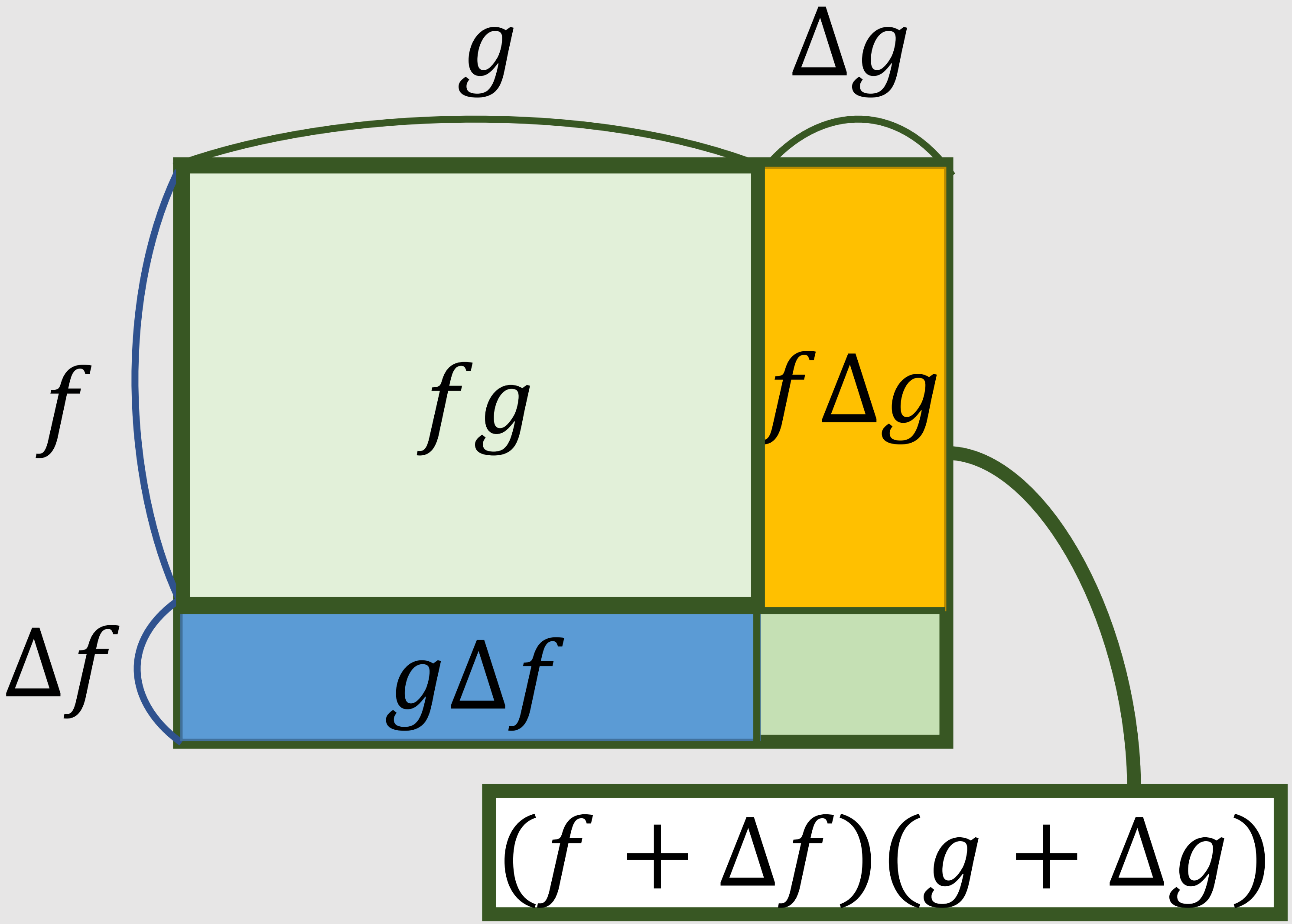
Vector Differentiation

