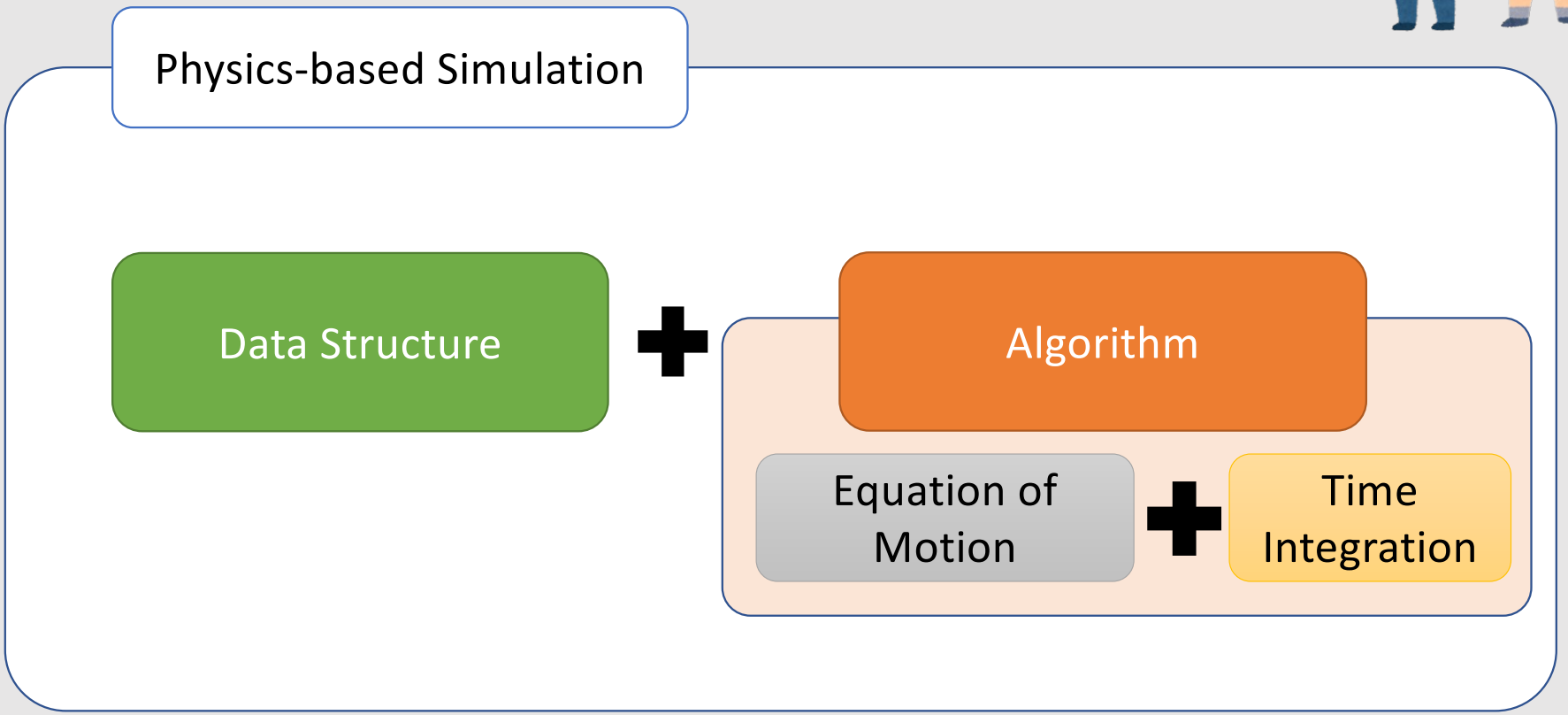
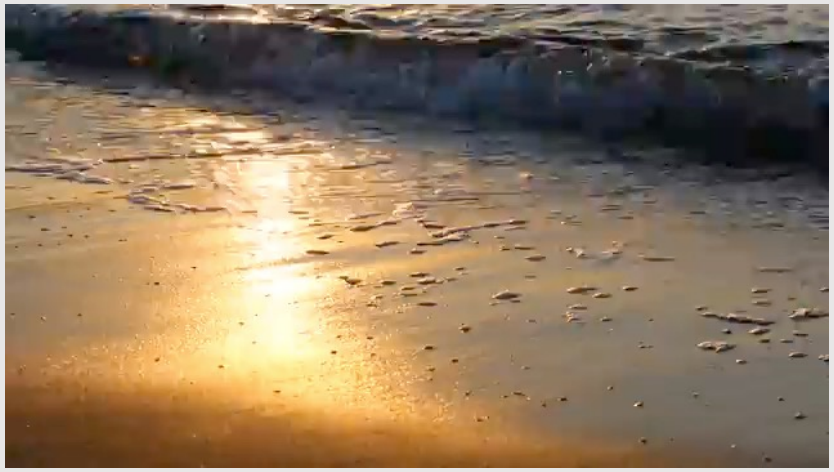
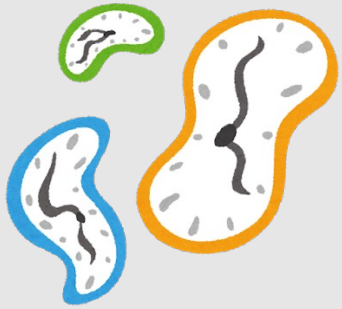
Spatial Discretization

