}000s} = 1020 m/s$$

Using the equation for centripetal acceleration, he then came up with the value of the Moon's acceleration

$$a_{\supset} = \frac{(1020 m/s)^2}{384{,}000{,}000 m} = 0.00271 m/s^2$$

He then compared the acceleration of objects near the Earth's surface with that of the Moon in orbit and found

$$\frac{a_{\oplus}}{a_{\supset}} = \frac{9.8 m/s^2}{0.00271 m/s^2} = 3600$$

He then realized that 3600 could well represent the ratio of the Moon's orbital radius to the Earth radius squared.

$$\frac{a_{\oplus}}{a_{\supset}} = 60^2 = \left(\frac{r_{\supset}}{r_{\oplus}}\right)^2$$

From this formulation, Newton surmised that the acceleration of an object (be it the Moon or an apple) is inversely proportional to its distance from the center of the Earth squared (and perhaps where he first realized that the Earth acts as though all its mass is concentrated in a point at its center). That is,

$$a \propto \frac{1}{r^2}$$

Given the fact that $F = ma$, Newton concluded that the force required to hold the Moon in its orbit around the Earth was also dependent upon the mass of the moon, m. That is,

$$F \propto \frac{m}{r^2}$$

Because gravity is responsible for the perceived weight of objects, and would likely be proportional to the mass of the Earth, M, as well as the moon, Newton further hypothesized that,

$$F \propto \frac{Mm}{r^2}$$

Inserting the proportionality constant, k, gave Newton his final formulation for the force due to gravity.

$$F = k\frac{Mm}{r^2}$$

It wasn't until the 1797-1798 experimental work of Henry Cavendish (1731-1810) that the value of k was determined. Once he did so, the k was replaced with a G giving us the now familiar expression

$$F = \frac{GMm}{r^2}$$

So, it should be evident from this work of induction that Newton's act of creative genius was in the fact that he was able to use observational evidence to formulate a relationship to determine the nature of the central force required to keep objects in orbital motion. Edmund Halley (1646-1742) used Newton's formulation of gravity and observations of an earlier bright comet to predict its return. That comet, now named Halley's Comet, returned as predicted in the year 1758. Later Urbain Leverrier (1811-1877) and John Couch Adams (1819-1892) independently used Newton's formulation of gravity to analyze the irregular motions of the planet Uranus, and predict the location of a hitherto unknown planet – Neptune – discovered in 1846. These cases used Newton's formulation of the force due to gravity to make predictions and, as such, are examples of deduction.

Principles and laws are inferences that result from the generalization of different types of data. Principles are general relationships between observable properties. As the day progresses and the land warms, warm air rises over the land and is replaced by cool breezes that blow from the sea to the land. We see that when air warms, it expands and thereby gaining buoyancy. We see that living organisms require energy in order to survive. We see the conservation of energy in its many forms. We see that objects fall to the ground when left unsupported. We conclude that light travels in straight lines. These are all principles of science. The empirical laws of science are more abstract than general principles in the sense that they typically incorporate mathematics in their expressions. Examples of laws in physics are numerous, and would include such things as the law of levers, the law of pulleys, the law of mechanical advantage, the laws of kinematics and dynamics, the laws of thermal expansion, the conservation laws in mass, energy, and charge, Newton's second law of motion, Ohm's law, the laws for series and parallel circuits, the thin lens formula, Snell's law, and the laws of relating to heat and change of state, Boyle's law and the ideal gas law. All relate mathematic variables in precise ways. These are all "simple" examples of induction based on experimentation.

There are many examples of more sophisticated forms of induction where scientists have linked areas of physics to arrive at a new and more meaningful understanding. Isaac Newton did this by linking motion to force; Michael Faraday did this by connecting electricity with magnetism; James Clerk Maxwell did this by unifying electromagnetism with light; Albert Einstein did this by interfacing time with space, mass with energy, and force with geometry. It was the ability of these scientists to make sense of information that gave value to their ideas, and allow us to call them genius.

Observation and *experimentation* are central to the inductive process. But physical laws, primarily those of classical physics, were initially derived with the use of experimentation. No amount of observation would have allowed a casual observer to discover any of the laws mentioned above. These are empirical relationships based controlled experimentation.

