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ATTENUATION OF STRONG GROUND MOTION IN SHALLOW EARTHQUAKES

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SUMMARY

In this paper a theoretical relation is presented to model attenuation of strong ground motion. The relation is based on the Brune spectra for the near and far-field and uses parameters obtained from acceleration records, such as seismic moment, spectral decay factor, duration and stress drop. Attenuation of horizontal peak ground acceleration as a function of epicentral distance is presented for both horizontal components. The theoretical model is applied to PGA data from two Icelandic earthquakes (M_w 6.6 and 6.5). For comparison the model is also applied to data from European and North-American earthquakes. The earthquakes are shallow (depth < 15 km) and with magnitude in the range M 6.4-6.6. The records chosen are from rock and stiff soil sites. The Icelandic earthquakes are strike-slip but the other data come from normal and oblique faults as well. The attenuation curves for the earthquakes are found to have a similar slope. The acceleration levels are lower by a factor of 0.7 for the Icelandic data. Empirical attenuation relations found in the literature are found to provide a poor fit to the data.

INTRODUCTION

Over the past decades many empirical attenuation relations, with the purpose of scaling strong-motion acceleration for engineering purposes, have been presented. These attenuation relations are in most cases similar in form, with magnitude and distance from source to site as the independent variables. The parameters are estimated by fitting the relations to the data (in most cases PGA) by means of regression analysis. When the different attenuations relations, that have been put forward in the literature, are examined, the disagreement between them is apparent (see, for example, Douglas [1] for a recent review). This disagreement is partly due to the data sets from which the models are derived and also due to different modelling, processing and estimation techniques.

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The attenuation model used in this paper is derived theoretically, although it has model parameters that can be determined empirically. It is based on Brune's near- and far-field models (see Brune [2, 3]) and extended with an exponential term to account for spectral decay at high frequencies (Olafsson and Sigbjörnsson [4, 5]). The theoretical model is derived using Parseval's theorem. A similar theoretical approach has also been derived for response spectra (Snaebjörnsson et al. [6]).

In this paper the theoretical model, for both near- and far-field, is presented and then a comparative study is made using data from shallow strike-slip earthquakes in Iceland, June 2000, of magnitude Mw 6.6 and 6.5 (see Sigbjörnsson et al. [7] and Thorarinsson et al. [8]). The model is also applied to PGA from 100 accelerograms obtained in 7 earthquakes in Europe and North-America. A comparison is then made of the results.

STRONG-MOTION MODELLING

The Brune model (Brune [2, 3]) has been applied successfully to analyse Icelandic earthquakes and strong-motion data (Ólafsson et al. [5, 9]; Ólafsson and Sigbjörnsson [4, 10]; Ólafsson [11]). This model will also be used to derive ground motion estimation equation in the following and used to analyse strong-motion data and near source effects. First some preliminaries of the modelling will be reviewed and the applied formulas stated.

The Brune model

Seismic shear waves have been modelled successfully by the so-called Brune model. It was was derived by considering the effective stress needed to accelerate the sides of a circular causative fault on which a stress pulse is applied instantaneously (Brune [2, 3]). It is commonly used to obtain fault dimensions from spectra of shear waves for small to moderate sized earthquakes (Udias [12]). The model describes near- and far-field displacement-time functions as well as spectra and includes the effect of fractional stress drop. The near-field amplitude displacement spectrum is given as (Brune [2]):

$$\left|D(\omega)\right| = \frac{\sigma\beta}{\mu}\frac{1}{\omega\sqrt{\omega^2+\tau^{-2}}} \tag{1}$$

while the far-field rms displacement spectrum can be expressed as follows (Brune [2]):

$$\left\langle D(\omega)\right\rangle = \left\langle R_{\theta\phi}\right\rangle \frac{\sigma\beta}{\mu}\frac{r}{R}\frac{1}{\omega^2+\omega_c^2} \tag{2}$$