Map of Physics-based Simulation



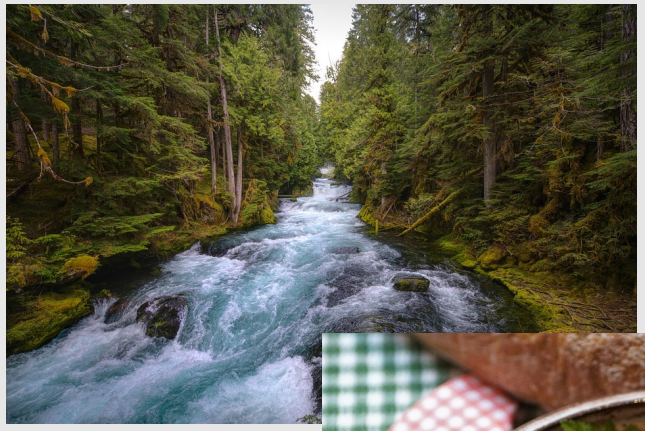
Temporal Discretization





How We can Discretize World?

- It is challenging to parameterizing everything



Ultimate Discretization: Atom

- Laplace's demon

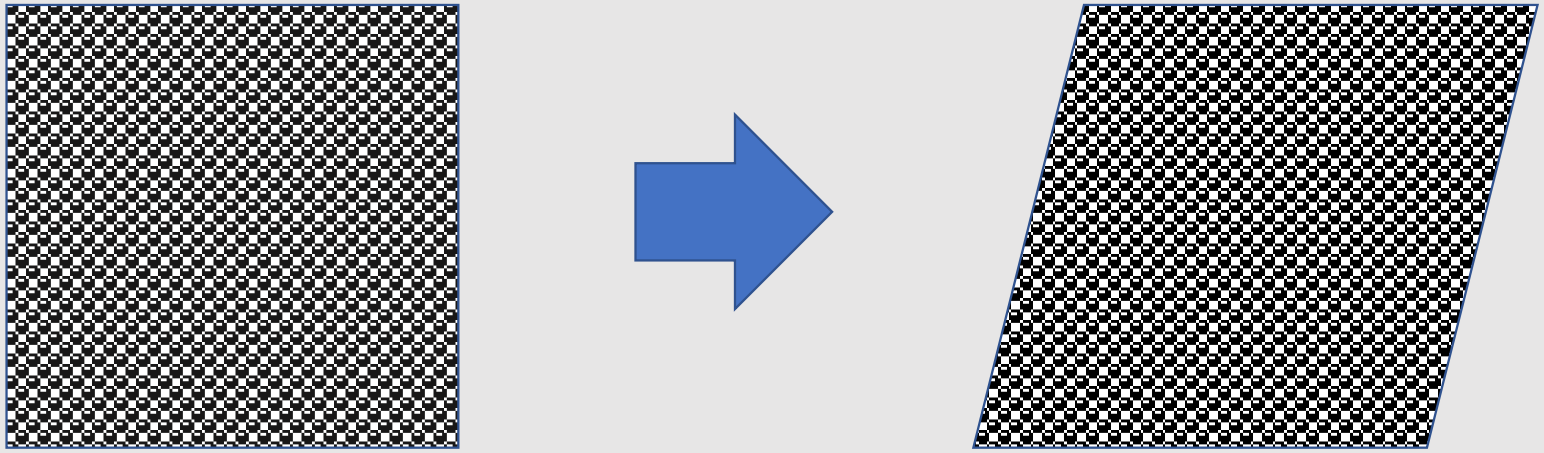
We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