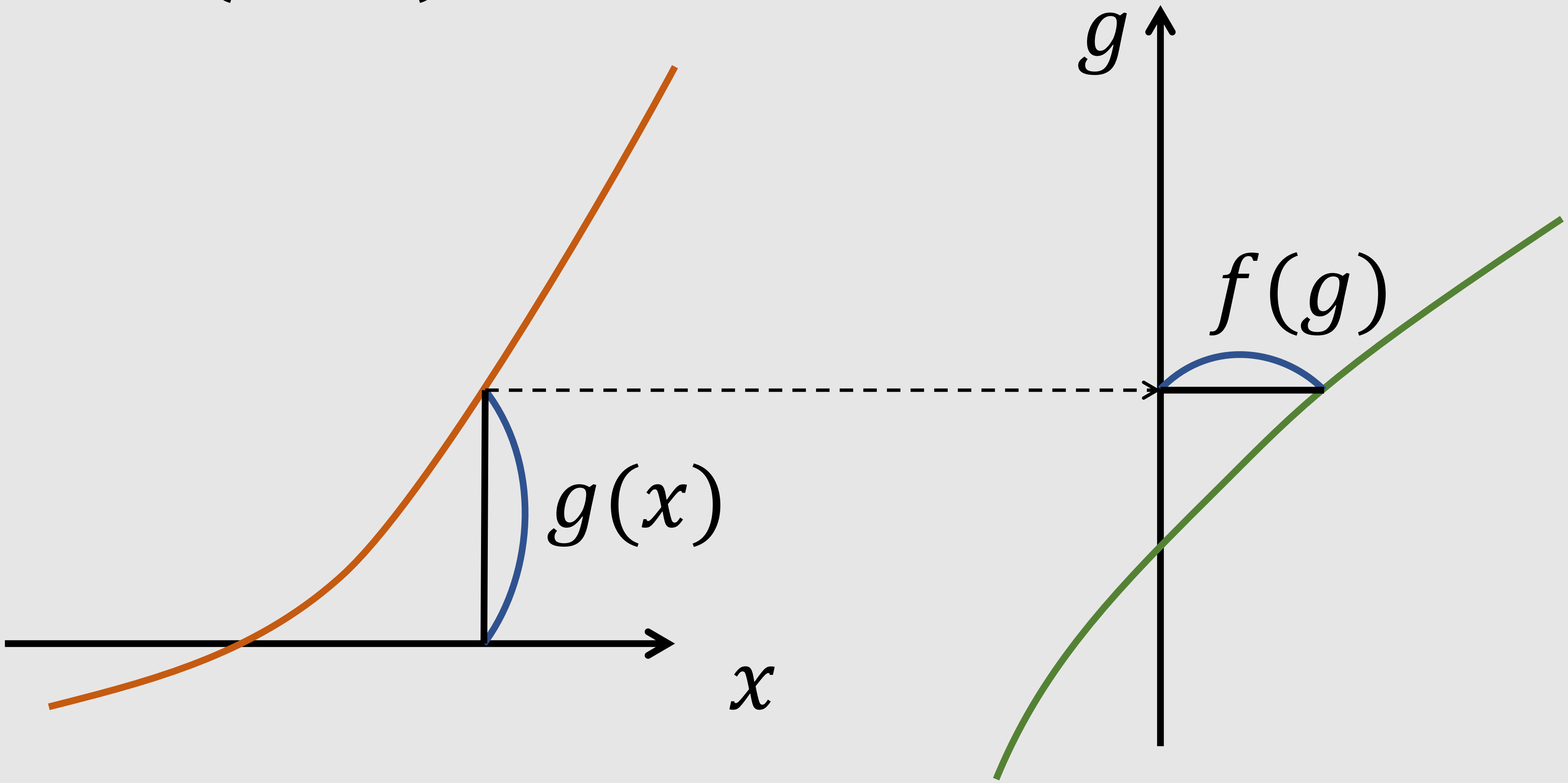
Review of Differentiation Rules: Multiple