Here, $\left\langle R_{\theta\phi}\right\rangle$ is the rms average of the radiation pattern, ω denotes frequency in rad/s, β is the shear wave velocity, μ is the shear modulus, r is the radius of the circular fault, R is the distance from source to site, ω_c and τ are model parameters, respectively, the corner frequency and rise-time, given as (Brune [2, 3]):

$$\omega_c = \sqrt{\frac{7\pi}{4}}\frac{\beta}{r} \tag{3}$$

and

$$\tau = O(r/\beta) \tag{4}$$

where the Landau symbol O indicates the properties of the functional relationship. Finally, σ denotes the effective stress. If the effective stress does not drop to zero but only to a fraction of complete stress drop, the rise-time and high-frequency spectra are modified especially in the long-period range (see Brune [2, 3]).

In the above mentioned studies of Icelandic earthquakes it is assumed that the effective stress equals the stress drop, i.e. $\sigma = \Delta\sigma$, where $\Delta\sigma$ denotes the stress drop. For a double couple source it can be shown that the stress drop is related to the seismic moment, M_o, through (see, for instance, Udias [12]):

$$\Delta\sigma = \frac{7}{16}\frac{M_o}{r^3} \tag{5}$$

Furthermore, it is assumed that the acceleration spectrum can be derived from the displacement spectrum by introducing an exponential term to account for spectral attenuation at high frequencies (Anderson and Hough [13]; Ólafsson [11]):

$$|A(\omega)| = \omega^2 |D(\omega)| \exp(-\tfrac{1}{2}\kappa\omega) \tag{6}$$

where κ is the so-called spectral decay parameter. The spectral decay parameter is related to the quality factor Q through the following equation:

$$\kappa = \frac{R}{\beta Q} \tag{7}$$

The quality factor, Q, is in this context assumed to represent the average scattering and anelastic attenuation over the whole path. Studies of Icelandic strong-motion data indicate that the spectral decay, κ, can be taken as constant (Ólafsson [11]), at least for moderate epicentral distances, where the seismic wave field is dominated by shear waves and the Brune model is assumed to hold as an engineering approximation. The slow increase of κ with distance found by Anderson and Hough [13] is not observed in the available Icelandic data (Ólafsson [11]). This implies that the quality factor Q varies approximately linearly with increasing distance from the source. This seems consistent with the fact that sites at great distance from the source are receiving shear waves that have penetrated through lower crustal layers with less attenuation than the upper layers.

Far-field approximation

The acceleration spectrum in the far-field can hence be expressed as follows, accounting for the free-surface effects and partitioning of the wave energy into two horizontal components (Ólafsson [11]):

$$|A(\omega)| = \frac{2C_P R_{\theta\phi} M_o}{4\pi\beta^3\rho R} \frac{\omega^2}{(1+(\omega/\omega_c)^2)} \exp(-\tfrac{1}{2}\kappa\omega) \tag{8a}$$

Here, C_P is the partitioning factor, $R_{\theta\phi}$ denotes the radiation pattern and ρ is the material density of the crust. The following expression is suggested for the geometrical spreading function (Ólafsson [11]):

$$R = \begin{cases} D_2^{1-n}\, D^n & D_1 < D \le D_2 \\ D & D_2 < D \le D_3 \end{cases} \tag{8b}$$

where $1 < n \le 2$ and R is a distance defined as:

$$D = \sqrt{d^2 + h^2} \tag{8c}$$

Here, d is the epicentral distance and h is a depth parameter. The parameters D_1, D_2 and D_3 are used to set the limits for the different zones of the spreading function. The first zone can be thought of as a crude approximation for the intermediate field. Hence, the quantity D_1 can be approximated by h; D_2 quantifies the size of the zone representing the intermediate field, which is related to the magnitude of the earthquake (as represented by the seismic moment) and the thickness of the seismogenic zone; while D_3 can be thought of as the distance where cylindrical waves begin to dominate the wave field.