— Pierre Simon Laplace, A Philosophical Essay on Probabilities 1814

I have no need of that hypothesis

Continuum Approximation

- Drastically reducing degrees of freedom (DoFs)
  - Drawback: fracture



What is a Good Discretization?

- No silver bullet. Discretization depends on the problem.

Efficiency (small memory footprint)

Simplicity (Regularity)

Naturally satisfy constraints
- Collision
- Incompressibility

Naturally preserves conserved quantities
- Mass
- Linear momentum
- Angular momentum
- Energy
- (Vorticity for fluids)

More important for realistic simulation

Lagrangian vs. Eulerian

Temperature of a River

- How to record the history of temperature of the flowing water?

Reference Frames

Lagrangian

Observation point is moving together with flow

Eulerian

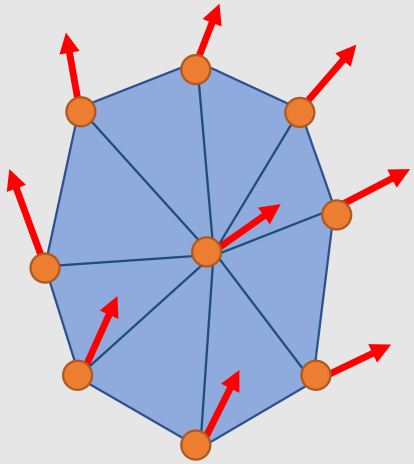
Observation point is fixed

Material Derivative

- Measuring the change of the temperature on the carousel

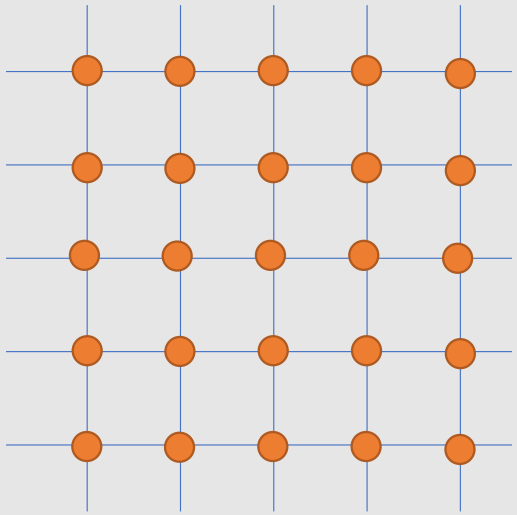
Data Structure for Continuum

Lagrangian
(e.g., deformable mesh)



Observation points moves over time

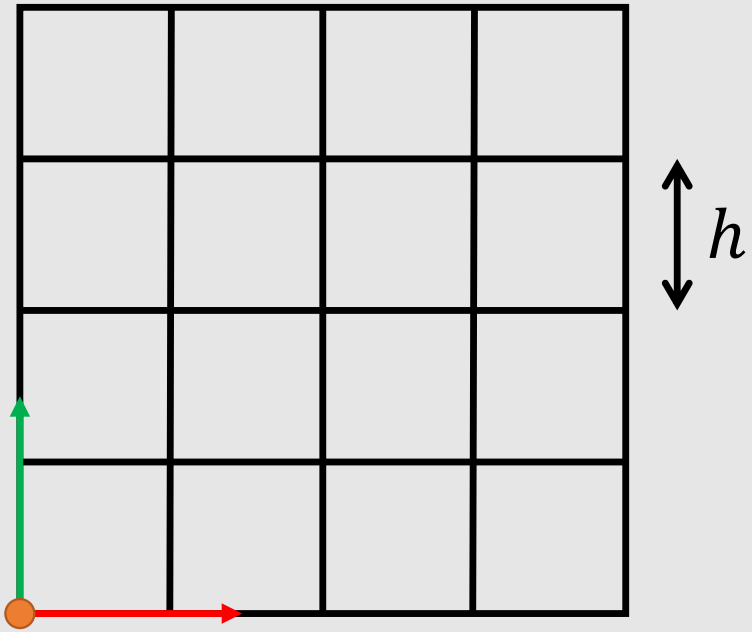
Eulerian
(e.g., regular grid)



Observation points don’t move

Regular Grids

- Most common discretization for spatial values



Let's find out the corresponding grid cell for (p_x, p_y)

Check it out!