Deduction in Science

One of the main goals of scientists and engineers is to perform deductive processes. Scientists use inductive processes to formulate principles, laws, hypotheses, and theories from which they can then deduce predictions. For example, applications of various empirical laws such as $\Sigma \boldsymbol{F} = m\boldsymbol{a}$, $\Delta V = IR$, and $\Delta L = \alpha L_o \Delta T$ can be used to predict future situations under certain conditions. One can, given the force on and mass of a vehicle, predict its acceleration.

Applying a voltage difference across an electrical network with a known resistance, one can predict the consequent current. Heating a particular rod of known length and composition by a certain amount, one can determine in advance what the change in length will be. Almost every piece of technology that we have today has been designed using the deductive process. This is true on a vast scale, from nanotechnology to an aircraft carrier.

Astronomers are observationalists par excellence and are very good at applying what they know from Earth-based studies to deduce knowledge about celestial objects. They cannot bring planets, comets, stars, nebulae, or galaxies into the laboratory for experimentation. They do, however, apply principles, laws, hypotheses, and theories to their observations in order to learn about celestial objects. For instance, Edwin Hubble was able to use the distances and motions of remote galaxies to determine the age of the cosmos. Using variants of the Hertzsprung-Russell diagram, astronomers were able to deduce how it is that stars are born, live out their lives, and die even though the process can take millions or billions of years. Using the laws of thermodynamics and nuclear theory, astronomers have been able to discover how it is that stars operate. Earlier than any of these examples, astronomers made use of Newton's universal law of gravitation and observations of an orbiting moon to deduce the mass of Jupiter. (See sidebar story 2.)

SIDEBAR STORY 2

Deduction of the Mass of Jupiter

A generation before Newton, Johannes Kepler (1571-1630) enunciated three planetary laws of motion based upon observations of the planet Mars made earlier by Tycho Brahe (1546-1601). Kepler stated these laws roughly as follows:

1. Planets move in elliptical orbits around the Sun with the Sun located at one of the foci.
2. The radius arm between a planet and the Sun sweeps out equal areas in equal time intervals.
3. The period of a planet expressed in years squared equals the semi-major axis of the orbit expressed in astronomical units (equal roughly to the average Earth-Sun distance) cubed. That is,

$$P^2 = r^3$$

If the units other than years and astronomical units are used (e.g., SI units), then the form of the equation would be expressed as

$$P^2 = (\text{constant})r^3$$

where the value and units of the constant would depend upon the units employed in the equation's other variables. At this point Newton, with his second law, the definition of centripetal acceleration, and his new formulation of gravity, was able to write

$$F = ma = \frac{mv^2}{r} = k\frac{Mm}{r^2}$$

Substituting for $v = (2\pi r/P)$ and simplifying the two rightmost components of this equation, Newton arrived at the following relationship

$$P^2 = \frac{4\pi^2 r^3}{kM} = (\text{constant})r^3$$

which is Kepler's third or harmonic law! Newton's formulation of the law of gravity therefore was able to explain the origin of the harmonic law– it's due to the fact that gravity is an inverse-squared force. Newton's hypothesis then, with this firm underpinning, was on its way to becoming theory.

It should be noted, too, that Newton's more detailed analysis of the central force problem resulted in a prediction of elliptical motion. That is, when gravitational force is assumed to drop off with in inverse-square of the distance, then elliptical motion results. This is precisely what Kepler observed. Newton's law of gravitation, $F = Gm_1m_2/r^2$, was also used to explain Kepler's law of equal areas. These derivations are beyond the scope of this book, but provide additional bases that led to the universal acceptance of his formulation of the law of gravitational force.

Note that the above formulation of Kepler's harmonic law is for the simple case that assumes purely circular motion. In reality, the solar system's moons and planets move with barycentric motion. That is, the sun and planets, the planets and the moons orbit the centers of mass in they systems. Taking this consideration into account (*and retaining our assumption of circular motion for simplicity*), Newton was able to derive a more precise form of the Harmonic law

$$(M + m)P^2 = \frac{4\pi^2 (R + r)^3}{k}$$

This relationship later was employed to measure the masses of various solar system bodies using solar mass units for mass and astronomical units for distance of measure long before the space age. For instance, if the mass of a moon of Jupiter, *m,* is taken to be very small in relation to the mass of Jupiter, *M,* and the distance of Jupiter from its barycenter (*R*) very small in relation to the distance of the moon from its barycenter (*r*), then we can simplify the above relationship

$$MP^2 = \frac{4\pi^2 r^3}{k} \quad (\text{assuming } m << M \text{ and } R << r)$$