$$(fg)' = f'g + fg'$$



Review of Differentiation Rules: Composite

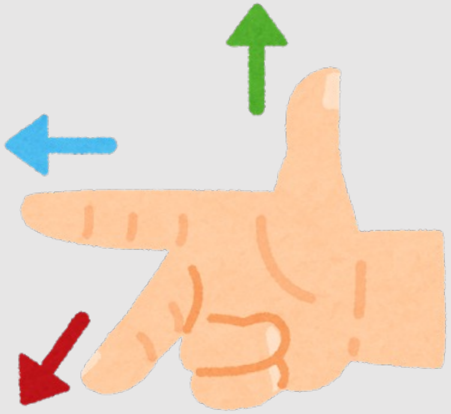
$$f'(g(x)) = f'\big(g(x)\big)g'(x)$$



Two Ways to Understand Vector

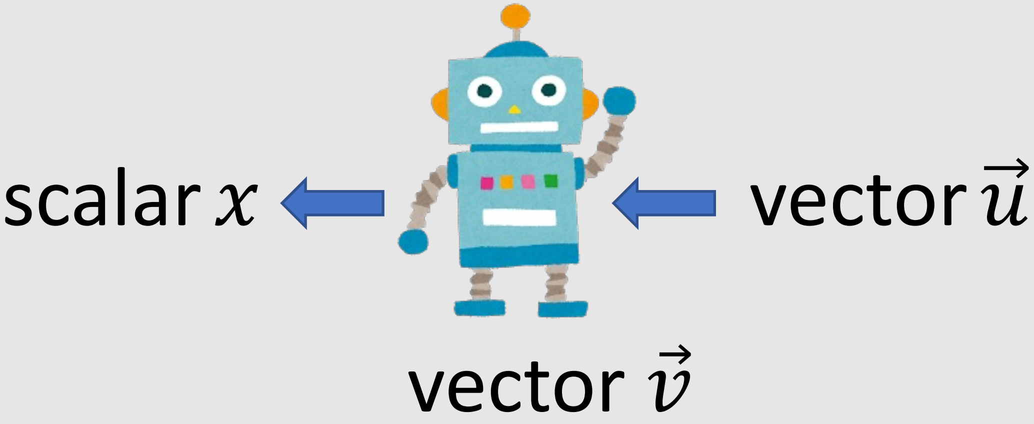
- Dot product define linear relationship between input & output

Spatial direction or position



Linear form

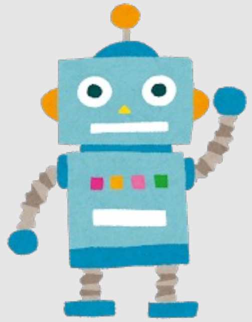
$$x = \vec{v} \cdot \vec{u}$$



Differential of a Scalar Function w.r.t. Vector

- Inner product with $d\vec{p}$ gives difference dW

$$dW = \frac{\partial W(\vec{p})}{\partial \vec{p}} \cdot d\vec{p}$$



vector

Since $d\vec{p}$ and dW change linearly, $\partial W(\vec{p})/\partial \vec{p}$ should be a vector

Differential of a Scalar Function w.r.t. Vector

- The differential can be written as

$$\frac{\partial W(\vec{p})}{\partial \vec{p}} = \frac{\partial W}{\partial p_1}\vec{e}_1 + \frac{\partial W}{\partial p_2}\vec{e}_2 + \frac{\partial W}{\partial p_3}\vec{e}_3$$

Let's Practice Differentiation!