Regular Grids Pros & Cons

- Advantages

  ☺Simple

  ☺Fast look-up

  ☺Hardware acceleration

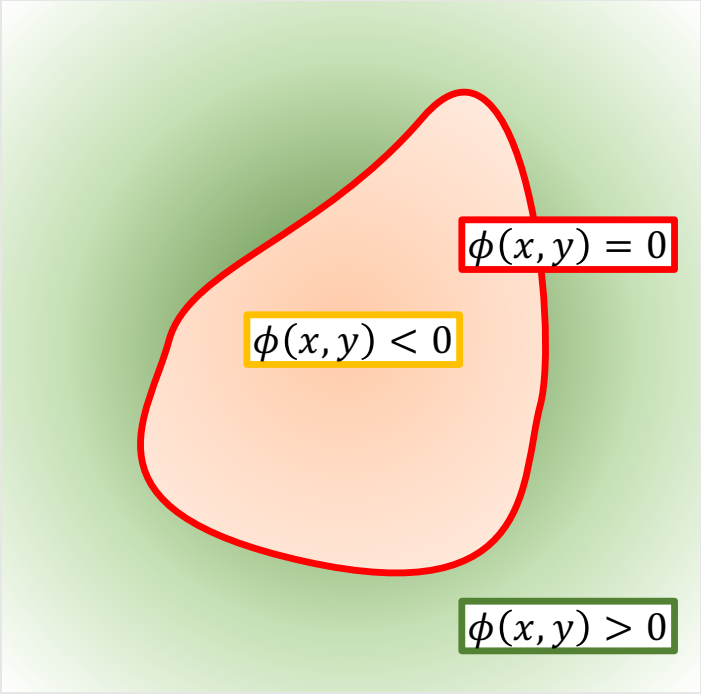
- Disadvantages

  ☹Difficult to track moving shape over time (i.e., mass conservation)

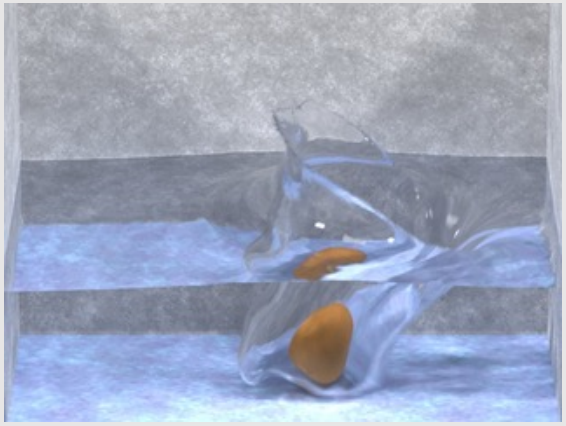
  ☹Difficult to handle non-grid-aligned boundaries

