Pixel size, focal ratio and seeing

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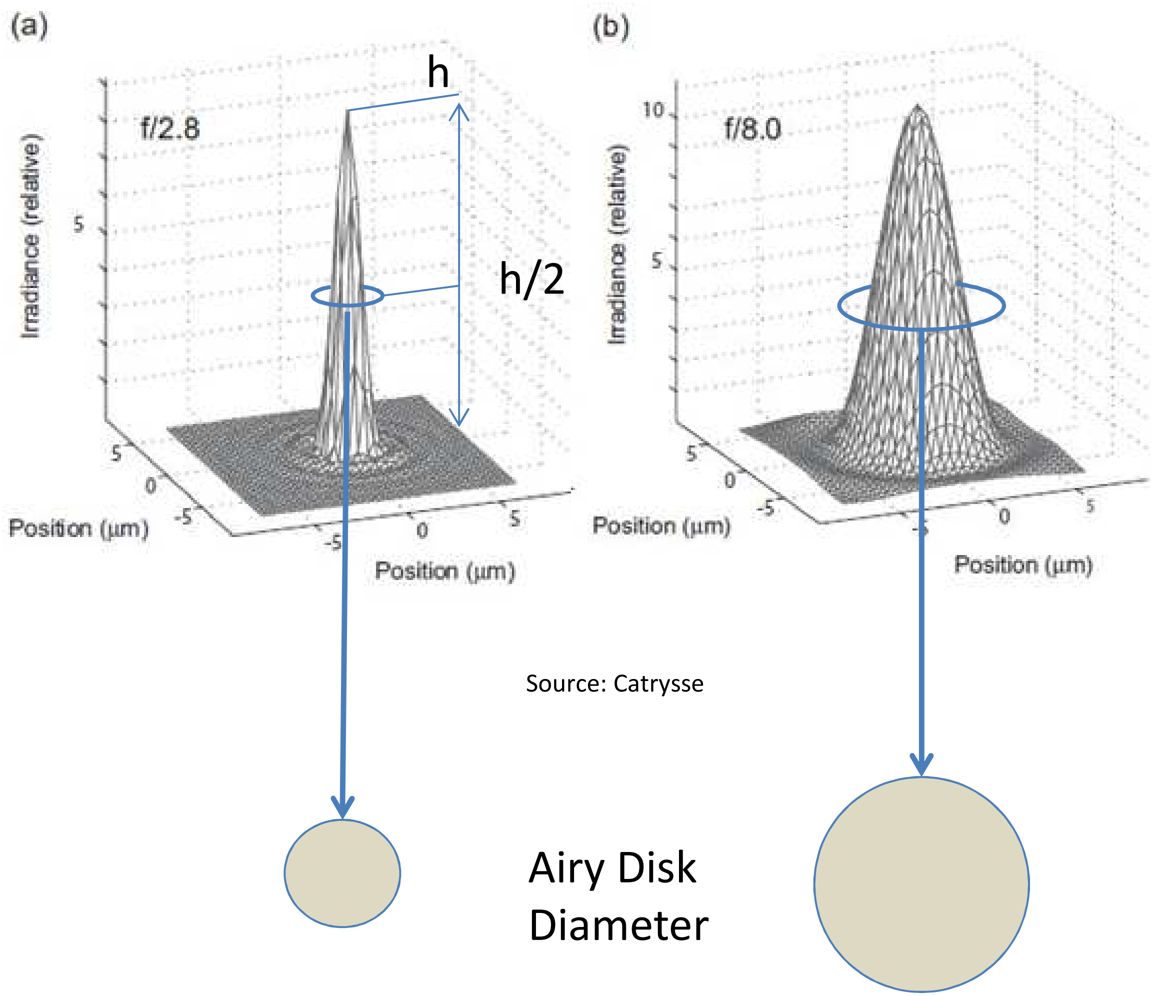
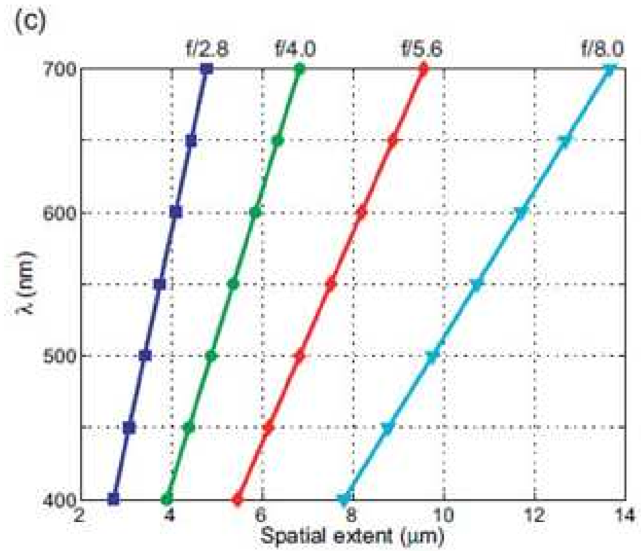
Optics and the Airy Disk:
Focal ratio: Sets spot size for diffraction limited optics



