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# CALCULATION OF THE MAGNETIC FIELD IN THE FERROMAGNETIC LAYER OF A MAGNETIC DRUM

BY

OLLE KARLQVIST

KUNGL. TEKNISKA HÖGSKOLAN
VETENSKAP OCH KONST

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# CALCULATION OF THE MAGNETIC FIELD IN THE FERROMAGNETIC LAYER OF A MAGNETIC DRUM

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ELANDERS BOKTRYCKERI AKTIEBOLAG

## 1. Introduction

A magnetic drum is used to store information. The information may come from an electronic computer or from a telephone dial. The magnetic drum consists of cylinder on the surface of which a ferromagnetic material is coated. It is equipped with reading and recording heads, which enables binary data to be recorded in the form of magnetized elements on its surface when the drum rotates. When a pulse is read in, it is stored as an elementary magnet with a north and a south pole. The length of this dipole is of the order of magnitude of 3 millimetres. The drum contains a number of channels, the distance between them being usually a few millimetres.

The drum has been proved to be a reliable store and it is used in most electronic computers. Drums have recently found applications in telephony (MALTHANER and VAUGHAN, 1953) and other branches where storing of information is necessary.

In this paper we study the field in the ferromagnetic layer and the variation of this field with permeability, airgap, layer thickness and other influencing factors. The problem is definitely non-linear and extremely difficult to solve. But the linear case gives a first approximation, which in some cases seems to be satisfactory. Here we solve the linear boundary value problem for the two-dimensional static field and the one-dimensional transient field. WALLACE 1953 has solved the stationary linear case by assuming a sinusoidal surface magnetizing field and has found very good agreement with measurements. In an unpublished report by D. HUNTER, WALLACE's method is generalized to space magnetizing and the demagnitizing effects are also studied. WESTMIJZE 1953 has made a very thorough study of the problem, but all parts of it are not yet published.

The drum for which the numerical computations have been made is that of the Besk, the Swedish electronic computer. This work was, however, done after this drum was designed. The pulse frequency is assumed low enough to neglect eddy current losses in the head and layer, that are made of a spinel material.

## 2. Main results

Below are given the analytical expressions for the magnetic field in three different cases. The numerical values of the magnetic field for special values of the parameters can be found in sections nr 6 and 7.
A recording head of normal construction is shown in fig. 1. Usually the most interesting region is around the gap and this is shown in fig. 2.

The three cases are:

1. The permeability in the layer is 1.
2. The permeability in the layer is greater than 1, but the layer is infinitely thick.
3. As 2, but the layer is finite.

The notations are

$d$ = layer thickness

$\mu$ = layer permeability

$N$ = half the pole distance

$b$ = distance head — layer

$B_0$ = induction in the pole gap measured in voltsecond per square metre

$B_0 = \mu_0 V/N$, where $V$ is the magnetic potential of the head.

$\mu_0 = 4\pi \cdot 10^{-7}$ in the *MKSA* system.

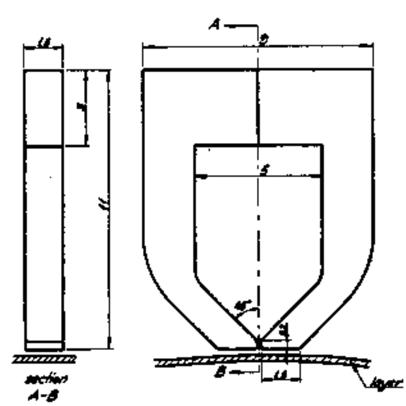


Fig. 1. Kernel to a recording head used in the Beak. The airgaps are exaggerated. Measurements in millimetres.

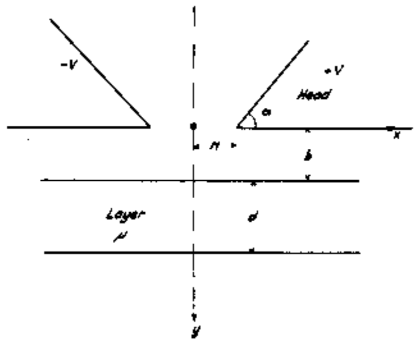


Fig. 2. The recording head in the neigbourhood of the airgaps.

The expressions in the three cases for the magnetic flux $B(x, y) = (B_x, B_y)$ at $y = b$ (inside the layer) are:

1.
$$B_x = -\frac{1}{\pi} B_0 \left( \operatorname{arctg} \frac{N + x}{y} + \operatorname{arctg} \frac{N - x}{y} \right)$$

$$B_y = \frac{1}{2\pi} B_0 \log \frac{y^2 + (N + x)^2}{y^2 + (N - x)^2}$$

2.
$$B_x = -\frac{2\mu}{\pi} B_0 \int_0^\infty \frac{dt}{t} \frac{\cos \frac{tx}{b} \sin \frac{tN}{b}}{\mu \sinh t + \cosh t}$$

$$B_y = \frac{2\mu}{\pi} B_0 \int_0^\infty \frac{dt}{t} \frac{\sin \frac{tx}{b} \sin \frac{tN}{b}}{\mu \sinh t + \cosh t}$$