In more modern form, the relationship can be written at follows:

$$M = \frac{4\pi^2 r^3}{GP^2} \quad \text{(assuming } m << M \text{ and } R << r\text{)}$$

A series of observations of the Jovian moon Ganymede shows that it has an orbital period of 618,100*s* (7.154 days) and a mean orbital radius of 1,070,000,000*m.* Putting these data into the equation with the proper value and units for *G* results in a mass for Jupiter of 1.89 x 10^{27}*kg*. Hence, the mass of Jupiter has been deduced from theoretical considerations integrated with observations. Fly-by missions to the planet later confirmed this deduction.

Deduction takes different forms, from the mundane to the complex. These extremes in this article are typified by using a formula to predict the outcome of a particular situation, to using observational evidence and a hypotheses or theory to determine the mass of Jupiter. Deductions – and some will say predictions – are characterized by two logical conditions (Nagel, 1961): (1) the premises must contain at least one universal law, hypothesis, or theory whose inclusion is essential for the deduction, and (2) the premises also must contain a suitable number of initial conditions. These latter conditions constitute an "if – then" combination. For instance, if the voltage difference is ΔV and the current I in an electrical circuit, then the effective resistance must be $\Delta V/I$.

Observations inform us about the past and present, and reason in the form of a logical deduction can be used to predict the future. The law of levers can be used to predetermine combinations of force and distance that will balance one another. In a more sophisticated sense, a knowledge of Newton's second law, $\Sigma \mathbf{F} = m\mathbf{a}$, can be used to predict the first and third laws as special cases of the more general form of the second law.

The knowledge of the past and present is known with relative certainty compared to knowledge of the future. Still, if we are willing to accept the assumptions about the nature of the universe (uniformity, causality, etc.), then we must conclude that the predictive methods of science are tenable, and we can in a sense foresee and foretell the future. The worth of any such prediction can only be measured in relation to its verification. If a prediction is verified, this lends credence to the universal law, hypothesis, or theory upon which the prediction was made.

The Hypothetico-deductive Method

Closely linked with the scientists' use of induction and deduction is the process of hypothetico-deduction. This is a simple and effective method of advancing the frontiers of science and, in many cases, increasing our understanding of nature. The basic gist behind this method is the formulation and testing of hypotheses. That is, hypotheses can be generated from simple observations. Hypotheses, tentative explanations, then result in predictions that necessarily must follow from a hypothesis, and if corroborated with empirical evidence, sustained. As Popper (1962) noted, scientific hypotheses are conjectures that have a potential for being refuted. If the evidence disconfirms the hypothesis, the hypothesis is either rejected or modified. Well-sustained hypotheses become theories, the value of which can be judged only in relation to their ability to make further predictions and explain more observations in order to account for diverse physical phenomena. Hypotheses are well thought out explanations that incorporate evidence, not mere guesses as is all too often implied by the use of this term in the vernacular. Also to be avoided is the phrase "educated guess" which a hypothesis clearly is not. Neither are hypotheses to be confused with predictions, as is too often the case in even the science classroom.

To help clarify the meaning of a hypothesis and relate it to predictions, consider the following very simple example. A physics student who has just completed a study of energy looks at the following kinematics relationship and thinks she "sees" a conservation principle contained within it.

$$v^2 - v_0^2 = 2ad$$

Working under the hypothesis that this kinematic law derived from observation has the form it does because it incorporates conservation of energy, the following prediction is made: If kinematic laws hold because they are based on the conservation of energy, then kinematic laws should be derivable from the statement W=ΔE, the work-energy theorem. The student sets to work.