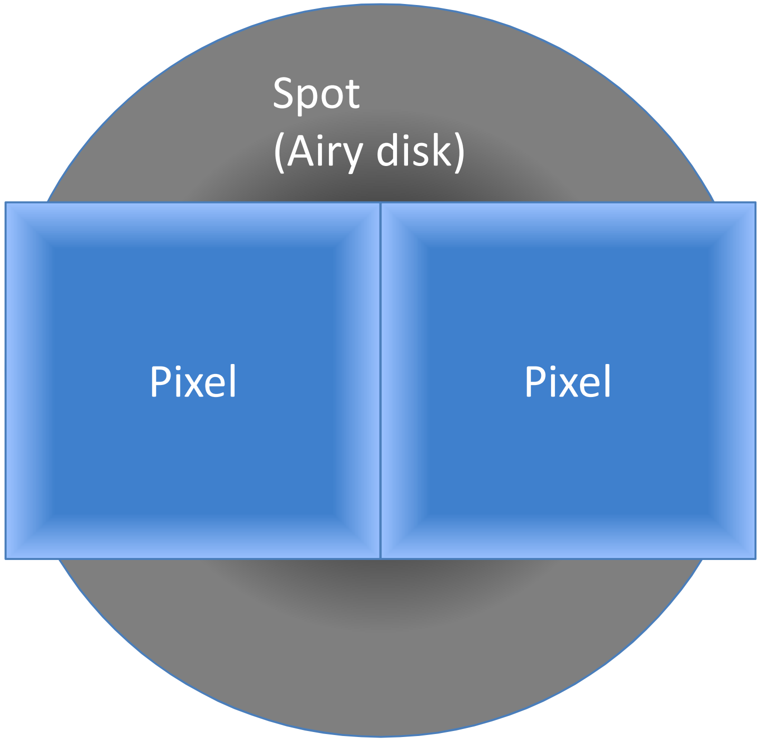


Airy Disk Diameter (microns)

~3 microns, f/2.8

~10 microns, f/8.0

Nyquist Sampling of Airy Disk
Pixel Pitch: Sized to fit Airy Disk (spot):



550 nm test wavelength			
Pixel size (microns)	Optimum F#	Airy Diameter (microns)	Optical resolution LP/mm
0.75	1.12	1.5	667
0.9	1.34	1.8	556
1	1.49	2	500
1.1	1.64	2.2	455
1.25	1.86	2.5	400
1.4	2.09	2.8	357
1.5	2.24	3	333
1.75	2.61	3.5	286
2	2.98	4	250
2.25	3.35	4.5	222
2.5	3.73	5	200
2.75	4.10	5.5	182
3	4.47	6	167
3.5	5.22	7	143
4	5.96	8	125

Nyquist Sampling Criteria:

$$Pixel\ size = 1.22 * \lambda * F\#$$

- Exact Nyquist Sampling: 2 pixels to cover Airy Diameter (spot)

For Seeing-Limited spot size, the FWHM of the seeing sets the spot size and should still be covered by two pixels for proper sampling.

An excerpt from Warren Smith's "Modern Optical Engineering"

Exploring the theory behind Illumination intensity profile, Critical focus zone and Resolution system. Figure 6.9a shows a schematic telecentric system. Note that the dashed principal ray is parallel to the axis to the left of the lens. If this system is used to project an image of a scale (or some other object), it can be seen that a small defocusing displacement of the scale does not change the height on the scale at which the principal ray strikes, although it will, of course, blur the image. Contrast this with Fig. 6.9b where the stop is at the lens, and the defocusing causes a proportional error in the ray height. The telecentric stop is also used where it is desired to project the image of an object with depth (along the axis), since it yields less confusing images of the edges of such an object.