Asymptotic expansions (see appendix 1 for definition) for the B-fields are:

$$B_x = -B_0 \frac{2Nb\mu^2}{\pi x^2} \left[ 1 - \frac{b^2}{x^2} \left( 6\mu^2 - \frac{N^2}{b^2} - 5 \right) + \ldots \right]$$

$$B_y = B_0 \frac{2N\mu}{\pi x} \left[ 1 - \frac{b^2}{x^2} \left( 2\mu^2 - 1 - \frac{N^2}{3b^2} \right) + \ldots \right]$$

$$3. \quad B_x = -B_0 \frac{4\mu}{\pi} \int_0^\infty \frac{dt}{t \cdot K} \cos\frac{tx}{b} \sin\frac{tN}{b} \left(\sinh\frac{td}{b} + \mu \cosh\frac{td}{b}\right)$$

$$B_y = B_0 \frac{4\mu}{\pi} \int_0^\infty \frac{dt}{t \cdot K} \sin\frac{tx}{b} \sin\frac{tN}{b} \left(\mu \sinh\frac{td}{b} + \cosh\frac{td}{b}\right)$$

$$K = (\mu + 1) \sinh t\left(1 + \frac{d}{b}\right) + (\mu - 1) \sinh t\left(1 - \frac{d}{b}\right) +$$
$$+ \mu(\mu + 1) \cosh t\left(1 + \frac{d}{b}\right) - \mu(\mu - 1) \cosh t\left(1 - \frac{d}{b}\right)$$

Asymptotic expansions are:

$$B_x = -B_0 \frac{2bN\mu}{\pi x^2} + \dots$$

$$B_y = B_0 \frac{2N}{\pi x} - \dots$$

The expressions in the three cases are derived by assuming linear magnetic potential along $y = 0$. Thus

$$\begin{array}{ll} v = -V & x < -N \\ v = V \cdot x/N & -N < x < N \\ v = +V & x > N \end{array}$$

This linear potential between the corners is the result of investigations made for the cases $\mu = 1$ and $\mu = \infty$ treated with conformal mapping.

The results may have their greatest interest when estimating the effect on the field due to the drum eccentricity, layer thickness, airgap, layer permeability and so on. In the manufacturing of drums this may be of interest in order to keep mechanical tolerances below certain values.

## 3. Idealization of the problem

The first approximation is to regard the drum surface as a plane. The variation of the distance $b$ (fig. 2) due to the curved surface is about 10 % for the interval $0 < x < 10\,N$. The quotient $b/N$ is usually between 0,5 and 2. The drum diameter is 120 millimetres and the gap is about 0,02 millimetres. The length of the head is about 100 times the gap width, so we assume that the head has infinite length.

The width of the head is also about 100 times the gap width and this shows that it is satisfactory to treat the two-dimensional problem only. An investigation shows that the field along a generatrice is very flat under the head.

The permeability of the head is about 1000 for the frequencies encounted, and this means that the lines of force leave the head nearly perpendicularly. The magnetic potential of the head is therefore assumed constant, and is $+V$ on the right half and $-V$ on the left half of the head.

In order to investigate how the pole length (0,3 mm in fig. 1) influences the field in the layer, the angle $\alpha$ is introduced. If $\alpha = 0$ this length is zero, and if $\alpha = 90°$ the length is infinite. This has already been studied by BOOTH 1952 and is included here only because we study the potential between the corners of the head.

## 4. The boundary value problem

The problem is to find the magnetic potential $v(x, y)$ in the region $y > 0$, $-\infty < x < \infty$ (fig. 2) when the potential along $y = 0$ is prescribed. The magnetizing vector is then

$$H = -\operatorname{grad} v(x, y) \tag{1}$$

The potential satisfies the equation:

$$\frac{\partial}{\partial x} \mu \frac{\partial v}{\partial x} + \frac{\partial}{\partial y} \mu \frac{\partial v}{\partial y} = 0 \tag{2}$$

If $\mu$, the permeability of the layer, is a constant, we get Laplace's equation:

$$\Delta v = 0 \qquad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{3}$$

Usually equation (2) is a non-linear equation. The boundary conditions along $y = b$ and $y = b + d$ read:

$$\frac{\partial v_1}{\partial y} = \mu \frac{\partial v_2}{\partial y} \tag{4}$$

$$\mu \frac{\partial v_3}{\partial y} = \frac{\partial v_4}{\partial y} \tag{5}$$

where

$v_1$ = the potential above the layer $(y = b - 0)$

$v_2$ = the potential in the layer $(y = b + 0)$

$v_3$ = the potential in the layer $(y = c - 0)$

$v_4$ = the potential below the layer $(y = c + 0)$

$\mu$ = the permeability of the layer

The non-stationary one-dimensional field can be computed from the equation ($\mu$ is a constant):

$$\frac{\partial^2 H}{\partial x^2} = \sigma \mu \mu_0 \frac{\partial H}{\partial t} \tag{6}$$

$\sigma$ = conductivity of the layer

Equation (6) is a parabolic equation, while equations (2) and (3) are elliptic equations. In the special cases $\mu = 1$ and $\mu = \infty$ equation (3) is solved by means of conformal mapping. The general case is solved by means of Fourier transforms. The non-stationary case is solved by means of Laplace transforms.

## 5. The special cases $\mu = 1$ and $\mu = \infty$

The four most interesting cases are treated by conformal mapping (Weber 1950). We use the notations

$$H_0 = V/N$$

$$H_1 = V/b$$

1. $\mu = \infty, \alpha = 90°$.

Because of the symmetry it is sufficient to consider the part ABCDEF in fig. 3.

