Angular Velocity

角速度

What is Skew Symmetric Matrix?

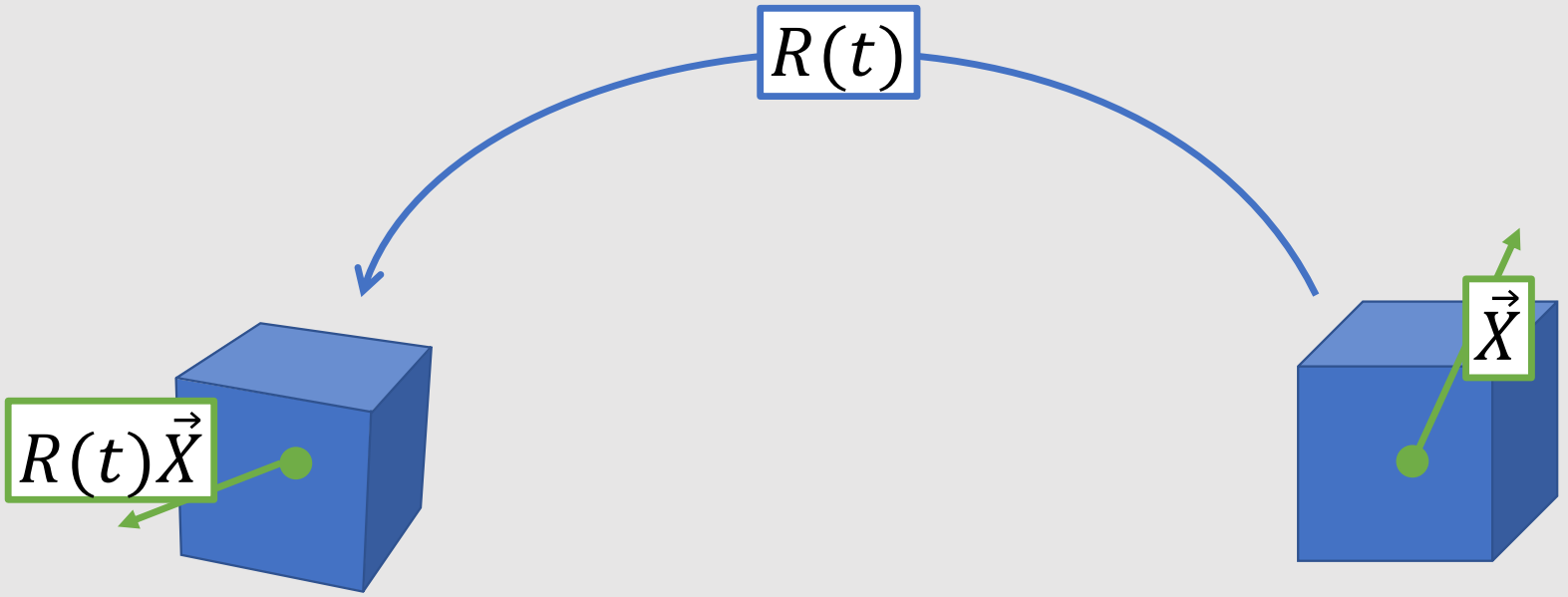
- 3x3 skew symmetric matrix represents a vector

$$A^T = -A$$

$$A = \text{Skew}(\vec{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\text{Skew}(\vec{a})\,\vec{b} = \vec{a} \times \vec{b}$$