The time domain properties of acceleration, a, can readily be derived from the Fourier spectrum, A, by applying the Parseval theorem. This gives:

$$I = \int_0^T a^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |A(\omega)|^2\, d\omega \tag{9}$$

The last integral can be evaluated after substituting Eq.(8). The result is:

$$I = \left(\frac{7}{16}\right)^{2/3} \left(\frac{2C_P\, R_{\theta\phi}\, \Delta\sigma^{2/3}}{\beta\rho R}\right)^2 \frac{\Psi}{\kappa} M_o^{2/3} \tag{10}$$

where Ψ denotes a dispersion function given by the following integral for which a closed form solution is readily obtained:

$$\Psi = \lambda \int_0^{\infty} \frac{\varpi^4}{(1+\varpi^2)^2} e^{-\lambda\varpi} d\varpi \tag{11a}$$

$$\Psi = 1 - \frac{1}{2}\lambda\, \mathrm{ci}(\lambda)\big(\lambda\cos(\lambda) + 3\sin(\lambda)\big) - \frac{1}{2}\lambda\, \mathrm{si}(\lambda)\big(\lambda\sin(\lambda) - 3\cos(\lambda)\big) \tag{11b}$$

Here, ci(·) and si(·) represent the cosine and sine integrals, respectively, $\varpi = \omega/\omega_c$ and:

$$\lambda = \kappa\omega_c \tag{12}$$

where ω_c is the corner frequency. The sine and cosine integrals applied in Eq.(11) are given, respectively, as follows:

$$si(\lambda) = -\frac{\pi}{2} + \int_0^{\lambda} \frac{\sin(t)}{t} dt$$
$$ci(\lambda) = \gamma + \ln(\lambda) + \int_0^{\lambda} \frac{\cos(t)}{t} dt \tag{13}$$

where γ is the Euler constant ($\gamma \approx 0.5772$).

Rms and PGA
The rms ground acceleration can be defined as follows:

$$a_{rms} = \sqrt{\frac{1}{T_d} \int_0^{T_d} a^2(t)\, dt} \tag{16}$$

where T_d is the duration of shaking. A closed form attenuation formula can be derived using the Parseval theorem and Eq. (16) and (10) above. It is:

$$\log_{10}(a_{rms}) = \log_{10}\left(\left(\frac{7}{16} \right)^{1/3} \frac{2C_P R_{\theta\phi} \Delta\sigma^{2/3}}{\beta\rho\sqrt{\kappa}} \right) + \frac{1}{2}\log_{10}\left(\frac{\Psi}{T_d} \right) + \frac{1}{3}\log_{10}(M_o) - \log_{10}(R) \tag{17}$$

The PGA can be related to the rms ground acceleration by applying the theory of locally stationary Gaussian processes (Vanmarcke and Lai [14]). The result is:

$$a_{peak} = p\, a_{rms} \tag{18}$$

where p is the so-called peak function, which depends on the strong motion duration, T_d, and the predominant period of the strong motion phase of the acceleration (Vanmarcke and Lai [14]; Hanks and McGuire [15]; Boore [16]). This relation has been shown to hold for the available Icelandic strong-motion data (see, for instance, Ólafsson [11]). Hence, the same functional form as used for the rms value can describe attenuation of the peak ground acceleration.

Near-field approximation
Rms and PGA
The model described in the previous section is not valid in the near-field and can, therefore, not be expected to describe the peak ground acceleration accurately close to the fault. To be able to obtain an approximation valid for shear waves in the near-fault area, it is suggested that the Brune near-field model, Eq.(1), is used. Hence, the near-field acceleration spectrum can be approximated as follows, accounting for the free surface and partitioning of the energy into two horizontal components:

$$\left| \mathrm{A}(\omega) \right| = \frac{7}{8} \frac{\mathrm{C_p M_o}}{\rho\beta \mathrm{r}^3} \frac{\omega}{\sqrt{\omega^2 + \tau^{-2}}} \exp(-\tfrac{1}{2}\kappa_o \omega) \tag{19}$$