The Schwarz-Christoffels integral gives the following expressions for the mapping function:

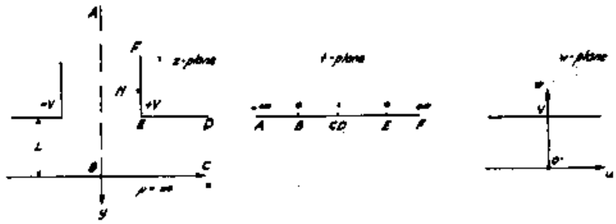

Fig. 3. The case $\mu = \infty$, $\alpha = 90°$.

$$\bar{z} = x - j\,y = \frac{2\,N}{\pi}\,\mathrm{arctg}\left(\frac{N}{b}\,\mathrm{tgh}\,p\right) + \frac{2\,b}{\pi}\,p \tag{7}$$

$$w = -\frac{v}{\pi}\log(1-t) \tag{8}$$

$$H = H_x + j\,H_y = j\,H_1\,\mathrm{tgh}\,p \tag{9}$$

The parameter $p$ is defined by

$$t = \frac{a\,N^2\,\mathrm{tgh}^2\,p}{b^2 + N^2\,\mathrm{tgh}^2\,p} \tag{10}$$

The field along BC has been computed for $b/N = 0{,}5$, 1 and 2 and is plotted in fig. 4.

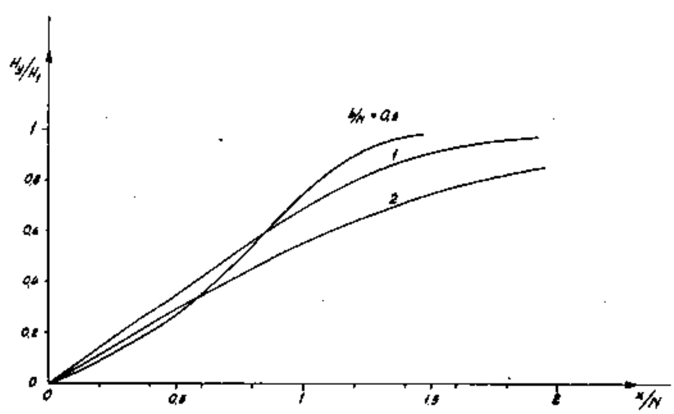

Fig. 4. The magnetic field on the surface of the layer. $\mu = \infty$, $\alpha = 90°$.

2. $\mu = \infty, \alpha = 0°$

The mapping function is:

$$\bar{z} = \frac{2\,b}{\pi}\left(\frac{a}{1-a}\,\text{tgh}\;p + p\right) \tag{11}$$

$$w = -\frac{V}{\pi}\log\left(1 - \text{tgh}^2\,p\right) \tag{12}$$

$$H = j\,H_1\,\frac{\sinh 2\,p}{\cosh 2\,p + \dfrac{1+a}{1-a}} \tag{13}$$

Here,

$$t = a \cdot \text{tgh}^2\,p \tag{14}$$

The parameter $a$ is to be solved from a transcendental equation.

The following values are obtained:

| $b/N$ | $a$ |
|---|---|
| 0,5 | 0,62056 |
| 1 | 0,34511 |
| 2 | 0,12821 |

The field along BC is shown in fig. 6.

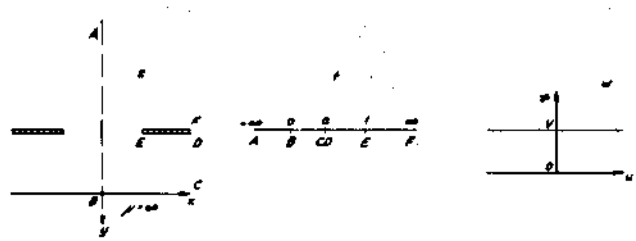

Fig. 5. The case $\mu = \infty$, $\alpha = 0°$.

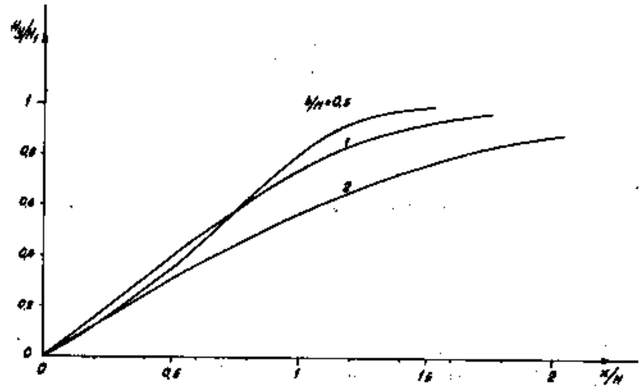

Fig. 6. The magnetic field when $\mu = \infty$, $\alpha = 0°$.

3. $$\mu = 1, \alpha = 90°$$

The mapping function is:

$$z = \frac{2N}{\pi} (\coth p - p) \tag{15}$$

$$w = -\frac{2V}{\pi} \log (\sinh p) \tag{16}$$

$$H = -H_0 \operatorname{tgh} p \tag{17}$$

$$t = \coth^2 p \tag{18}$$

The mapping function has been used in order to calculate the potential between the two corners in fig. 7. To compute the field for different $b/N$ as in the cases above is a tremendous task. It is much easier to compute the field due to linear potential between the corners and then compute the difference field due to the difference between the linear potential and the actual potential between the corners.

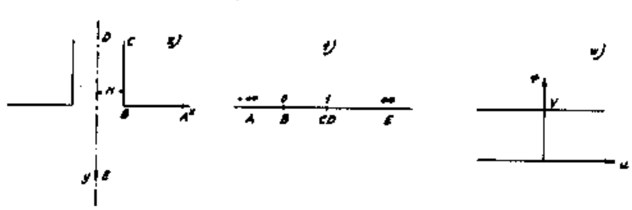

Fig. 7. The case $\mu = 1$, $\alpha = 90°$.

