ANDEAN CALCULATORS

Despite the general conviction that the data recorded on *quipus* were calculated by means of *yupanas* (Mackey et al., 1990, Urton, 2002: 119, Sempat, 2002:124; Quilter, 2002: 215) (while doing reckoning, making and untying knots would be really too difficult….), not only have these *yupanas* had hard being recognized as the main instruments of calculations in the Andean world, mostly outside Peru, but, actually they have often been presented as nothing less than fortress models even toys, too.

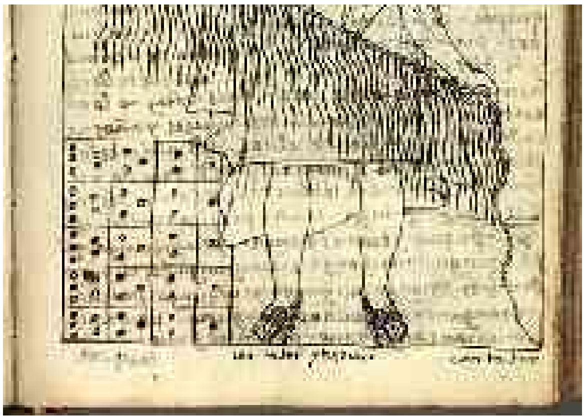
The acknowledgement of their cultural function is not yet clear at all because, typical though they are of the Late Horizon, they might also be considered previous or pan-Andean as some typologies of theirs have been found in different areas and contexts far from one another and without any particular gradients from Cuzco towards the outskirts.

So far *yupanas* haven't been rewarded the attribution they actually deserve because:

- firstly, unlike *quipus*, their different shapes evoke stylised and idealized models of architectural buildings instead of mathematical instruments;
- secondly, they have never been quoted in any chronicles apart from a dictionary by Gonzales Holguin where some sentences associate only the word *yupana*, not the object itself, to calculations: "*Yupani.* Reckoning / *Yupay.* Calculations / *Yupani triuspa.* Summing up or making brief summaries / *Yupana.* Letter, the figures of numbers" (Gonzales Holguin, 1607:371);
- finally the passages where the Incas' counting system is described are quite ambiguous.

With regard to this matter, as on other occasions, Acosta has proved himself to be one of the most accurate writers of the seventeenth century . Here is what he wrote about *yupanas*: *"…watching them in another type of quipus, where corn grains are used to count, is a fascinating experience, because these Indians can do the same accounts that a very expert accountant is to do by means of a pen, some ink and a lot of complicated operations to know, for example, the true amount of a tribute he is due. With the help of their grains, they put <u>one seed here, three seeds there and eight seeds I don't know where</u> and, after moving one of them from here and changing down three of them, they succeed in doing their accounts without the smallest error and so they can decide the exact amount that each person is to pay much better than we can do by means of our pens and ink. Whoever can judge whether these people are men or beasts. What I affirm with dead certainty is that these people excel us by far in what they work hard"* (Acosta, 1954:190).

Since the thirties, after the public exhibition of a very famous drawing by Guaman Poma (1936:362 (360)) from the *Nueva Corónica y Buen Gobierno,* where a tablet with black and white small circles was drawn next to " *a contador mayor y tesorero*", numerous scientists have tried to explain how *yupana* worked. In fact, the tablet was soon considered to be the frame of a *yupana* and, moreover, the author himself wrote the words "*cuentan en tablas*" to comment his drawing and hinted at the use of the *yupana* as a calendar (Guaman Poma 1936:361-363 (359-363)).



Nevertheless, also the most recent interpretation attempts (Mackey et al., 1990) didn't succeed in explaining exhaustively either the difference between the white and the black small circles of the *yupana* by Guaman Poma or, more generally, how *yupana* worked because the redundant systems created inevitably developed into unacceptable calculation bases (81, 12 and 11) (Wassén, 1990; Radicati, 1990; Ansiòn, 1990).
An Italian mathematician, Nicolino De Pasquale, has recently proposed a new reading of the *yupana* by Guaman Poma . His interpretation has not only allowed us to discover how *yupanas* worked, but also to recognize the numbers represented by the black circles.
As a confirmation of the validity of his interpretation, a calculating machine was also built that uses the same system of calculation he has discovered and which does actually work (Maurizio Orlando 2002).