6.7 Apertures and Image Illumination—*f*-Number and Cosine-Fourth

f-Number

When a lens forms the image of an extended object, the amount of energy collected from a small area of the object is directly proportional to the area of the clear aperture, or entrance pupil, of the lens. At the image, the illumination (power per unit area) is inversely proportional to the image area over which this object is spread. Now the aperture area is proportional to the square of the pupil diameter, and the image area is proportional to the square of the image distance, or focal

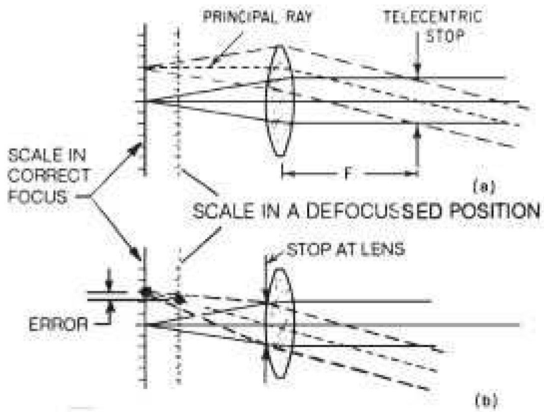


Figure 6.9 The telecentric stop is located at the focal point of the projection system shown, so that the principal ray is parallel to the axis at the object. When the object is slightly out of focus (dotted) there is no error in the size of the projected image as there is in the system with the stop at the lens, shown in the lower sketch.

Non-uniform illumination across the field of view of the detector is normal

This is corrected by flat-fielding

length. Thus, the square of the ratio of these two dimensions is a measure of the relative illumination produced in the image.

The ratio of the focal length to the clear aperture of a lens system is called the relative aperture, *f*-number, or "speed" of the system, and (other factors being equal), the illumination in an image is inversely proportional to the square of this ratio. The relative aperture is given by:

$$f\text{-number} = \text{efl/clear aperture} \tag{6.1}$$

As an example, an 8-in focal length lens with a 1-in clear aperture has an *f*-number of 8; this is customarily written *f*/8 or *f*:8.

Another way of expressing this relationship is by the *numerical aperture* (usually abbreviated as N.A. or NA), which is the index of refraction (of the medium in which the image lies) times the sine of the half angle of the cone of illumination.

$$\text{Numerical aperture} = NA = n' \sin U' \tag{6.2}$$

Numerical aperture and *f*-number are obviously two methods of defining the same characteristic of a system. Numerical aperture is more conveniently used for systems that work at finite conjugates (such as microscope objectives), and the *f*-number is appropriately applied to systems for use with distant objects (such as camera lenses and telescope objectives). For aplanatic systems (i.e., systems corrected for coma and spherical aberration) with infinite object distances, the two quantities are related by:

$$f\text{-number} = \frac{1}{2\text{NA}} \tag{6.3}$$

The terms "fast" and "slow" are often applied to the *f*-number of an optical system to describe its "speed." A lens with a large aperture (and thus a small *f*-number) is said to be "fast," or to have a high "speed." A smaller aperture lens is described as "slow." This terminology derives from photographic usage, where a larger aperture allows a shorter (or faster) exposure time to get the same quantity of energy on the film and may allow a rapidly moving object to be photographed without blurring.

It should be apparent that a system working at finite conjugates will have an object-side numerical aperture as well as an image-side numerical aperture and that the ratio NA/NA′ = (object-side NA)/(image-side NA) must equal the absolute value of the magnification. The term "working *f*-number" is sometimes used to describe the numerical aperture in *f*-number terms. If we use the terms "infinity *f*-number" for the *f*-number defined in Eq. 6.1, then the image-side

Fnumber = F# =
Focal length/aperture

In special cases:

Numerical Aperture =
NA = 1/2F#

working *f*-number is equal to the infinity *f*-number times $(1 - m)$, where m is the magnification.

Another term that is occasionally encountered is the *T-stop*, or *T*-number. This is analogous to the *f*-number, except that it takes into account the transmission of the lens. Since an uncoated, many-element lens made of exotic glass may transmit only a fraction of the light that a low-reflection coated lens of simpler construction will transmit, such a speed rating is of considerable value to the photographer. The relationship between *f*-number, *T*-number, and transmission is

$$T\text{-number} = \frac{f\text{-number}}{\sqrt{\text{transmission}}} \tag{6.4}$$

Cosine-to-the-fourth

For off-axis image points, even when there is no vignetting, the illumination is usually lower than for the image point on the axis. Figure 6.10 is a schematic drawing showing the relationship between exit pupil and image plane for point A on axis and point H off axis. The illumination at an image point is proportional to the solid angle which the exit pupil subtends from the point.

The solid angle subtended by the pupil from point A is the area of the exit pupil divided by the square of the distance OA. From point H, the solid angle is the projected area of the pupil divided by the square of the distance OH. Since OH is greater than OA by a factor equal to $1/\cos\theta$, this increased distance reduces the illumination by a factor of $\cos^2\theta$. The exit pupil is viewed obliquely from point H, and its projected area is reduced by a factor which is *approximately* $\cos\theta$. (This is a fair approximation if OH is large compared to the size of the pupil; for high-speed lenses used at large obliquities, it may be subject to significant errors. See Example A in Chap. 8 for an exact expression.)