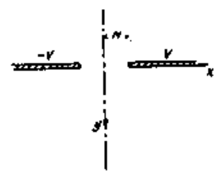

Fig. 8. The case $\mu = 1$, $\alpha = 0°$.

4. $$\mu = 1, \alpha = 0°$$

The field can be explicitely expressed in $z$:

$$H = -\frac{2 H_0}{\pi \sqrt{1 - \frac{z^2}{N^2}}} \tag{19}$$

The variation of the potential between the corners of the head is computed for the four cases and is plotted in fig. 9. The linear potential is also drawn, and the question is whether this linear potential can be a good approximation for computation of the field. We have for the linear potential

$$\begin{aligned} v &= -V & x &< -N \\ v &= V \cdot x/N & -N < x &< N \\ v &= V & x &> N \end{aligned} \tag{20}$$

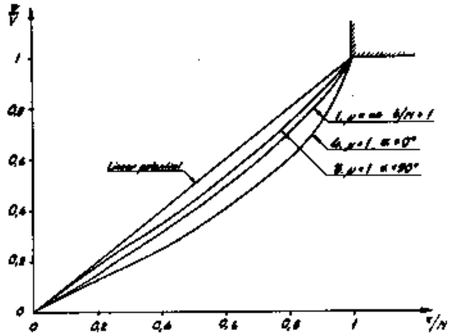


Fig. 9. The potential between corners for different cases.

The potential implies that we get a discontinuity in $H_x$ instead of a singularly at the point $(0, N)$. $H_y$ has still a singularity at that point. In practice we have no singularities because the corners are rounded off. The influence of rounded corners has been studied by COCKROFT 1928.

The potential $v(x, y)$ due to a known potential $f(x)$ along the line $y = 0$ can be computed from:

$$v(x, y) = \frac{1}{2b} \sin \frac{\pi y}{b} \int_{-\infty}^{\infty} f(t) \frac{dt}{\cosh \frac{\pi (t - x)}{b} - \cos \frac{\pi y}{b}} \tag{21}$$

If $b$ is infinite we use the formula:

$$v(x, y) = \frac{y}{\pi} \int_{-\infty}^{\infty} f(t) \frac{dt}{y^2 + (t - x)^2} \tag{22}$$

Evaluating the integral (21) for the function defined by equation (20) we get the field for $\mu = \infty$.

$$H_y(x, b) = H_0 \frac{1}{\pi} \log \frac{\cosh \frac{\pi (x + N)}{2b}}{\cosh \frac{\pi (x - N)}{2b}} \tag{23}$$

The field for different $b/N$ is shown in fig. 10. If we use equation (22) we get the field for $\mu = 1$, and this is shown in fig. 11. The field is always computed from equation (1).

The field is given by the eqs.

$$H_x(x, y) = -H_0 \frac{1}{\pi} \left( \operatorname{arctg} \frac{N + x}{y} + \operatorname{arctg} \frac{N - x}{y} \right) \tag{24}$$

$$H_y(x, y) = H_0 \frac{1}{2\pi} \log \frac{y^2 + (N + x)^2}{y^2 + (N - x)^2} \tag{25}$$

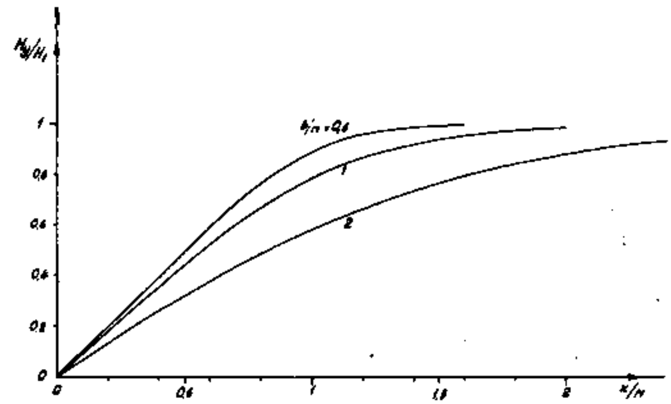

Fig. 10. The magnetic field when $\mu = \infty$ and linear potential between corners.

In the last three eqs. we have the field given explicity as simple functions of $x$ and $y$, and it is very easy to compute an actual field. The approximation is found to be satisfactory for $y$-values greater than $0{,}5 \cdot N$. An estimation of the error involved can be done by using the integrals (21) or (22), where the function $f$ is taken to be the difference between the linear potential and the actual potential between the corners. Another method is to solve Laplace's difference equation instead of the differential equation (Karlqvist 1952) and as this is often favourable for numerical computation, it has been used here.

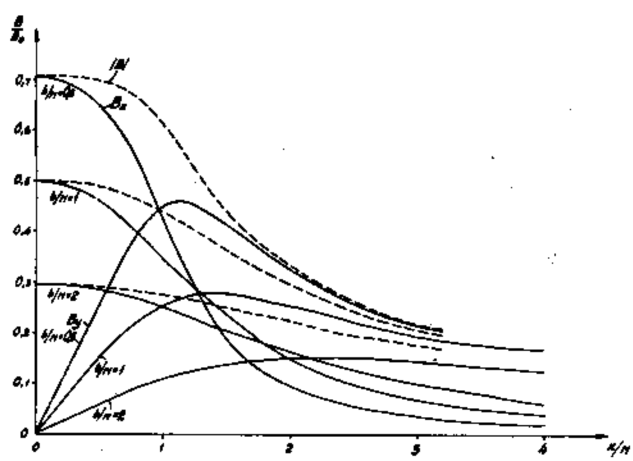

Fig. 11. The magnetic field when $\mu = 1$ and linear potential between corners.