According to the model by De Pasquale, in the *yupana* of the Nueva Corònica, the first numbers of Fibonacci's series (which the Incas had probably discovered before the influence of the after-Conquest) were given a position value in a sort of abacus where the first 39 numbers were placed in the lowest row, the numbers from 40 to 1560 in the above row and so on.
The resulting system is a base 40 one presenting the following values: 40^0, 40^1, 40^2,40^3, 40^4, in the right column. (In the field of ethnography, the importance of number 40 in the Andean world, particularly in textile manufacture, was also proved by Urton,1997:121-124).
The advantage of De Pasquale's model lies not only in the fact that it is in keeping with what Acosta affirmed in some of his passages (1954: 190): *" one grain here, three there and eight ones I*

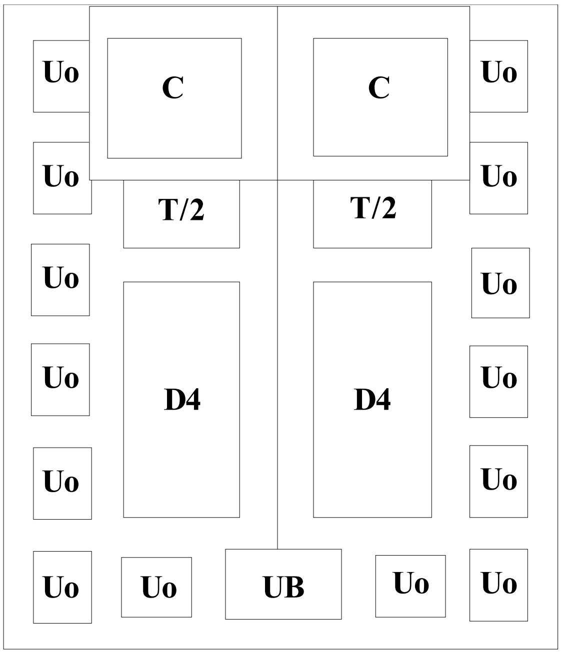
don't know where…by moving a grain from here and changing round three grains " and with his hints at the mechanism of carried out numbers and at the absence of number eight, but it also actually constitutes the most coherent, functional and ergonomic mathematical system that has been proposed to explain the *yupana* pictured in Guaman Poma so long.

C	T	D	U
$C_{2560000}$	$T_{2560000}$	$D_{2560000}$	$U_{2560000}$
C_{64000}	T_{64000}	D_{64000}	U_{64000}
C_{1600}	T_{1600}	D_{1600}	U_{1600}
C_{40}	T_{40}	D_{40}	U_{40}
C_{1}	T_{1}	D_{1}	U_{1}

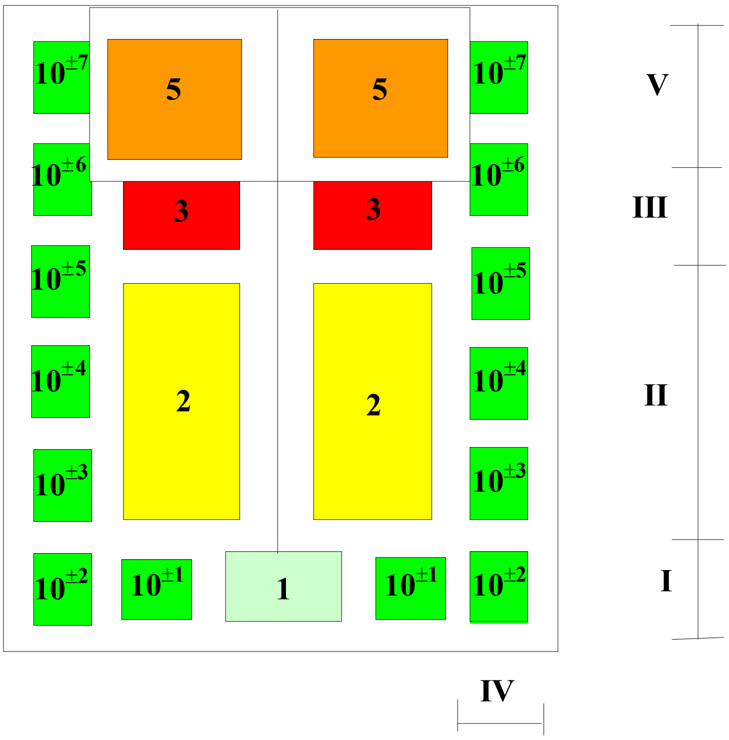
C	T	D	U	
12800000*5	**7680000*3**	**5120000*2**	**2560000**	**40^4**
320000*5	**192000*3**	**128000*2**	**64000**	**40^3**
8000*5	**4800*3**	**3200*2**	**1600**	**40^2**
200*5	**120*3**	**80*2**	**40**	**40^1**
5*5	**3*3**	**2*2**	**1**	**40^0**

The fifteen *yupana* listed below have been examined, on the basis of these results, working on the assumption that the same system of calculation had been applied to the other *yupanas*, too, and, in particular, to the finding of MNAAHP exhibited in Lima (some of these *yupanas* have been personally viewed, others have been examined by means of photographic materials).