Thus the illumination at point H is reduced by a factor of $\cos^3\theta$. This is, however, true for illumination on a plane normal to the line OH

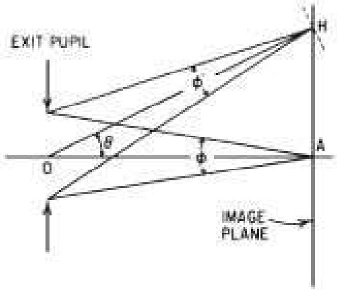


Figure 6.10 Relationship between exit pupil and image points, used to demonstrate that the illumination at H is $\cos^4\theta$ times that at A.

Light intensity roll-off is proportional to the Cos^4(off-axis angle)

This is corrected by flat fielding

(indicated by the dashed line in Fig. 6.10). We want the illumination in the plane AH. An illumination of x lumens per square foot on the dashed plane will be reduced on plane AH because the same number of lumens is spread over a greater area in plane AH. The reduction factor is $\cos \theta$, and combining all the factors we find that

$$\text{Illumination at } H = \cos^4 \theta \text{ (illumination at } A\text{)} \tag{6.5}$$

The importance of this effect on wide-angle lenses can be judged from the fact that $\cos^4 30° = 0.56$, $\cos^4 45° = 0.25$, and $\cos^4 60° = 0.06$. It can be seen that the illumination on the film in a wide-angle camera will fall off quite rapidly.

Note that the preceding has been based on the assumption that the pupil diameter is constant (with respect to θ) and that θ is the angle formed in image space (although many people apply it to the field angle in object space). The "cosine fourth law" can be modified if the construction of the lens is such that the apparent size of the pupil increases for off-axis points, or if a sufficiently large amount of barrel distortion is introduced to hold θ to smaller values than one would expect from the corresponding field angle in object space. Certain extreme wide-angle camera lenses make use of these principles to increase off-axis illumination. The $\cos^4$ effect is in addition to any illumination reduction caused by vignetting. It should be remembered that the cosine-fourth effect is *not* a "law" but a collection of four cosine factors which may or may not be present in a given situation.

6.8 Depth of Focus

The concept of depth of focus rests on the assumption that for a given optical system, there exists a blur (due to defocusing) of small enough size such that it will not adversely affect the performance of the system. The *depth of focus* is the amount by which the image may be shifted longitudinally with respect to some reference plane (e.g., film, reticle) and which will introduce no more than the acceptable blur. The *depth of field* is the amount by which the object may be shifted before the acceptable blur is produced. The size of the acceptable blur may be specified as the linear diameter of the blur spot (as is common in photographic applications) (Fig. 6.11) or as an angular blur, i.e., the angular subtense of the blur spot from the lens. Thus, the linear and angular blurs (B and β, respectively and the distance D are related by

$$\beta = \frac{B}{D} = \frac{B'}{D'} \tag{6.6}$$

Depth of focus (aka critical focus zone) is a function of focal ratio (numerical aperture) and pixel size (as is shown later)

Optimum pixel size set by spot size (seeing limited typically or diffraction limited in best case)

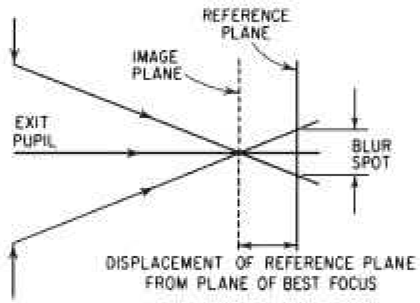


Figure 6.11 When an optical system is defocused, the image of a point becomes a blurred spot. The size of the blur is determined by the relative aperture of the system and the focus shift.

for a system in air, where the primed symbols refer to the image-side quantities.

Angular depth of focus

From Fig. 6.12, it can be seen that the depth of field δ for a system with a clear aperture A can be obtained from the relationship

$$\frac{\delta}{\beta\,(D \pm \delta)} = \frac{D}{A}$$

This expression can be solved for the depth of field, giving

$$\delta = \frac{D^2\beta}{(A \pm D\beta)} = \frac{DB}{(A \pm B)} \tag{6.7}$$

Note that the depth of field *toward* the optical system is smaller than that *away* from the system. When δ is small in comparison with the distance D, this reduces to

$$\delta = \frac{D^2\beta}{A} = \frac{D\beta}{A} \tag{6.8}$$

For the image side, the relationship is

$$\delta' = \frac{D'^2\beta}{A} = \frac{F^2\beta}{A} = F\beta\,(f/\#) = B'(f/\#) \tag{6.9}$$

where the second, third, and fourth forms of the right-hand side apply when the image is at the focal point of the system, and F is the system focal length.