## 6. The general case with finite μ but infinite layer thickness

In the last section it was shown that the linear potential is a good approximation to the special cases $\mu = 1$ and $\mu = \infty$, when $y > 0{,}5\,N$. Since the magnetic field is a continuous function of $\mu$ we obtain a good approximation also when $\mu$ is finite.

The equations of the general case are solved in appendix 2. In this section we consider the case when the layer thickness is infinite, which gives easier analytic expressions than having finite thickness. The equations for the field components along the layer surface $y = b$ are according to appendix 2:

$$B_{2x} = -B_0 \frac{2\mu}{\pi} \int\limits_0^\infty \frac{dt}{t} \frac{\cos\frac{tx}{b}\sin\frac{tN}{b}}{\mu \sinh t + \cosh t}$$

$$= -B_0 \frac{2\mu}{\pi(\mu+1)} \sum_0^\infty \left(\frac{\mu-1}{\mu+1}\right)^n \left(\operatorname{arctg}\frac{x+N}{b(2n+1)} - \operatorname{arctg}\frac{x-N}{b(2n+1)}\right) \quad (26)$$

$$B_{2y} = B_0 \frac{2\mu}{\pi} \int\limits_0^\infty \frac{dt}{t} \frac{\sin\frac{tx}{b}\sin\frac{tN}{b}}{\mu \sinh t + \cosh t}$$

$$= B_0 \frac{\mu}{\pi(\mu+1)} \sum_0^\infty \left(\frac{\mu-1}{\mu+1}\right)^n \log\frac{(N+x)^2 + b^2(2n+1)^2}{(N-x)^2 + b^2(2n+1)^2} \quad (27)$$

$$B_0 = \mu_0 H_0$$

The field $B_{2y}$ is measured inside the layer.

Figs. 12 and 13 show the fields for different $\mu$ and $b/N$.

The asymptotic expansions can be used to compute the tails of the fields and this gives information of the interaction of the pulses on the drum. The total field is

$$|B| = \sqrt{B_x^2 + B_y^2} \quad (28)$$

and are the dotted lines in fig. 10, where the fields for the case $\mu = 1$ are plotted.

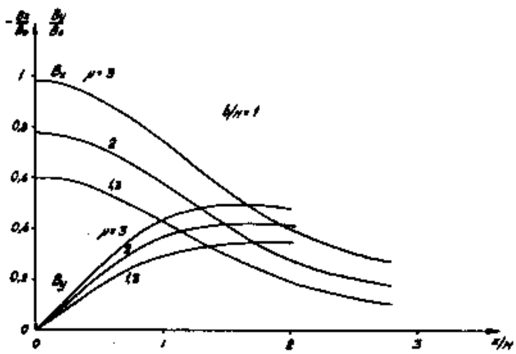

Fig. 12. The magnetic field in the layer for various $\mu$ and $b = N$.

The asymptotic expansions are according to appendix 2:

$$B_{2x} = -B_0 \frac{2 N b \mu^2}{\pi x^2} \left[1 - \frac{b^2}{x^2} \left(6 \mu^2 - \frac{N^2}{b^2} - 5\right) + \dots\right] \quad (29)$$

$$B_{2y} = B_0 \frac{2 N \mu}{\pi x} \left[1 - \frac{b^2}{x^2} \left(2 \mu^2 - 1 - \frac{N^2}{3 b^2}\right) + \dots\right] \quad (30)$$

At the point $x = 0$ the infinite sum in equation (26) can be put in closed form, namely:

$$B_{2x}(0, b) = -B_0 \frac{2 N \mu}{\pi b \sqrt{\mu^2 - 1}} \operatorname{arcosh} \mu$$

which shows that the $B$-field along the line $x = 0$ $(y > b)$ approaches infinity in the same way as $\log \mu$. This means that the field approaches infinity very slowly with an increase in $\mu$.

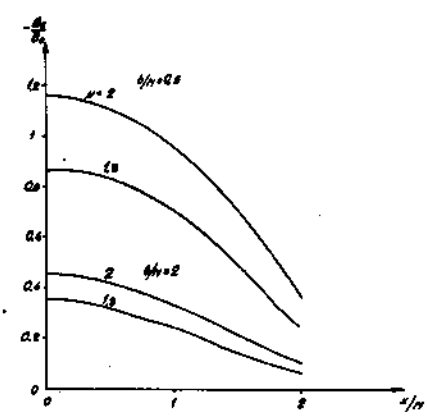

Fig. 13. The magnetic field in the layer for various $\mu$ and $b/N = 0{,}5$ and 2.