$$\Delta E = W$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = Fd = mad$$

and, after multiplying both sides by $\frac{2}{m}$, she gets

$$v^2 - v_0^2 = 2ad$$

So, this supports the basic assertion that kinematic laws hold because they are based on the conservation of energy. But does this derivation "prove" anything? Not necessarily. The outcome is merely consistent with the assumed basis for this particular kinematic relationship. Now, if conservation of energy is the basis of kinematic relationships (assumed free from resistance), then conservation of energy should also be visible in all other kinematic laws as well. We should be able to derive kinematic relationships from the work-energy theorem and visa versa. Consider the following derivation:

$$d = d_0 + v_0 t + \frac{1}{2}at^2$$

$$(d - d_0) = v_0 t + \frac{1}{2}at^2$$

$$F(d - d_0) = mav_0 t + \frac{1}{2}m(at)^2$$

and given that $v - v_0 = at$

$$W = mav_0 t + \frac{1}{2}m(v - v_0)^2$$

$$W = mav_0 t + \frac{1}{2}m(v^2 - 2vv_0 + v_0^2)$$

$$W = mav_0 t + \frac{1}{2}mv^2 - mvv_0 + \frac{1}{2}mv_0^2$$

$$W = mav_0 t + \frac{1}{2}mv^2 - m(v_0 + at)v_0 + \frac{1}{2}mv_0^2$$

$$W = \frac{1}{2}mv^2 - mv_0^2 + \frac{1}{2}mv_0^2$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$W = \Delta E$$

The working hypothesis that kinematic relationships hold due to conservation of energy appears to be borne out. The fact of the matter is that even the definitions of acceleration and average velocity shown in the relationships $v = v_0 + at$ and $d - d_0 = \overline{v}(t - t_0)$ also can be derived from the work-energy theorem and visa versa, but these derivations are left for the student. (See the results of the anticipated student work at the end of this document.)

The insight that conservation of energy is responsible for the form of kinematic equations is crucial for their appropriate application. They are valid only so long as energy is conserved. To the extent that energy is not conserved in a particular situation (e.g, friction), the kinematic equations are invalid. While this is a very simplistic example of the hypothetico-deductive method, it suffices to show how the process works and to explain some of the understanding that can be derived from such an approach.

Perhaps a better example of the formulation of a hypothesis in physics would be in developing an explanation of the source of the buoyant force (F_B) experienced by objects immersed in a fluid of density ρ. Noting that law that states that pressure (p) increases with depth ($p = \rho gd$), one can calculate the differences in the forces due to a fluid on the top and bottom surfaces of an imaginary cube of dimension A ($F = pA$) at different depths. This difference in these two forces amounts to the buoyant force experienced, and can even predict the value of the buoyant force from the relationship so derived. That is, $F_B = \rho Vg$. (See sidebar story 5 in Wenning (2005) for a detailed explanation.)

Empiricism in Science

Scientific knowledge is belief based on reason and empirical evidence; while it is tentative, it is still quite durable and, in most cases of established science treated in high school, unlikely to change. A scientific understanding of nature is an understanding that has been tested against the empirical evidence that nature provides, and not found wanting; a scientific law, hypothesis, and theory can be tested against empirical evidence with the use of predictions.

Nature itself is the final arbiter in any disagreement between principles, laws, hypotheses, and theories developed by scientists. Prior to the scientific revolution, scientific knowledge was based upon ancient authorities, especially Aristotle. Religious dogmas, particularly those proposed by Thomas Aquinas (1225-1274 AD), also played a pivotal role in the establishment of knowledge that intruded upon the 1633 trial of Galileo. After the scientific revolution, facts, principles, laws, hypotheses, and theories were subject to objective judgment in the light of empirical evidence.

Galileo's telescopic observations during the early part of the 17th century showed Ptolemy's model of the solar system to be wrong, but did not confirm that the model proposed by Copernicus was correct. In fact, later observations showed that even Copernicus was incorrect. Neither did Galileo's observations eliminate a competing model of the solar system, the Tychonic system, which quite admirably accounted for Galileo's observations. In this model, the Earth was at the center of the known universe and the Sun orbited the Earth daily. The planets in turn orbited the Sun. Galileo's observations were not inconsistent with this alternative model. It wasn't until adequate observations were made that it became clear that the Keplerian model of the solar system that dispensed with the perfect circular motion of Copernicus and replace it with elliptical motion, was correct. Incontrovertible empirical evidence of the Earth's motion wasn't obtained until Bradley observed the aberration of starlight (1729), Bessel discovered the parallax of the double star 61 Cygni (1838), and later empirical evidence in the mid to late 19th century such as Doppler shifts in stellar spectra and deflections of falling bodies came to bear.