The depth of focus in terms of linear blur-spot size B can be obtained by substituting Eq. 6.6 into the above. Also, note that the depth of field δ and the depth of focus δ' are related by the longitudinal magnification of the system, so that

Depth of field is asymmetric about the focus point: less moving toward the aperture and more moving away. Seeing doesn't change the extremes of the depth of focus: but "softens" the spot size at best focus point

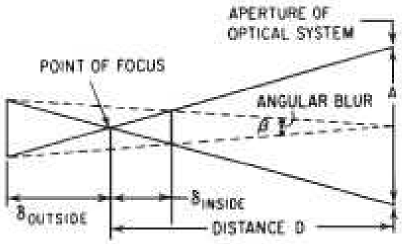


Figure 6.12 Relationships used to determine the longitudinal depth of field in terms of a tolerable angular blur.

$$\delta' = \overline{m} \approx m^2\delta \tag{6.10}$$

The *hyperfocal distance* of a system is the distance at which the system must be focused so that the depth of field extends to infinity. If $(D + \delta)$ equals infinity, then ß is equal to A/D, so that

$$D\ (\text{hyperfocal}) = \frac{A}{\beta} = \frac{F\Delta}{B} \tag{6.11}$$

Key Concept For Critical Focus Zone

The photographic depth of focus

The photographic depth of focus is based on the concept that a defocus blur which is smaller than a silver grain in the film emulsion will not be noticeable. This concept also can be applied to pixel size in, for example, a charge-coupled device (CCD). If the acceptable blur diameter is B, then the depth of focus (at the image) is simply

$$\delta' = \pm B(f\text{-number})$$
$$\delta' = \pm \frac{B}{2\text{NA}} \tag{6.12}$$

The corresponding depth of field (at the object) is from D_{near} to D_{far}, where

$$D_{near} = \frac{fD\ (A + B)}{(fA - DB)} \tag{6.13}$$

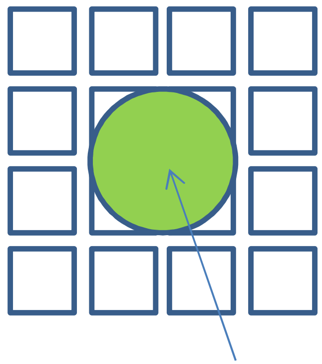
$$D_{far} = \frac{fD\ (A - B)}{(fA + DB)} \tag{6.14}$$

and the hyperfocal distance is simply

$$D_{hyp} = \frac{-fA}{B} \tag{6.15}$$

where D = the nominal distance at which the system is focused (note that, by our sign convention, D is normally negative)

Critical Sampling (per Nyquist)



Optical Blur (spot size)

A = the diameter of the entrance pupil of the lens
f = the focal length of the lens

Note that there are several false assumptions here. We assume that the image is a perfect point, with no diffraction effects. We also assume that the lens has no aberrations and that the blurring on both sides of the focus is the same. None of these assumptions is correct, but the equations above *do* give a usable model for the depth of focus. In practice, the acceptable blur diameter B is usually determined empirically by examining a series of defocused images to decide the level of acceptability; the equations above are then fitted to the results.

Defocusing is a function of position of image plane, seeing doesn't change the defocusing characteristics but can overwhelm the diffraction limited spot size variation as a function of image plane location

6.9 Diffraction Effects of Apertures

Even if we assume that an infinitely small point source of light is possible, no lens system can form a true point image, even though the lens be perfectly made and absolutely free of aberrations. This results from the fact that light does not really travel in straight-line rays, but behaves as a wave motion, bending around corners and obstructions to a small but finite degree.

According to Huygen's principle of light-wave propagation, each point on a wave front may be considered as a source of spherical wavelets; these wavelets reinforce or interfere with each other to form the new wave front. When the original wave front is infinite in extent, the new wave front is simply the envelope of the wavelets in the direction of propagation. At the other extreme, when the wave front is limited by an aperture to a very small size (say, to the order of a half wavelength), the new wave front becomes spherical about the aperture. Figure 6.13 shows a plane wavefront incident on a slit AC, which is in front of a perfect lens. The lens is focused on a screen, EF. We wish to determine the nature of the illumination on the screen. Since the lens of Fig. 6.13 is assumed perfect, the optical path lengths AE, BE, and CE are all equal and the waves will arrive in phase at E, reinforcing each other to produce a bright area. For Huygen's wavelets