Reference & Current Configuration



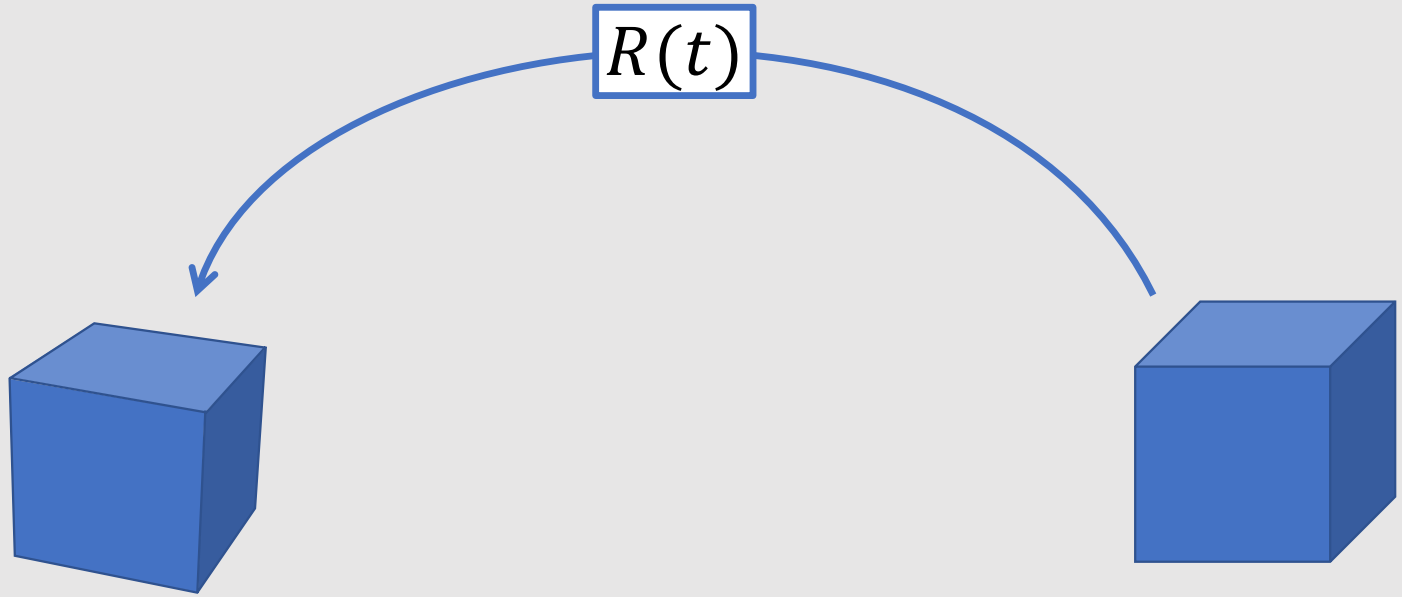
current configuration

reference configuration

Constraint on the Rotation

walking on the edge!