$$\mathrm{W}(\vec{p}) = \vec{a} \cdot \vec{p}$$

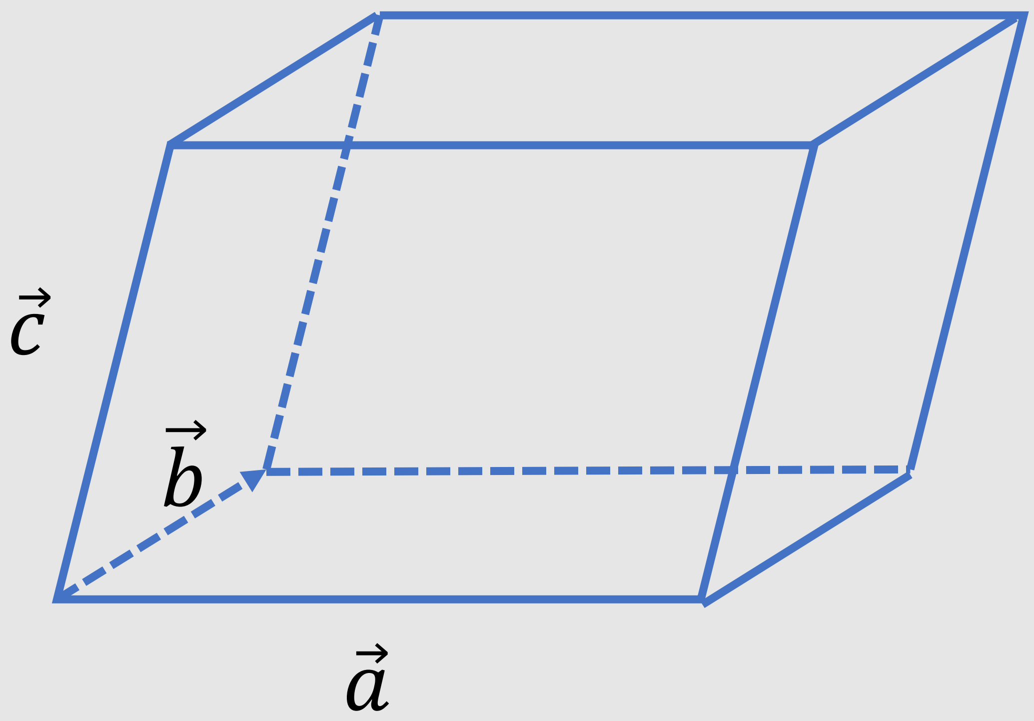
$$W(\vec{p}) = |\vec{p}|^2$$

$$W(\vec{p}) = {}^{1}/_{|\vec{p}|}$$

check it out!

Scalar Triple Product (スカラー三重積)