Over the course of the years human ingenuity and reason have triumphed over ignorance. Humans have interacted with nature in a variety of forms – the formulations of principles and laws from observations, the creation and development of hypothesis, and ultimately theory formation. These all require creativity and increasingly sophisticated forms of observation that includes technology, and give rise to a more and more sophisticated understanding of nature. This is in no way more true than in the development of theories. Theories are the hallmark of scientific understanding. They are consistent with established knowledge, they unify data and account for hitherto unexplained data, they sometimes point to relationships that previously have gone unnoticed, they explain and often predict. These are all hallmarks of Darwin's theory of Evolution, Mendeleev's periodic table, Wegener's theory of plate tectonics, Einstein's theory of Special Relativity, and Watson and Crick's Double Helix model of DNA. The theories of science represent the pinnacle of scientific knowledge, yet they all are subject to judgment and revision in light of new scientific evidence.

Scope and Limitation of Scientific Knowledge

Scientific knowledge, because its conclusions ultimately are based on empirical evidence, cannot provide answers to questions that do not have an empirical basis. Science cannot, for instance, determine the number of angels that can dance upon the head of a pin; neither can it prove nor disprove the existence of a god. It cannot deal with questions of faith or morals, or controversial subject topics such as eugenics, stem cell research, abortion, and so forth. It cannot be used to make human value judgments. It can, however, inform these decisions by providing appropriate information that can be used in making decisions about these issues. As science teachers, we must be careful not to overstep the bounds established by reliance on human reason and empirical evidence. We must be careful to avoid letting our students feel as through science can solve all problems.

Some statements that scientists accept as correct at first appear to be scientific but are not because they can be shown to be falsifiable. (Note that a statement does not have to be correct to be scientific under Popper's principle of falsifiability. See Popper, 1963.) For instance, consider the following statement derived from induction, "All copper conducts electricity". As surprising as it might seem, this is not a scientific statement because it cannot be refuted. This statement can be proven if and only if all copper everywhere in the universe has been tested. This is a practical impossibility. The statement that all copper conducts electricity can be refuted with but a single case – which has yet to be found. Still, to find this single case might take an untold amount of time. Pragmatic vindication of induction, however, is possible. Scientists have decided to believe that the results of induction are correct because we presume that the entire population has the same traits as exhibited in a sample. This is the Uniformity of Nature principle, and is a presumption upon which all scientific knowledge rests.

Even simple scientific laws such as $\Delta V=IR$ have their limitations, but these limitations are often left unstated. Consider, for instance, a 750-Watt bread toaster. At 120 volts this toaster draws 6.25 amperes implying an internal resistance of 19Ω. Could one reasonably expect to use a standard 9-volt battery to power this toaster? Why or why not? If one were to use a 9-volt battery, it would have to supply nearly ½ amp of current, something far beyond the capacity of the battery to provide. A battery of this type in this situation would be considered "non-Ohmic" as Ohm's law fails to hold for this combination of circuit elements. Similarly, a light bulb filament – as it passes from a non-glowing state to a glowing state – has a significant change of resistance during the "turn on" phase. The tungsten that makes up the bulb has a resistance that is temperature dependent. Hence, a statement of the resistance of a length of filament L and cross section A whose resistivity is ρ would be more complex than the commonly stated law

$$R = \frac{\rho L}{A}$$

Likewise, experimental test results that corroborate a hypothesis or theory do not prove that it is correct; rather, what it implies is that the hypothesis or theory has not yet been shown to be false. When experimental evidence shows that predictions turn out to be wrong, then the hypothesis or theory from which they are generated is shown to be either incomplete or wrong. Like the principles or laws, corroboration of a hypothesis or theory has nothing to do with its confirmation.

The verification process used in science is much more extensive than in the example with apples. Scientific verification procedures are intentional, intense, and international in scope. All laws generated through induction must be put to every conceivable test and under varying conditions on a universal basis before it is said to be worthy of such a name. Even so, statements derived from induction will always be subject to doubt and can never provide us with absolute certainty. Nonetheless, we apply principles, laws, hypotheses and theories as though they are correct beyond any reasonable doubt. This pragmatic approach is taken because work on a day-to-day basis does not necessarily depend upon absolute certainty. Suffice it to say that established scientific opinion is an adequate basis for most action as evidence has shown.