Implicit Surface Representation

- Surface is where level set function is zero $\phi(x, y) = 0$



Suitable for
open boundary

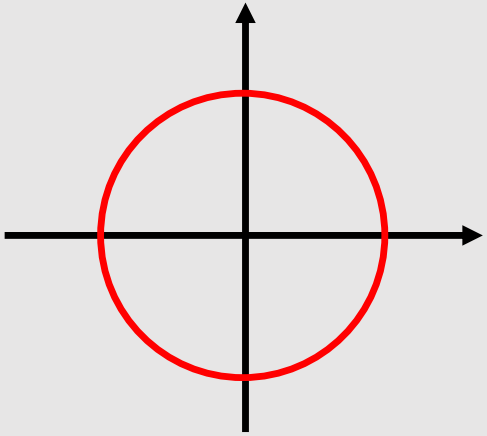


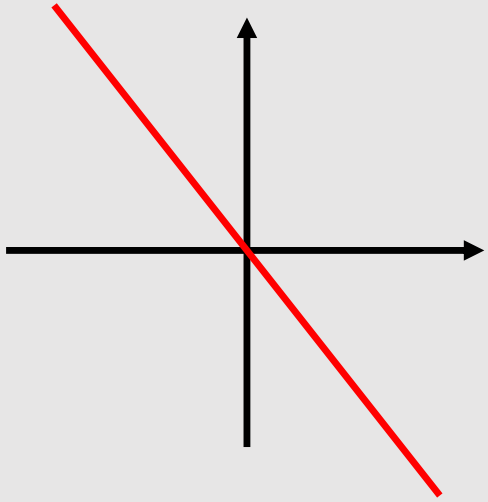
[Enright et al. 2002]

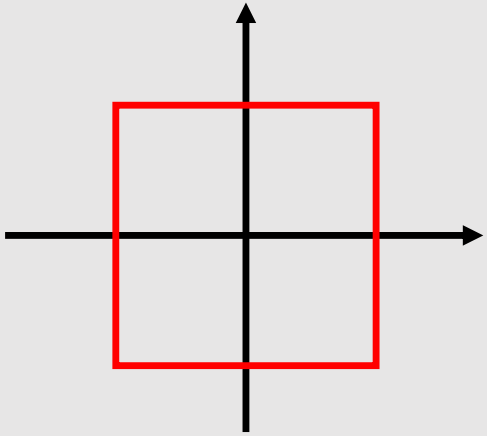
Level-set Function Practice

- What function become on the red curves 0?

Check it out!