Rotation $R(t)$ changes under constraint $R(t)^T R(t) = I$

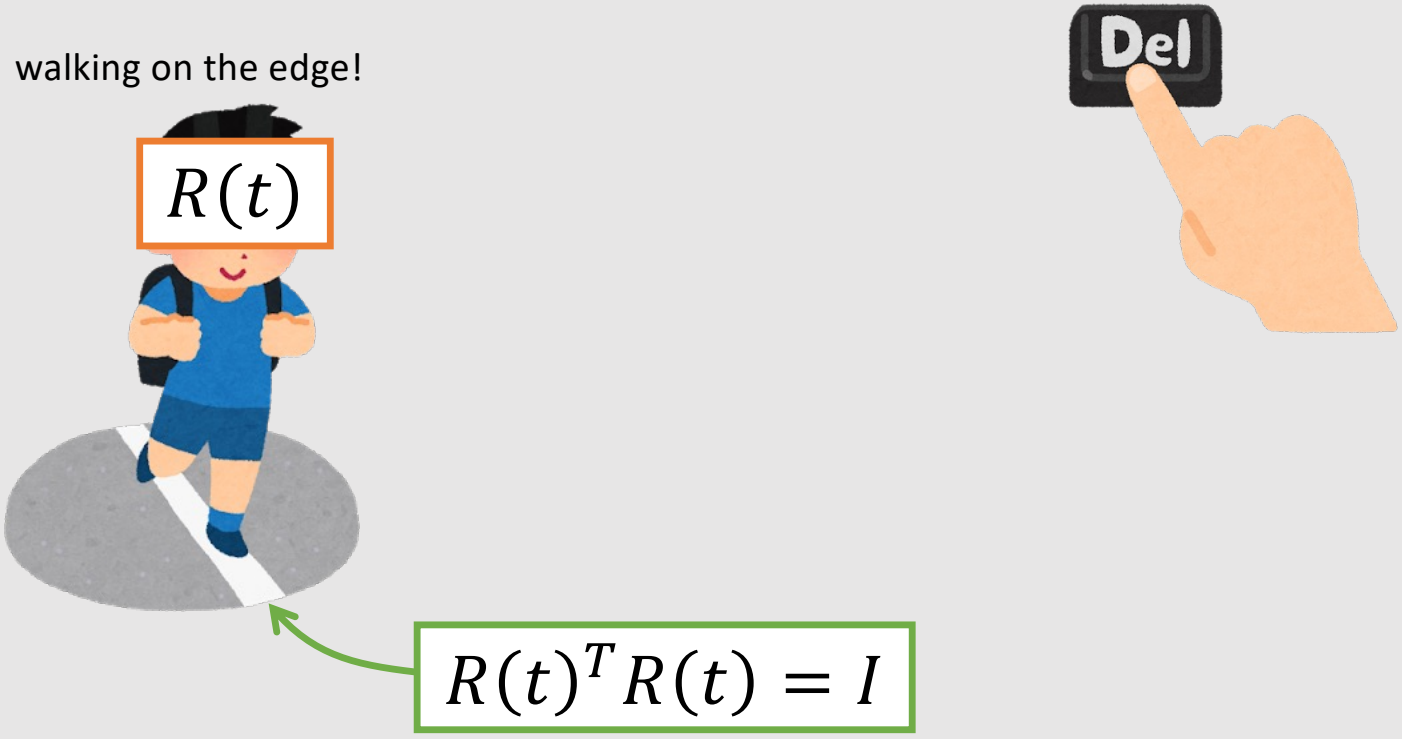


current configuration

reference configuration

Differentiation of Rotation Matrix

Continuous change under constraints $R(t)$ → DoF elimination



Differentiation of Rotation Matrix

Continuous change under constraints $R(t)$ → DoF elimination

$$\frac{d}{dt}(RR^T) = 0$$

$$\dot{R}R^T + R\dot{R}^T = 0$$

$$\dot{R}R^T + \left(\dot{R}R^T\right)^T = 0$$

$\dot{R}R^T$ is skew-symmetric

$$\dot{R}R^T = \text{Skew}(\vec{\omega})$$

$$\dot{R} = \text{Skew}(\vec{\omega})R$$

Another Differentiation of Rotation Matrix

Continuous change under constraints $R(t)$ → DoF elimination

$$\frac{d}{dt}(RR^T) = 0$$

$$\dot{R}R^T + R\dot{R}^T = 0$$

$$\dot{R}R^T + \left(\dot{R}R^T\right)^T = 0$$

$\dot{R}R^T$ is skew-symmetric

$$\dot{R}R^T = \text{Skew}(\vec{\omega})$$

$$\dot{R} = \text{Skew}(\vec{\omega})R$$

$$\frac{d}{dt}(R^TR) = 0$$

$$\dot{R}^TR + R\dot{R} = 0$$

$$\left(R\dot{R}\right)^T + R\dot{R} = 0$$

$R\dot{R}$ is skew-symmetric

$$R^T\dot{R} = \text{Skew}(\vec{\Omega})$$

$$\dot{R} = R\text{Skew}(\vec{\Omega})$$

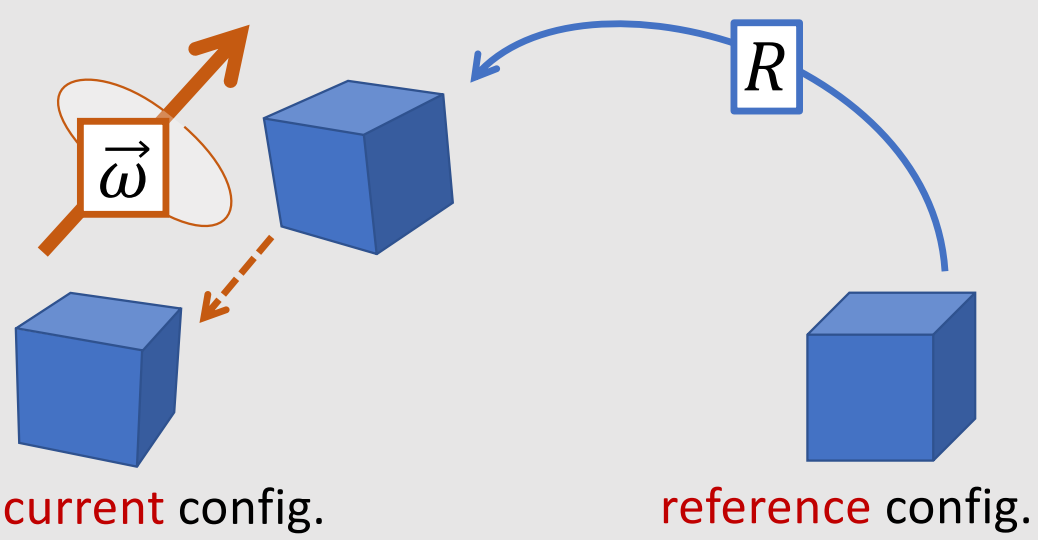
Angular Velocities are Vector Values

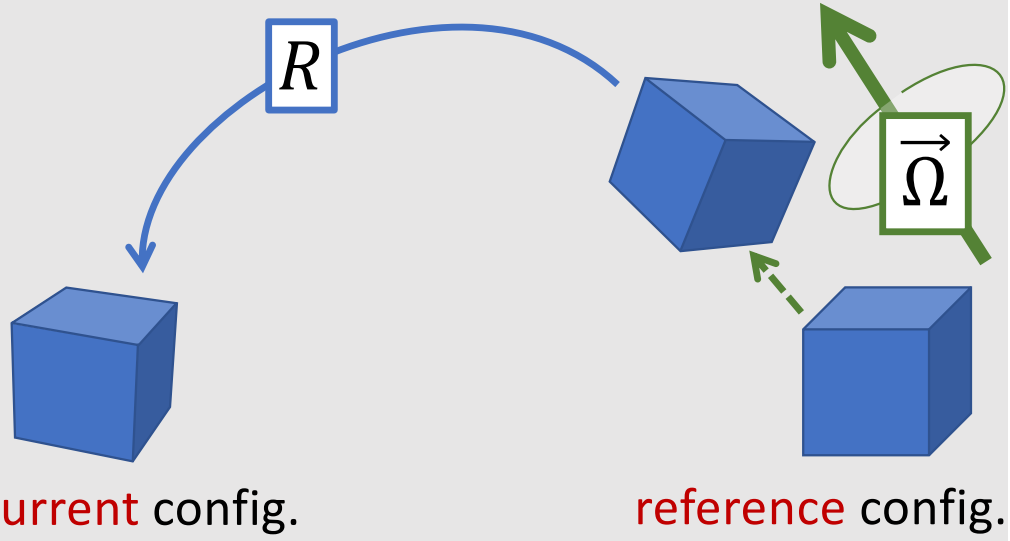
$\vec{\omega}$: velocity in current config.