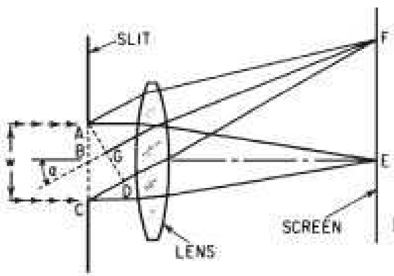


Figure 6.13

starting from the plane wave front in a direction indicated by angle α, the paths are different; path AF differs from path CF by the distance CD. If CD is an integral number of wavelengths, the wavelets from A and C will reinforce at point F. If CD is an odd number of half wavelengths, a cancellation will occur. The illumination at F will be the summation of the contributions from each incremental segment of the slit, taking the phase relationships into account. It can be readily demonstrated that when CD is an integral number of wavelengths, the illumination at F is zero, as follows: if CD is one wavelength, then BG is one-half wavelength and the wavelets from A and B cancel. Similarly, the wavelets from the points just below A and B cancel and so on down the width of the slit. If CD is N wavelengths, we divide the slit into $2N$ parts (instead of two parts) and apply the same reasoning. Thus, there is a dark zone at F when

$$\sin \alpha = \frac{\pm N\lambda}{w}$$

where N = any integer
λ = the wavelength of the light
w = the width of the slit

Thus, the illumination in the plane EF is a series of light and dark bands. The central bright band is the most intense, and the bands on either side are successively less intense. One can realize that the intensity should diminish by considering the situation when CD is 1.5λ, 2.5λ, etc. When CD is 1.5λ, the wavelets from two-thirds of the slit can be shown (as in the preceding paragraph) to interfere and cancel out, leaving the wavelets from one-third of the aperture; when CD is 2.5λ, only one-fifth of the slit is uncanceled. Since the "uncanceled" wavelets are neither exactly in nor exactly out of phase, the illumination at the corresponding points on the screen will be less than one-third or one-fifth of that in the central band.

For a more rigorous mathematical development of the subject, the reader is referred to the references following this chapter. The mathematical approach is one of integration over the aperture, combined with a suitable technique for the addition of the wavelets which are neither exactly in nor exactly out of phase. This approach can be applied to rectangular and circular apertures as well as to slits.

For a rectangular aperture, the illumination on the screen is given by

$$I = I_0 \frac{\sin^2 m_1}{m_1^2} \cdot \frac{\sin^2 m_2}{m_2^2} \tag{6.16}$$

$$m_i = \frac{\pi w_i \sin \alpha_i}{\lambda} \qquad i = 1,2 \tag{6.17}$$

In these expressions λ is the wavelength, w the width of the exit aperture, α the angle subtended by the point on the screen, m_1 and m_2 correspond to the two principal dimensions, w_1 and w_2, of the rectangular aperture and I_0 is the illumination at the center of the pattern.

When the aperture is circular, the illumination is given by

$$I = I_0\left[1 - \frac{1}{2}\left(\frac{m}{2}\right)^2 + \frac{1}{3}\left(\frac{m^2}{2^2 2!}\right)^2 - \frac{1}{4}\left(\frac{m^3}{2^3 3!}\right)^2 + \frac{1}{5}\left(\frac{m^4}{2^4 4!}\right)^2 - \cdots\right]^2$$

$$= I_0\left[\frac{2J_1(m)}{m}\right]^2 \qquad (6.18)$$

where m is given by Eq. 6.17 with the obvious substitution of the diameter of the circular exit aperture for the width, w, and $J_1(\ )$ is the first-order Bessel function. The illumination pattern consists of a bright central spot of light surrounded by concentric rings of rapidly decreasing intensity. The bright central spot of this pattern is called the *Airy disk.*

We can convert from angle α to Z, the radial distance from the center of the pattern, by reference to Fig. 6.14. If the optical system is reasonably aberration-free, then

$$l' = \frac{-w}{2\sin U'}$$

and to a close approximation, when α is small

$$Z = \frac{l'\alpha}{n'} = \frac{-\alpha w}{2n'\sin U'} \qquad (6.19)$$

The table of Fig. 6.15 lists the characteristics of the diffraction patterns for circular and slit apertures. The table is derived from Eqs. 6.16 and 6.18, but the data is given in terms of Z and $\sin U'$ rather than α and w. Note that $n' \sin U'$ is the numerical aperture NA of the optical system.

Notice that 84 percent of the energy in the pattern is contained in the central spot, and that the illumination in the central spot is almost 60 times that in the first bright ring. Ordinarily the central spot and

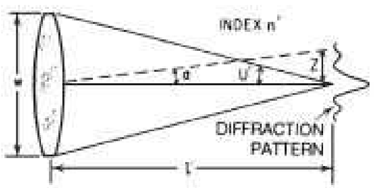


Figure 6.14

By convention the diffraction limited spot size is called the Airy disk. They are mathematically related to the first order Bessel Function.