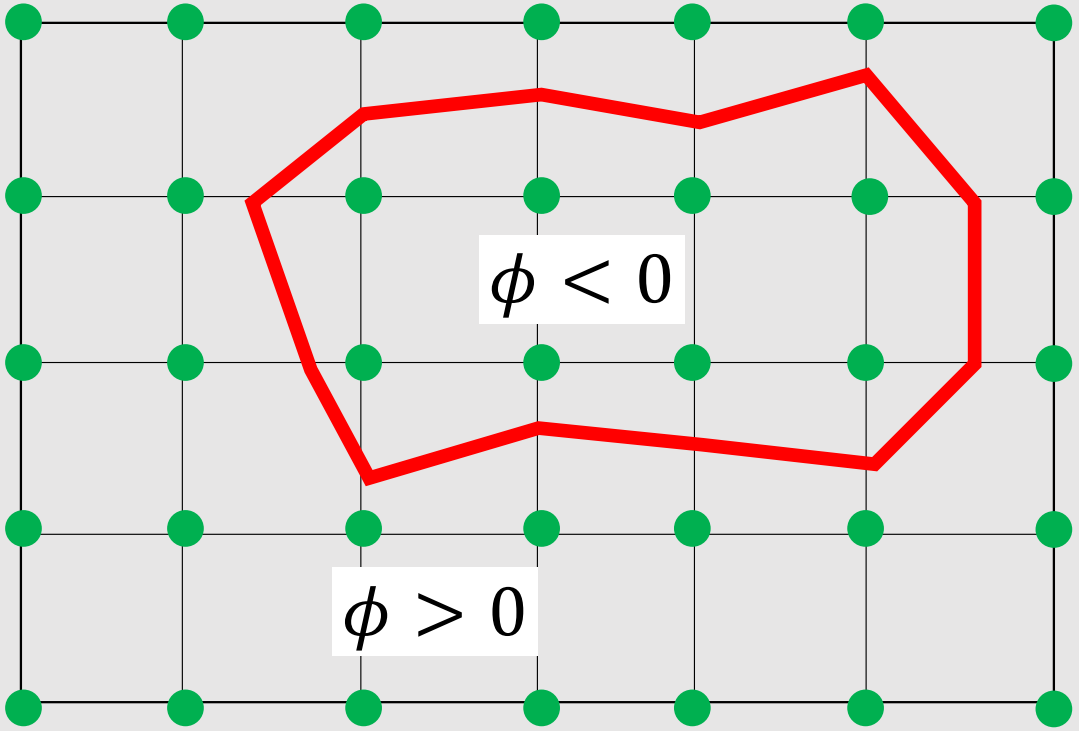






Level-set Function on a Regular Grid

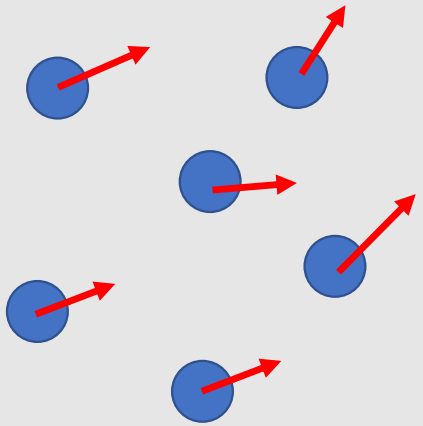
- Define value on a vertices of the grid