## 7. Finite μ and layer thickness

In the general case the field are given by:

$$B_{2x} = - B_0 \frac{4\mu}{\pi} \int\limits_0^\infty \frac{dt}{t \cdot K} \cos\frac{tx}{b} \sin\frac{tN}{b} \left(\sinh\frac{td}{b} + \mu \cosh\frac{td}{b}\right) \quad (31)$$

$$B_{2y} = B_0 \frac{4\mu}{\pi} \int\limits_0^\infty \frac{dt}{t \cdot K} \sin\frac{tx}{b} \sin\frac{tN}{b} \left(\mu \sinh\frac{td}{b} + \cosh\frac{td}{b}\right) \quad (32)$$

where $B_{2x}$ and $B_{2y}$ are the fields at $y = b + 0$, that is, the field inside the layer. At the other side of the layer $y = b + d - 0$ we have

$$B_{3x} = - B_0 \frac{4\mu^2}{\pi} \int\limits_0^\infty \frac{dt}{t \cdot K} \cos\frac{tx}{b} \sin\frac{tN}{b} \quad (33)$$

$$B_{3y} = B_0 \frac{4\mu}{\pi} \int\limits_0^\infty \frac{dt}{t \cdot K} \cdot \sin\frac{tx}{b} \sin\frac{tN}{b} \quad (34)$$

The function $K$ is defined by:

$$K = (\mu + 1) \sinh t \left(1 + \frac{d}{b}\right) + (\mu - 1) \sinh t \left(1 - \frac{d}{b}\right) +$$

$$+ \mu (\mu + 1) \cosh t \left(1 + \frac{d}{b}\right) - \mu (\mu - 1) \cosh t \left(1 - \frac{d}{b}\right)$$

The corresponding asymptotic expansions are:

$$B_{2x} = - B_0 \frac{2bN\mu}{\pi x^2} + \dots \quad (35)$$

$$B_{2y} = B_0 \frac{2N}{\pi x} - \dots \quad (36)$$

$$B_{3x} = - B_0 \frac{2bN\left(\mu + \frac{d}{b}\right)}{\pi x^2} + \dots \quad (37)$$

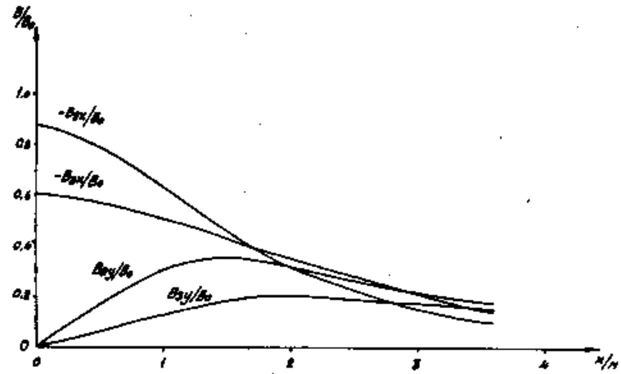


Fig. 14. The magnetic field for $\mu = 2$ and $d = b$.

$$B_{3y} = B_0 \frac{2N}{\pi x} - \ldots \tag{38}$$

The next term in the asymptotic expansions is found in appendix 2. In fig. 14 are plotted the fields when $b/N = 1$ and $d/N = 1$, $\mu = 2$.

The deviation from the case when $\mu = 1$ is not very large. If the field is to be computed for many values of $b/N$ it is convenient to write the factor

$$\cos \frac{t\,x}{b} \cdot \sin \frac{t\,N}{b}$$

in the form

$$\frac{1}{2}\left(\sin \frac{(N+x)\,t}{b} + \sin \frac{(N-x)\,t}{b}\right)$$

At a large distance from the origin the field is determined by $B_{2y}$ eq. (36). This formula shows that the field is independent of $\mu$ and depends of $x/N$ only. The same conclusion applies to $B_{3y}$ and thus to the whole layer.

The linear case where $\mu$ is a constant can be used as a first approximation to the nonlinear case. The hysteresis loop for the material is shown in fig. 15 and if we use the initial magnetizing curve, we find that the $\mu$-values vary from 1,1 at the origin to 2 at $x = 4\,N$. The induction $B_0$ was then 2500 gauss. The material is thus saturated for $x = \pm\, 4\,N$.

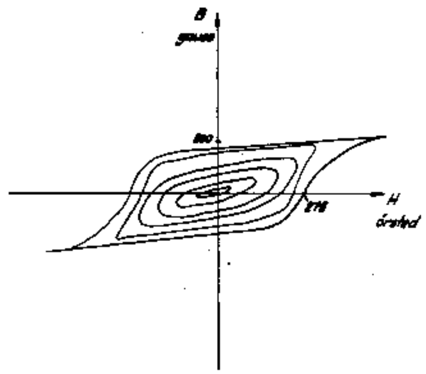


Fig. 15. The hysteresis loop for the layer material.

## 8. Pole and layer inductance

The magnetic flux is divided into two parts: the first part containing the flux going through the layer and the second part going through the pole gap (0,3 mm in fig. 1). The corresponding inductances are called $L_l$ and $L_p$.

By integrating round the head we readily found:

$$L_p = \frac{n^2 A \, \mu_0 \, \mu_h}{l_h + 2 \, N \, \mu_h}$$

where

$A$ = the area of the pole = 0,45 mm² for the head of fig. 1.
$n$ = number of turns
$l_h$ = the mean circumference round the head
$\mu_h$ = the permeability of the head.

The inductance $L_l$ can be computed from eq. (24) if $\mu$ is small. Numerical computations shows that the contribution to this inductance due to the layer is very small and can normally be neglected.

We have

$$L_l = \frac{n^2 \, aN \, \mu_0 \, \mu_h}{\pi \, l_h + 2 \, \pi \, N \, \mu_h} \ln \left(1 + \frac{b^2}{N^2}\right) \frac{N^2 + (N + b)^2}{N^2 + (d + b)^2}$$

$a$ = the width of the head (1,5 mm in fig. 1)

Actual values gives the result

$$L_p = 3 \text{ mH}$$

$$L_l = 0{,}3 \text{ mH}$$

This shows that we could save power if we keep the pole inductance low. The total inductance was 4,2 mH, that also includes the inductances from the sides of the head.