Notwithstanding the obvious statistical differences occurring among the various *yupanas*, this work has led to the lay out of a classification system and to the outcome that the area of the square boxes, their height and their rowing order allow us to define their common way of working and to affirm that all the *yupanas* can be traced back to two types of base systems of calculation: a base 10 system of calculation and a base 40 one (the latter allowed quicker calculations that were easily reconverted into the decimal system, which was, as we know, the basis of the social structure of the Incas and of most *quipus*).

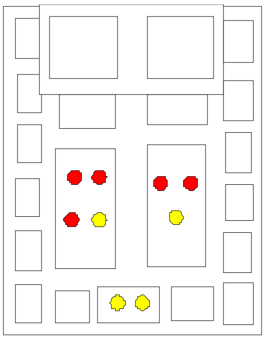


In the yupana exhibited in Lima, which uses a base 10 system of calculation, the quite rough average areas of its square boxes (we must consider that the Incas didn't have the instruments of an industrialized society) and their placement have permitted us to identify a frame made up of 4 central strips with the following square boxes: **UB**, **D4**, **T/2**, **C**, respectively corresponding to the working spaces for numbers **1**, **2**, **3**, **5**, and a side, L shaped, strip, which, by means of the square boxes **Uo**, allows to work with the tens, the hundreds, the thousands, etc. or with the tenths, the hundredths, the thousandths, etc.



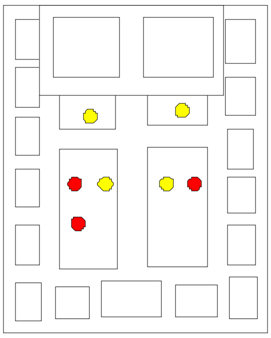
The ratio between the areas of the different square boxes gives us the following results:

U=1, unit value;
Uo> 1, value referred to the orders of tens, hundreds, tenths, hundredths, etc.;
UB=2U, base unit working with number 1. It can contain two grains or number markers (the corn grains named by Acosta) to perform operations like 1+1;
D4=8U, double unit working with number 2, which can contain up to four grains to represent numbers 2,4,6,8;
T/2=3/2 U, triple unit working with number 3 (only during calculations);
C=5U, quintet unit working with number 5, by means of its twin square box, for the carrying activity: 5+5=10.
Just like the *yupana* by Guaman Poma, also this yupana doesn't permit the representation of number zero as it is based on an exponential world centred on number 1. The absence of number zero finds a meaningful confirmation in the absence of a corresponding word in the quechua language. In this *yupana* the smallest number that can be represented is **1/10'000'000**, whilst the biggest one, resulting from the various arithmetical operations, is **99'999'999.9999999**: they are numbers that arrive to the greatest sensational theoretical precision of **1/1'000'000'000'000'000** (one of a million of billiards), which is extraordinary even if compared with the performances of modern computers.
The stressed symmetry as against the vertical direction allows to place two numbers a time on the *yupana* before starting the operations. If you want to make the operation **9+7**, for example, you'll have to follow the lay out where the yellow coloured grains are to be moved.



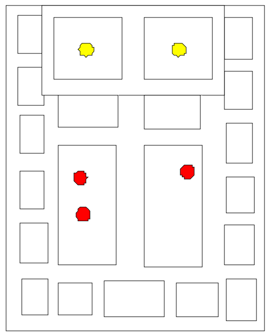
a

=



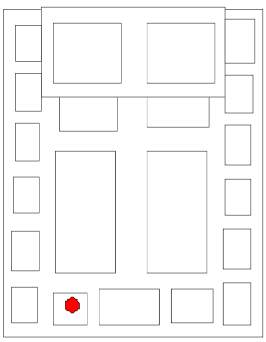
b

=



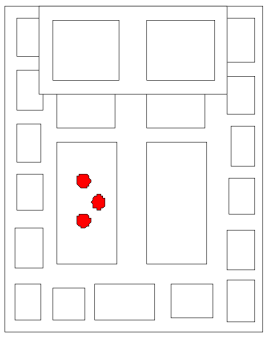
c

=



d

+



e

Put four seeds in the **D4** square box , on the left side (**2+2+2+2**) and one seed in **UB** (**1**), paying attention to place it on the left side, too, so as to obtain the first addend (**2*4+1=9**);secondly place three seeds in the **D4** square box on the right side (**2+2+2**) and one more seed in **UB** (**1**)-but on the right side, this time!- to obtain the second addend (**2*3+1=7**) (**a**) (don't use the **T/2** square box during this phase). Then, during the calculations, the **T/2** square box will be activated, both on the left and the right sides, by a seed with the value of two and another one with the value of one (**b**)

and you'll have both of the **C** square boxes working with a seed with the value of 3 and another one with the value of 2 (**c**).Now the carrying activity can start (**d**), corresponding to a ten that will complete the result by means of the six units left. (**e**). As to the numbers containing more orders, besides the units, calculations will obviously continue gradually, first working with the units and lately considering the carried numbers, the tens, the hundreds and so on.
As a matter of fact, these proceedings are the same ones used in the other *yupanas* (always starting from numbers 1,2,3,5) for both base 10 and base 40 systems It would be impossible, here, to explain how they work, but the example offered by the *yupana* in Lima is sufficient to show that all *yupanas* are simple and powerful abacuses that, thanks to their very essential shape (the one of the drawing by Guaman Poma), didn't necessarily need to be built because they could be drawn on the earth. This can, may be, explain the ambiguities and the uncertainties of the chronicle reporters.