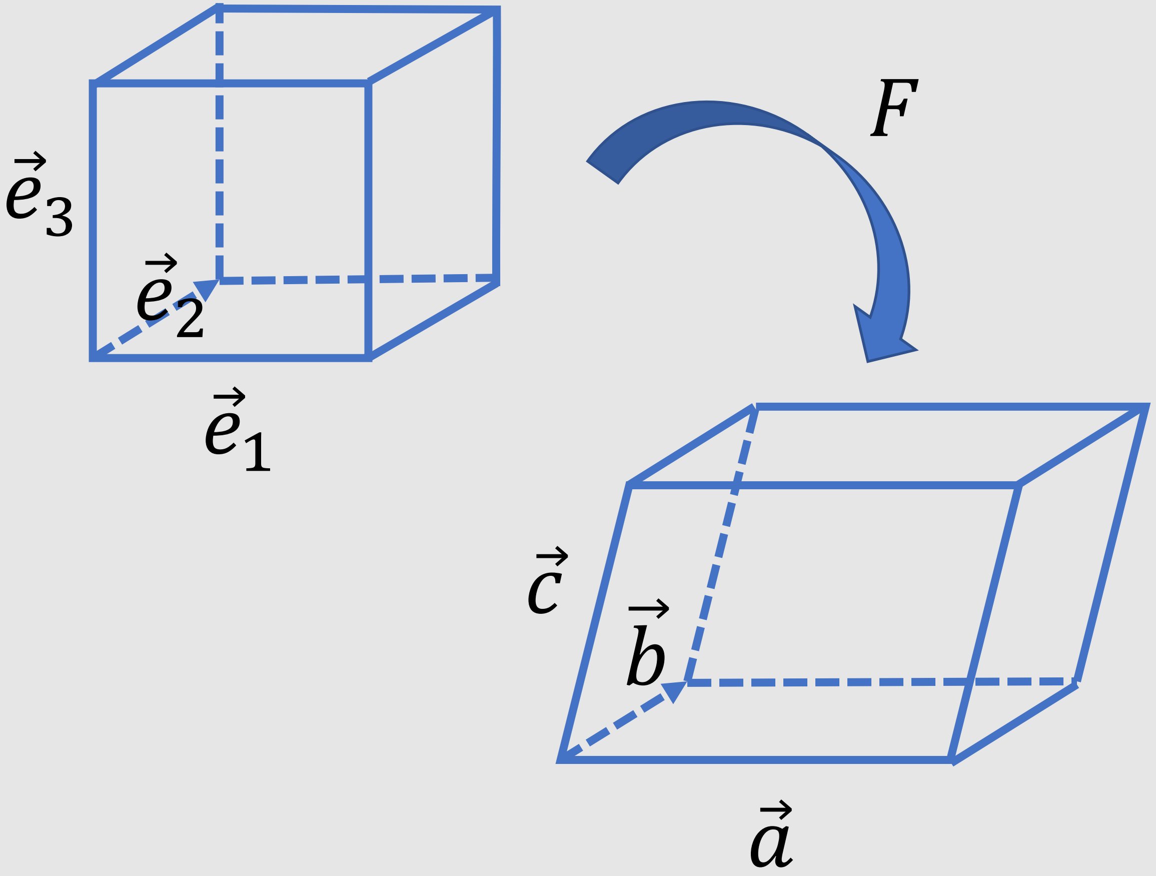
- Volume of parallelepiped



$$W = \vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$$

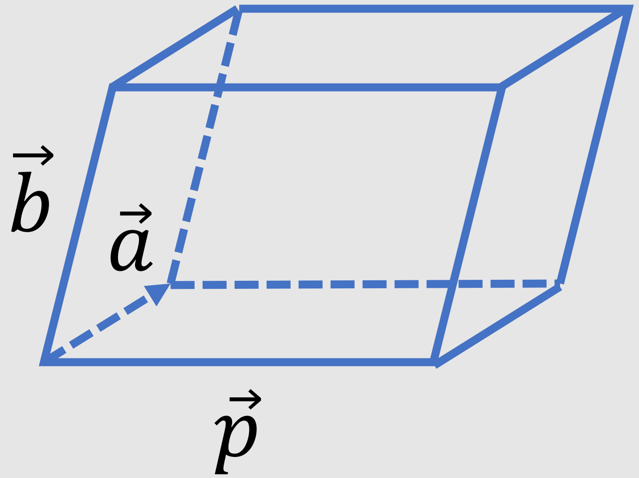
Scalar Triple Product and Determinant

- Scalar triple product is related to the volume change ratio



$$\vec{a}\cdot\left(\vec{b}\times\vec{c}\right) = det\underbrace{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}}_{F}$$

Differential of Scalar Triple Product



$$W(\vec{p}) = \vec{b} \cdot (\vec{a} \times \vec{p})$$

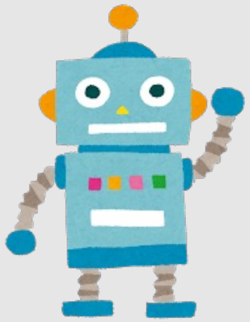
$$\frac{\partial W}{\partial \vec{p}} = ?$$

check it out!

Differentiation of Vector w.r.t. Vector

- Inner product with $d\vec{p}$ gives difference $d\vec{F}$

$$d\vec{F} = \frac{\partial \vec{F}(\vec{p})}{\partial \vec{p}} \cdot d\vec{p}$$



matrix

Since $d\vec{F}$ and $d\vec{p}$ change linearly, $\partial \vec{F}(\vec{p})/\partial \vec{p}$ should be a matrix

Differentiation of Vector w.r.t. Vector

- Inner product with $d\vec{p}$ gives difference $d\vec{F}$

$$\frac{\partial \vec{F}}{\partial \vec{p}} = \frac{\partial F_1}{\partial p_1}\vec{e}_1 \otimes \vec{e}_1 + \frac{\partial F_1}{\partial p_2}\vec{e}_1 \otimes \vec{e}_2 + \frac{\partial F_2}{\partial p_1}\vec{e}_2 \otimes \vec{e}_1 \cdots$$

$$= \sum_i \sum_j \frac{\partial F_i}{\partial p_j}\vec{e}_i \otimes \vec{e}_j$$

Let’s Practice Differentiation!

$$\vec{F} = A \cdot \vec{p}$$

$$\vec{F} = \vec{p}$$

$$\vec{F} = (\vec{a} \cdot \vec{p})\vec{b}$$

$$\vec{F} = \vec{p}/\|\vec{p}\|$$

check it out!

Differentiation w.r.t Vectors

- Transform the equation into a polynomial

$$W(\vec{\omega}) = \cdots$$
$$= \cdots$$
$$= a + \vec{b}^T \vec{\omega} + \vec{\omega}^T C \vec{\omega} + \cdots$$

Gradient: $\frac{\partial W}{\partial \vec{\omega}} = \vec{b}$

Hessian: $\frac{\partial^2 W}{\partial \vec{\omega}^2} = C^T + C$