Extract surface using the marching-cube method

Point Representation

points



mass, position and velocity

Particles Pros & Cons

- <u>Advantages</u>

  ☺Simple

  ☺Easy to preserve mass & momentum

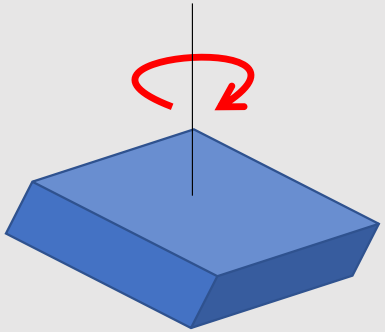
- <u>Disadvantages</u>

  ☹Difficult to find neighbors

  ☹Difficult to perform integration

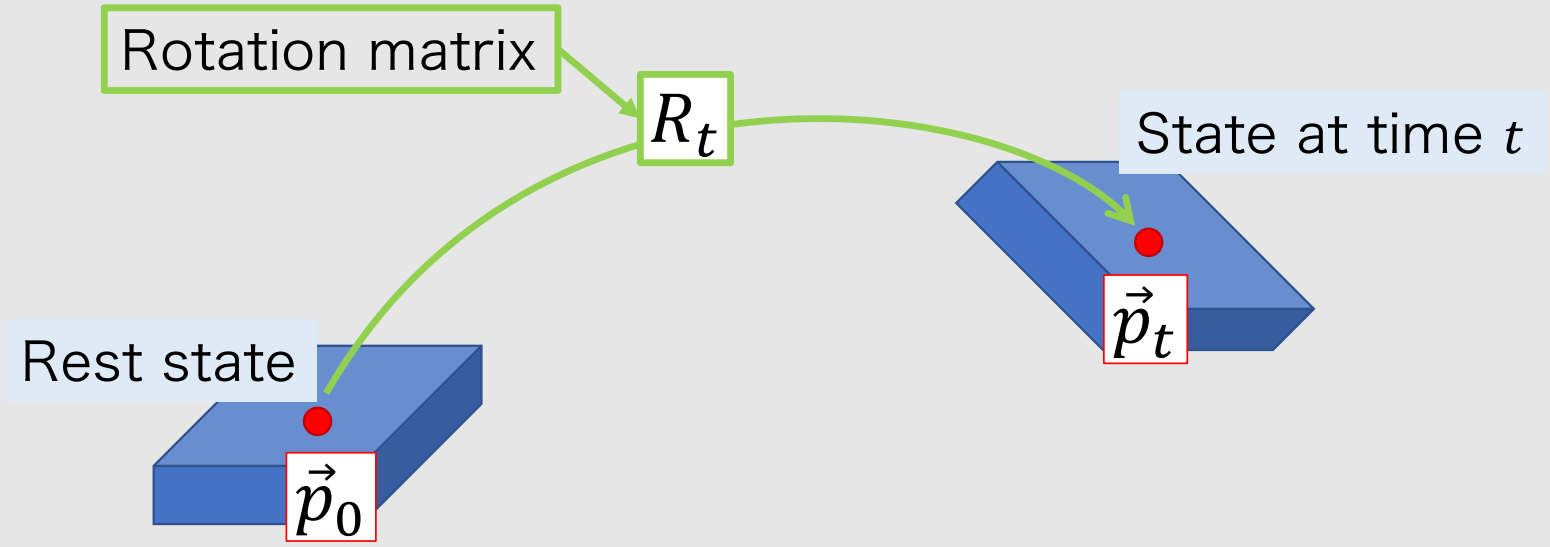
Rigid Body Representation

rigid body



Position, Orientation,
Mass, Rotational Inertia
Velocity, Angular velocity

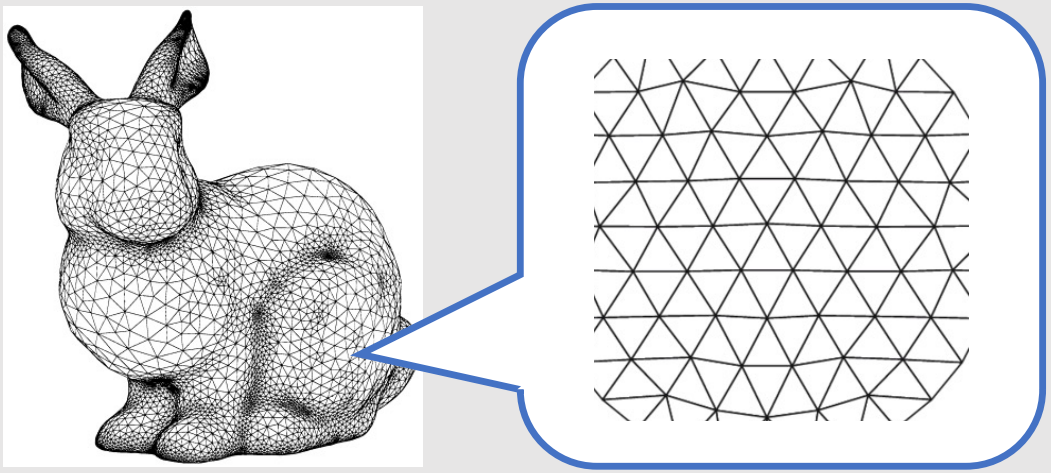
Representation of Rigid Body



(write equation here)

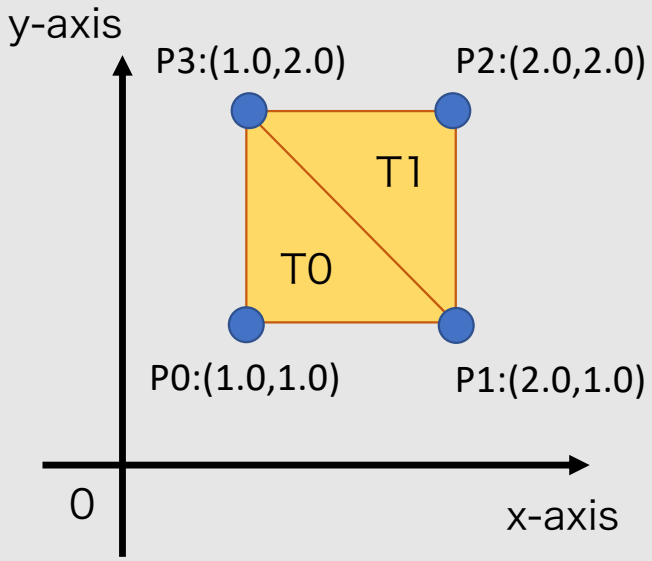
Meshes (Simplicial Complexes)

- Represent shape by triangles connecting points
- The most popular shape representation



Mesh Representation

- Coordinates of the points and their connectivity



Coordinates		
	X	Y
P0	1.0	1.0
P1	2.0	1.0
P2	2.0	2.0
P3	1.0	2.0

Displacement		
	X	Y
P0	-0.01	0.00
P1	0.02	-0.-1
P2	0.05	0.04
P3	0.03	-0.03

Connectivity			
	Vtx. 1	Vtx. 2	Vtx. 3
T0	0	1	3
T1	1	2	3

Mesh is Difficult

"I hate meshes. I cannot believe how hard this is. Geometry is hard."
— David Baraff, Senior Research Scientist, Pixar Animation Studios

https://graphics.pixar.com/people/deb/index.html

Some of Advanced Topics

- Hybrid Lagrangian Eulerian Approach
  - Moving grid (ALE)
  - Particles in regular grid
- Adaptive approach
- Frequency domain approach