A list of the *yupanas* viewed:

1. Caraz Yupana (Radicati,1990:219-230)
2. Chordeleg Yupana (Radicati, 1990: 219-234)
3. Pallasca Yupana (Radicati, 1990:219-234, Purin,1990 II:243)
4. Goteborg Yupana (Radicati, 1990: 219-234)
5. Radicati's Yupana (Radicati, 1990: 219-234)
6. Ica MRI's Yupana 1 (Radicati, 1990: 219-234)
7. Ica MRI's Yupana 2 (Radicati, 1990: 219-234)
8. Ica MRI's Yupana 3 (Radicati, 1990: 219-234)
9. Milan Yupana 1
10. Milan Yupana 2
11. MNAAHP's Yupana
12. Geiger's Yupana 1 (Geiger *et al.*, 2003:208)
13. Geiger's Yupana 2 (Geiger *et al.*, 2003:209)
14. Holm Yupana (Radicati, 1990: 219-234)
15. Chacas Yupana (Laurencich and Wegner, 2001:49).

A lay out of a typical classification of yupanas:

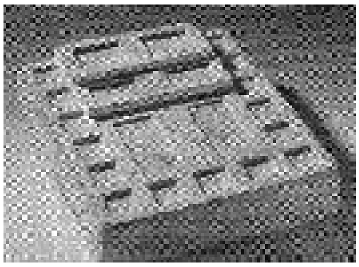
decimal yupanas:

1 level:
Ica MRI's Yupana 1
Ica MRI's Yupana 2
Ica MRI's Yupana 3
Holm yupana

2 levels:
Lima MNAAHP's yupana

3 levels:
Geiger's Yupana 2

DECIMAL *YUPANAS*

1 level	2 levels	3 levels
 Ica MRI n° 1 Ica MRI n° 2 Ica MRI n° 3 Holm	 **Lima MNAAHP**	 **Geiger n° 2**

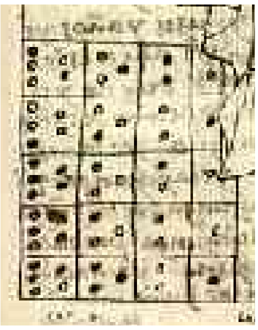
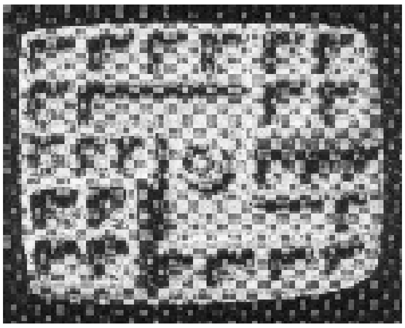
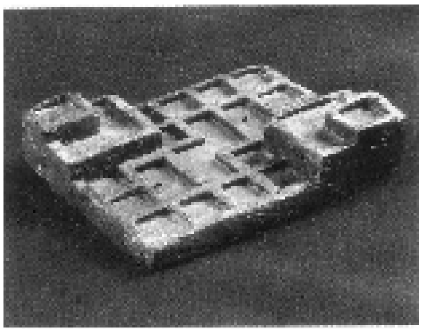
decimal and fortiethal yupanas:

<u>1 level</u> with equal square boxes:
They aren't found among the ones viewed. They are the simplest ones and might be drawn everywhere . Substantially, they look like Guaman Poma's base 40 one, but, when they work with a reduced number of seeds, they use the fourth column, the one of the quintets, which usually happens only during the operations of carried numbers. Working this way, they are reconverted and become base 10.

<u>2 levels</u>, with eight evident square boxes at the second level:
Milan Yupana 2
Chacas Yupana
Geiger's Yupana 1

<u>3 levels</u>, with two twenties at the third level:
Caraz Yupana
Chordoleg Yupana
Pallasca Yupana
Goteborg Yupana
Radicati's Yupana
Milan Yupana 1

DECIMAL AND FORTIETHAL *YUPANAS*

1 level	2 levels	3 levels
 **Guaman Poma**	 Milan n° 2 Chacas **Geiger n° 1**	 Caraz Chordoleg Pallasca **Goteborg** Radicati Milan n° 1

Antonio Aimi, Nicolino De Pasquale
(translated by **Franca Del Bianco)**