In order to verify the result we measured the small contribution to the inductance $L_l$ when layer is present and not present. The voltage from the head was put in a bridge that was balanced when layer was not present. The unbalance when layer was present was found by THEVENIN's theorem to correspond to an inductance change of 0,04 mH. The computed value according to the formulas for the field was 0,036 mH. The quotient $b/N$ was 1. Measurements was also made for greater values of this quotient and the result was fairly satisfactory.

## 9. The transient field

In this section we solve equation (6) for infinite layer and for finite layer. The $x$-coordinate in (6) is to be replaced by $y$.

We assume that a polarized electro-magnetic wave with the components $H_x$ and $E_z$ comes perpendicular to the layer. The wave is applied suddenly at $t = 0$ and the airgap $b$ is assumed to be zero. The initial value problem is for infinite layer:

$$H\,(0, t) = H_0; \quad H\,(\infty, t) = 0; \quad H\,(y, 0) = 0;$$

Then the Laplace transform of the equation (6) can be written:

$$h = \frac{1}{s} \cdot H_0\, e^{-k y \sqrt{s}}$$

$$k^2 = \sigma\, \mu\, \mu_0$$

The corresponding time function is

$$H\,(y, t) = H_0 \operatorname{erfc} y \sqrt{\frac{\sigma\, \mu\, \mu_0}{4\, t}}$$

erfc is defined in appendix 1.

Considering the finite layer, we must add the condition that $E_z$ is continuous at the point $y = d$ (= other side of the layer).

$$\lim_{y \to d - o} \sigma_2 \cdot \frac{\partial H}{\partial y} = \lim_{y \to d + o} \sigma_1 \frac{\partial H}{\partial y}$$

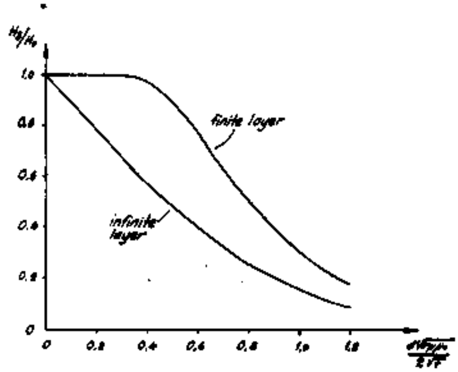


Fig. 16. The magnetic field in the transient case.

where $\sigma_1$ and $\sigma_2$ represent the conductivity on each side of the boundary. If the conductivity $\sigma_1$ of the layer is zero (spinel material) compared with the drum material (usually brass) we have the Laplace transform

$$h = H_0 \frac{e^{-(y-d)k\sqrt{s}}}{\cosh k d \sqrt{s}}$$

and the time function

$$H(d, t) = 2 H_0 \sum_{n=0}^{\infty} (-1)^n \operatorname{erfc}\left[(2n+1) d \sqrt{\frac{\sigma \mu \mu_0}{4t}}\right]$$

Fig. 16 shows the two cases, infinite layer and finite layer, from which one can compute the transient time for a given material.

## 10. Summary

The magnetic field in the ferromagnetic layer on a magnetic drum is calculated having finite airgaps and finite permeability in the layer. The layer is assumed to be a spinel material for which the permeability is low. The special cases when the permeability is one and infinite is treated by conformal mapping. The results from this investigation suggest a linear potential distribution between the corners of the recording head. This approximation gives explicit expressions for the field, and the method is generalized to finite permeability. Expressions are given for the field on each side of the layer, and asymptotic expressions are given for the field at a large distance from the pole gap of the recording head. The inductance of the head is calculated, and measurements of the inductance change have been made when the permeability is increased from 1 to $\mu$. The transient field is computed for the one-dimensional case, assuming the resistivity of the layer to be very large. The results can be used to analyze the influence on the field from permeability and geometric shape of the head.

## 11. Acknowledgements

The writer wishes to express his thanks to Messrs G. DAHLQUIST and E. STEMME of the Swedish Board for Computing Machinery, and to Mr D. WILLIS Cambridge, England, who has made many suggestions for solving the problem. He is also indebted to the Swedish Board for Computing Machinery for running some of the computations on the Swedish relay machine Bark. Professor T. LAURENT at the Royal Institute of Technology, Stockholm, has in many respects stimulated the writer through his interest in recent developments in the application of magnetic drums to automatic telephony.

## 12. Appendix 1

The sine-transform $g(x)$ of the function $G(t)$ is defined by

$$g(x) = \int_0^\infty \sin x t \, G(t) \, dt$$

Its inverse is

$$G(t) = \frac{2}{\pi} \int_0^\infty \sin t x \, g(x) \, dx$$

The Laplace-transform $f(s)$ of the function $F(t)$ is defined by

$$f(s) = \int_0^\infty e^{-st} F(t) \, dt$$

The error integral is defined by

$$\operatorname{erf} x = 1 - \operatorname{erfc} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

An asymptotic series diverges for all $x$, but can be used in numerical computation if a finite number of termes is included. Cf. WHITTAKER and WATSON 1927.

## 13. Appendix 2. Derivation of the field formulas in the general case

The boundary value problem defined by the equations (3), (4) and (5) is solved in the following way[1]).

Let $g(x)$ be the linear potential (20). Along the lines $y = b$ and $y = c$ (each side of the layer) the $H_y$ field makes a jump according to equations (4) and (5).