Lastly, we must be careful to properly understand an authentic meaning of the word "explanation" in science. Sometimes it is stated that the reason an object at rest remains at rest or an object in motion retains the same state of motion unless some unbalanced force is acting upon is it due to inertia. At other times it is noted that bodies gravitate toward one another due to gravitational forces. Both "inertia" and "gravity" are pseudo-explanations. These terms are just different labels for the facts stated in the principles so expressed. Explanations must in a sense be "more general" than the phenomena being explained (Nagel, 1961).

Implications for Teaching High School Physics

So what does scientific epistemology have to do with teaching high school physics, or any other science at this level? The author has heard this question from both physics teacher candidates and inservice physics teachers. The answer to this question is very important, and should not be left to the inference of the reader. Simply put, the answer is this. An understanding of scientific epistemology should have an influence on the way one teaches.

Consider the traditional lecture-based physics classroom. What do we see? In many cases the course mostly appears to revolve around two teaching/learning strategies, lectures by the teacher and reading of the textbook by the student. If one is lucky in such a classroom, every once in a while there will be a demonstration or a confirmatory lab in which students replicate an experiment following explicit instructions showing that the instructor or textbook is "correct". Now, compare this to religion. Typically learning is based on teaching from sacred texts (e.g., Torah, Bible, Koran, etc.) and a preacher (rabbi, minister or priest, mullah, etc.) explaining the content therein. When science teachers base student learning primarily on a textbook and lecture, aren't they essentially preaching "faith" in science based upon authority rather than science as an active mode of inquiry? Science is both a body of knowledge and a way of knowing. To teach the content of science without the process is to teach history, not an active pursuit of scientific knowledge.

If a teacher is to teach in a way that is consistent with scientific ways of knowing, then he or she must help students to construct knowledge and understanding from their experiences. The teacher's method should consistent largely of asking questions, and guiding students in such a way as to find answers to their questions. The students will learn when their attention is directed to certain points focusing on relevant information, and drawing conclusions. It's only when one helps another to see things with his own eyes that he can be said to be a teacher. Still, we must be careful not to allow the educational pendulum swing too far one way. Science teaching should not be thought of as an either/or situation, inquiry-oriented versus transmission-oriented instruction. Both have their place in implementation of the curriculum.

Still, teaching on the basis of authority, even in science, has its benefits. Nowhere more clearly can this seen than in post-introductory courses in science. It would be unreasonable in these courses to think that every result should be based on first-hand experiences and experiments. At some point students have to understand that the converged opinion of institutional science is, in the main, quite credible, but this should not be done in an introductory course where teachers need to instruct students in both the content and processes of science.

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Solutions of problems "left to the student".

$v = v_0 + at$ $vt = v_0 t + at^2$ now, $d = d_0 + v_0 t + \frac{1}{2}at^2$ $2(d - d_0) = 2v_0 t + at^2$ $2(d - d_0) - 2v_0 t = at^2$ hence, $vt = v_0 t + 2(d - d_0) - 2v_0 t$ $vt = 2(d - d_0) - v_0 t$ $vt + v_0 t = 2(d - d_0)$ $F\frac{vt}{2} + F\frac{v_0 t}{2} = F(d - d_0)$ $\frac{mavt}{2} + \frac{mav_0 t}{2} = W$ $\frac{mat}{2}(v + v_0) = W$ now, $v - v_0 = at$ $\frac{m}{2}(v - v_0)(v + v_0) = W$ $\frac{m}{2}(v^2 + vv_0 - vv_0 - v_0^2) = W$ $\frac{m}{2}(v^2 - v_0^2) = W$ $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = W$ $\Delta E = W$	$d - d_0 = \bar{v}(t - t_0)$ $d - d_0 = \frac{(v + v_0)}{2}t$ where $t_0 = 0$ $F(d - d_0) = \frac{ma(v + v_0)}{2}t$ $W = \frac{1}{2}mvat + \frac{1}{2}mv_0 at$ but, $v - v_0 = at$ $W = \frac{1}{2}mv(v - v_0) + \frac{1}{2}mv_0(v - v_0)$ $W = \frac{1}{2}mv^2 - \frac{1}{2}mvv_0 - \frac{1}{2}mv_0^2 + \frac{1}{2}mvv_0$ $W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ $W = \Delta E$