Ring (or band)	Circular Aperture			Slit Aperture	
	Z	Peak Illumination	Energy in Ring	Z	Peak Illumination
Central maximum	0	1.0	83.9%	0	1.0
1st dark ring	$0.61\ \lambda/n'\ \sin U'$	0.0		$0.5\ \lambda/n'\ \sin U'$	0.0
1st bright ring	$0.82\ \lambda/n'\ \sin U'$	0.017	7.1%	$0.72\ \lambda/n'\ \sin U'$	0.047
2d dark ring	$1.12\ \lambda/n'\ \sin U'$	0.0		$1.0\ \lambda/n'\ \sin U'$	0.0
2d bright ring	$1.33\ \lambda/n'\ \sin U'$	0.0041	2.8%	$1.23\ \lambda/n'\ \sin U'$	0.017
3rd dark ring	$1.62\ \lambda/n'\ \sin U'$	0.0		$1.5\ \lambda/n'\ \sin U'$	0.0
3rd bright ring	$1.85\ \lambda/n'\ \sin U'$	0.0016	1.5%	$1.74\ \lambda/n'\ \sin U'$	0.0083
4th dark ring	$2.12\ \lambda/n'\ \sin U'$	0.0		$2.0\ \lambda/n'\ \sin U'$	0.0
4th bright ring	$2.36\ \lambda/n'\ \sin U'$	0.00078	1.0%	$2.24\ \lambda/n'\ \sin U'$	0.0050
5th dark ring	$2.62\ \lambda/n'\ \sin U'$			$2.5\ \lambda/n'\ \sin U'$	0.0

Figure 6.15 Tabulation of the size of and distribution of energy in the diffraction pattern at the focus of a perfect lens.

the first two bright rings dominate the appearance of the pattern, the other rings being too faint to notice. The illumination in a diffraction pattern is plotted in Fig. 6.16. One should bear in mind the fact that these energy distributions apply to perfect, aberration-free systems with circular or slit apertures which are uniformly transmitting and which are illuminated by wave fronts of uniform amplitude. The presence of aberrations will, of course, modify the distribution as will any nonuniformity of transmission or wave-front amplitude (see, for example, Sec. 6.11).

6.10 Resolution of Optical Systems

The diffraction pattern resulting from the finite aperture of an optical system establishes a limit to the performance which we can expect from even the best optical device. Consider an optical system which images two equally bright point sources of light. Each point is imaged as an Airy disk with the encircling rings, and if the points are close, the diffraction patterns will overlap. When the separation is such that it is just possible to determine that there are two points and not one, the points are said to be resolved. Figure 6.17 indicates the summation of the two diffraction patterns for various amounts of separation. When the image points are closer than $0.5\lambda/NA$ (NA is the numerical aperture of the system and equals $n'\ \sin\ U'$), the central maxima of both patterns blend into one and the combined patterns may appear to be due to a single source. At a separation of $0.5\lambda/NA$ the duplicity of the image points is detectable, although there is no minimum between the maxima from the two patterns. This is *Sparrow's criterion* for resolution. When the image separation reaches $0.61\lambda/NA$, the maximum

The numerical aperture is useful for calculating the best case angular resolution

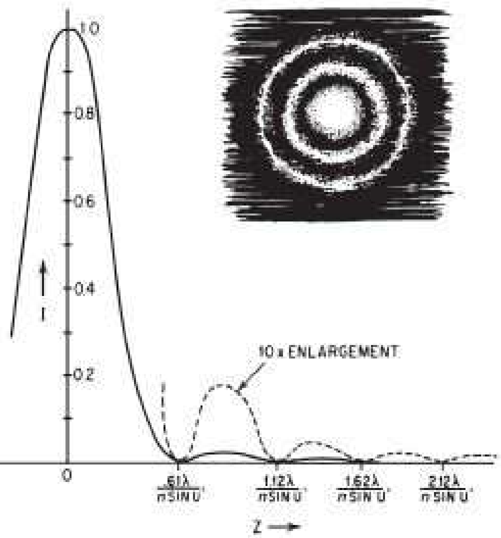


Figure 6.16 The distribution of illumination in the Airy disk. The appearance of the Airy disk is shown in the upper right.

of one pattern coincides with the first dark ring of the other and there is a clear indication of two separate maxima in the combined pattern. This is *Lord Rayleigh's criterion* for resolution and is the most widely used value for the limiting resolution of an optical system.*

From the tabulation of Fig. 6.15, we find that the distance from the center of the Airy disk to the first dark ring is given by

$$Z = \frac{0.61\lambda}{n' \sin U'} = \frac{0.61\lambda}{NA} = 1.22\lambda\ (f/\#) \tag{6.20}$$

This is the separation of two image points corresponding to the Rayleigh criterion for resolution. This expression is widely used in determining the limiting resolution for microscopes and the like. For resolution at the image, the NA of the image cone is used; for resolution at the object, the NA of the object cone is used.