$$\dot{R} = \text{Skew}(\vec{\omega})R$$

$\vec{\Omega}$: velocity in reference config.

$$\dot{R} = R\,\text{Skew}(\vec{\Omega})$$





Relationship between Two Angular Velocities

$\vec{\omega}$: velocity in current config.

$\vec{\Omega}$: velocity in reference config.

$$\dot{R} = \text{Skew}(\vec{\omega})R$$

$$\dot{R} = R\,\text{Skew}(\vec{\Omega})$$

$$\text{Skew}(\vec{\omega}) = R\,\text{Skew}(\vec{\Omega})R^T$$

$$\vec{\omega} = R\,\vec{\Omega}$$

I'm an anglerfish!

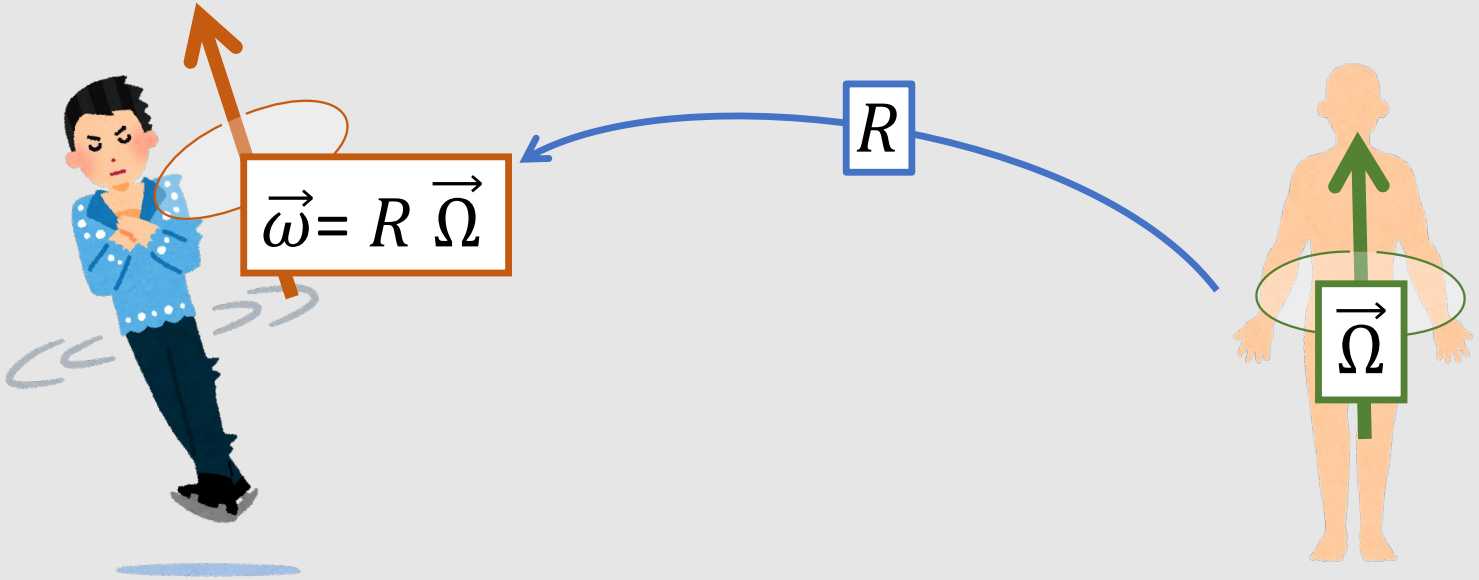
Angular Velocities are Vector Values

$\vec{\omega}$: velocity in current config.

$$\dot{R} = \mathrm{Skew}(\vec{\omega})R$$

$\vec{\Omega}$: velocity in reference config.

$$\dot{R} = R\,\mathrm{Skew}(\vec{\Omega})$$



current config.

reference config.