Put

$$g(x) = \int_0^\infty \sin xt \cdot G(t)\, dt$$

where

$$G(t) = \frac{2\,V \sin Nt}{\pi\, N t^2}$$

We want to determine three layers each with boundary value potentials $f_1(x)$, $f_2(x)$ and $f_3(x)$ at the lines $y = 0$, $y = b$ and $y = c$. Their potentials should be equal to the function $v(x, y)$. Set

$$f_1(x) = \int_0^\infty F_1(t) \sin xt\, dt$$

$$f_2(x) = \int_0^\infty F_2(t) \sin xt\, dt$$

$$f_3(x) = \int_0^\infty F_3(t) \sin xt\, dt$$

$$v(x, y) = \int_0^\infty V(t, y) \sin xt\, dt$$

[1]) The original proof assumed Cesàro summability of the sine transform of $g(x)$. This derivation due to Mr G. DAHLQUIST, avoids this difficulty.

Now we consider the function $u\,(x, y)$, defined by

$$u\,(x, y) = \int_0^\infty e^{-t\,|y|} \sin x\,t\, F_1\,(t)\,dt$$

This function satisfies Laplace's equation

$$\Delta\, u = 0$$

This shows that a layer with the boundary potential $f_1\,(x)$ at the line $y = 0$ gives a potential that is the sine transform of

$$e^{-t\,|y|} \cdot F_1\,(t)$$

and similar for $F_2$ and $F_3$

Then we have

$$V\,(t, y) = F_1\,(t)\,e^{-t\,|y|} + F_2\,(t)\,e^{-t\,|b-y|} + F_3\,(t)\,e^{-t\,|c-y|}$$

Now we express the boundary conditions for the transforms, which gives a linear system with three eqs. Solving this system we get

$$F_1\,(t) = (r^2 - e^{-2\,t\,(c-b)})\,\frac{G}{D}$$

$$F_2\,(t) = (e^{-t\,(2\,c-b)} - r\,e^{-t\,b})\,\frac{G}{D}$$

$$F_3\,(t) = e^{-t\,c}\,(r-1)\,\frac{G}{D}$$

where

$$r = \frac{\mu - 1}{\mu + 1}$$

$$D = r^2 - r\,(e^{-2\,t\,b} - e^{-2\,t\,c}) - e^{-2\,t\,(c-b)}$$

We can now form the various derivatives of $v\,(x, y)$ and easily compute the field components given by the eqs. (26), (27) ($c$ infinite), (31), (32), (33) and (34) ($c$ finite).

To prove the asymptotic formulas, we observe that the integral

$$f(x) = \int_0^\infty F(t) \sin x t \, dt$$

can be expanded by repeated partial integration in a series of $1/x$:

$$f(x) = \frac{F(0)}{x} - \frac{F''(0)}{x^3} + \dots$$

In the same manner we expand the integral

$$g(x) = \int_0^\infty G(t) \cos x t \, dt$$

$$g(x) = -\frac{G'(0)}{x^2} + \frac{G'''(0)}{x^4} - \dots$$

In our case the series are divergent for all $x$ but are still useful for large $x$ (cf. appendix 1).
The derivatives are computed recurrently. For example, the integrand of the formula (26) is $S/T$, where

$$S = \frac{1}{t} \sin \frac{t N}{b}$$

$$T = \mu \sin h t + \cos h t$$

Writing

$$G T = S$$

we have

$$G' T + G T' = S'$$

and so on.

Proceeding in this way, we get the asymptotic expansions (29), (30) ($c$ infinite) and (35), (36), (37) and (38) ($c$ finite).

The two first terms in the last four expansions are:

$$B_{2x} = -B_0 \left[ \frac{2 b N \mu}{\pi x^2} + \frac{2 b^3 N \mu}{\pi x^4} \left( 12 \mu q + p^2 - 1 + 6 q^2 - \right. \right.$$
$$\left. \left. - \frac{6 q (2 \mu + r)}{\mu^2} \right) + \dots \right]$$

$$B_{2y} = B_0 \left[ \frac{2\,N}{\pi\,x} - \frac{2\,b^2\,N}{\pi\,x^3\,\mu^2} (\mu^2 + 4\,q\,\mu - 2\,q^2\,\mu^2 - p^2\,\mu^2/3 - \right.$$

$$\left. - 4\,q\,\mu^3 + 2\,q^2) + \ldots \right]$$

$$B_{3x} = -\,B_0 \left[ \frac{2\,b\,N\,(\mu + q)}{\pi\,x^2} + \frac{4\,b^4\,\mu}{\pi\,x^4}\,G''\,(0) + \ldots \right]$$

$$B_{3y} = B_0 \left[ \frac{2\,N}{\pi\,x} - \frac{2\,b^2\,N}{3\,\pi\,x^3\,\mu^2} (3\,\mu^2 - p^2\,\mu^2 + 12\,q\,\mu + 6\,q^2 - \right.$$

$$\left. - 3\,q^2\,\mu^2 - 6\,q\,\mu^3) + \ldots \right]$$

$$p = N/b$$

$$q = d/b$$

$$G''\,(0) = \frac{p}{2\,\mu^4} \left[ 12\,q\,\mu^4 + 15\,q^2\,\mu^3 + (p^2 - 1)\,\mu^3 + p^2 q\,\mu^2 - \right.$$

$$\left. - 5\,q^3\,\mu^2 - 15\,q\,\mu^2 - 18\,q\,\mu - 6\,q^3 \right]$$

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