*The diffraction pattern of two point images will always differ somewhat from the diffraction pattern of a single point. It is thus possible to detect the presence of two points (as opposed to one) even in cases where the two points cannot be visually resolved or separated. This is the source of the occasional claims that a system "exceeds the theoretical limit of resolution." In Chap. 11 it is shown that there is a true limit on the resolution of a sinusoidal *line* target; the limit on the spatial frequency is $v_0 = 2\text{NA}/\lambda = 1/\lambda(f/\#)$.

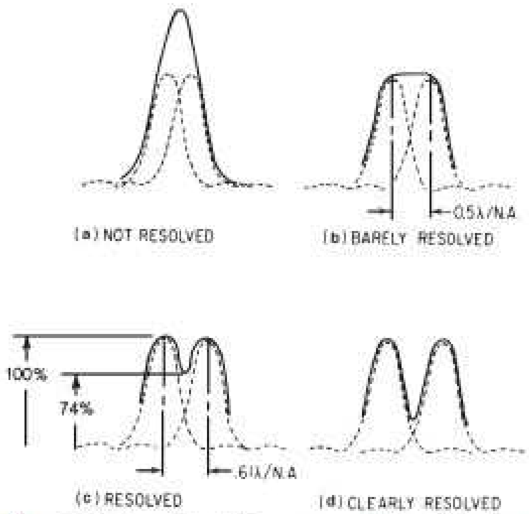


Figure 6.17 The dashed lines represent the diffraction patterns of two point images at various separations. The solid line indicates the combined diffraction pattern. Case (b) is the Sparrow criterion for resolution. Case (c) is the Rayleigh criterion.

Lambda and omega must be in same units: radians are "dimensionless"

To evaluate the performance limits of telescopes and other systems working at long object distances, an expression for the angular separation of the object points is more useful. Rearranging Eq. 6.19 and substituting the limiting value of Z from Eq. 6.20, we get, in radian measure,

$$\alpha = \frac{1.22\lambda}{w} \text{ radians} \tag{6.21}$$

For ordinary visual instruments, λ may be taken as 0.55 μm, and using $4.85 \cdot 10^{-6}$ radians for 1 second of arc, we find that

$$\alpha = \frac{5.5}{w} \text{ seconds of arc} \tag{6.22}$$

Omega here is in units of inches

when w is the aperture diameter expressed in inches. By a series of careful observations, the astronomer Dawes found that two stars of equal brightness could be visually resolved when their separation was $4.6/w$ seconds. Notice that if the Sparrow criterion is used instead of the Rayleigh criterion in Eq. 6.22, the limiting resolution angle is $4.5/w$ seconds, which is in close agreement with Dawes' findings.

It is worth emphasizing here that the *angular* resolution limit is a direct function of wavelength and an inverse function of the aperture of the system. Thus, the limiting resolution is improved by reducing

Limiting resolution is a function of aperture diameter for optics used at infinity (such as a telescope) the wavelength or by increasing the aperture. Note that focal length or working distance do not directly affect the angular resolution. The *linear* resolution is governed by the wavelength and the numerical aperture (NA or *f*-number), and *not* by the aperture diameter.

In an instrument such as a spectroscope, where it is desired to separate one wavelength from another, the measure of resolution is the smallest wavelength difference, $d\lambda$, which can be resolved. This is usually expressed as $\lambda/d\lambda$; thus, a resolution of 10,000 would indicate that the smallest detectable difference in wavelength was 1/10,000 of the wavelength upon which the instrument was set.

For a prism spectroscope, the prism is frequently the limiting aperture, and it can be shown that when the prism is used at minimum deviation, the resolution is given by

$$\frac{\lambda}{d\lambda} = B\,\frac{dn}{d\lambda} \tag{6.23}$$

where B is the length of the base of the prism and $dn/d\lambda$ is the dispersion of the prism material.

A diffraction grating consists of a series of precisely ruled lines on a clear (or reflecting) base. Light can pass directly through a grating, but it is also diffracted. As with the slit aperture discussed above, at certain angles the diffracted wavelets reinforce, and maxima are produced when

$$\sin\alpha = \frac{m\lambda}{S} \pm \sin I \tag{6.24}$$

where λ is the wavelength, I is the angle of incidence, S is the spacing of the grating lines, m is an integer, called the *order* of the maxima, and the positive sign is used for a transmission grating, the negative for a reflecting. (Note that a sinusoidal grating has only a first order.) Since α depends on the wavelength λ, such a device can be used to separate the diffracted light into its component wavelengths. When used as indicated in Fig. 6.18, the resolution of a grating is given by

$$\frac{\lambda}{d\lambda} = mN \tag{6.25}$$

where m is the order and N is the total number of lines in the grating (assuming the size of the grating to be the limiting aperture of the system).