Here, κ_o is the spectral decay of the near-field spectra. Otherwise the same notation is used as above. An approximation for the rms and PGA is now obtained by applying the Parseval theorem and, then, carrying out the integration. The result is:

$$\log_{10}(a_{rms}) = \log_{10}\left(\frac{1}{\sqrt{\pi}}\frac{7}{8}\frac{C_p}{\rho\beta r^3\sqrt{\kappa_o}}\right) + \frac{1}{2}\log_{10}\left(\frac{\Psi_o}{T_o}\right) + \log_{10}(M_o) \qquad (20)$$

Here, the duration is denoted by T_o and Ψ_o is a dispersion function given as:

$$\Psi_o = \lambda\int_0^\infty \frac{\varpi^2}{1+\varpi^2} e^{-\lambda\varpi} d\varpi \qquad (21a)$$

$$\Psi_o = 1 - \lambda(ci(\lambda)\sin(\lambda) - si(\lambda)\cos(\lambda)) \qquad (21b)$$

where $\lambda = \kappa_o / \tau$. It is seen that the PGA predicted by this equation is independent of the epicentral distance and hence should give an estimate on the upper-bound of PGA. Another result, which emerges when Eq.(5) is substituted into Eq.(20), is that the RMS acceleration is directly proportional to the stress drop. That is:

$$a_{rms} = \frac{2}{\sqrt{\pi}}\frac{C_p \Delta\sigma}{\rho\beta\sqrt{\kappa_o}}\sqrt{\frac{\Psi_o}{T_o}} \qquad (22)$$

This indicates that assuming constant stress drop the ground acceleration in terms of the rms value can decrease with increasing earthquake magnitude.

NUMERICAL RESULTS

In the following the presented model is compared to two different datasets: (1) Icelandic strong-motion data from South Iceland earthquakes 2000 and (2) shallow earthquakes from Europe and North-America, chose according to the conditions of depth < 15 km and any magnitude in the range 6.4 to 6.6. The records chosen are from rock or stiff soil sites.

The data from the Icelandic earthquakes is composed of 98 horizontal components of accelerations from two earthquakes occuring on June 17th (M_w 6.6) and June 21st (M_w 6.5) 2000 (see Thorarinsson et al. [7]). The data from the earthquakes are available in the ISESD database (Ambraseys et al. [16]). The data from the European data are obtained from ISESD and the North American data is obtained from the PEER database [17].

Icelandic strong-motion data

The data from the earthquakes on 17 and 21 of June are applied but have been scaled to fit the seismic moment of the 17 June earthquake. The far- and intermediate-field model is represented by the black curve while the near-field model is represented by the horizontal dashed curve. Following data are assumed: shear wave velocity, β = 3.5 km/s; density of rock, ρ = 2.8 g/cm^3; stress drop, $\Delta\sigma$ = 100 bar; average radiation pattern, $R_{\theta\phi}$ = 0.63; partitioning parameter, $C_P = 1/\sqrt{2}$; peak factor, p = 2.94; spectral decay in the far-field, κ = 0.04 s, characteristic dimension of the intermediate-field, R_2 = 30 km; depth parameter, h = 9 km; exponent describing attenuation in the intermediate-field, n = 2; spectral decay in the near-field, κ_o = 0.02 s; characteristic fault dimension (radius), r = 7.0 km; duration used in near-field model, $T_o = 3\cdot r/\beta$. These source data give average slip equal to 1.5 m, which seems in fair accordance with more refined estimates. The necessity of accounting for the above-mentioned near-source effects, which are significant for distances shorter than 10 km, is obvious

The results obtained for the horizontal PGA are displayed in Figure 1 including data from the two above-mentioned earthquakes. The model for the far- and intermediate-fields, represented by the solid black curves, is seen to fit the data reasonably well. Furthermore, the near-field model, given by the solid horizontal line, appears to give sensible values for the near-fault accelerations. The error or the standard deviation, a measure of the uncertainty, has a value $\sigma = 0.2830$. The mean value of the attenuation given by the theoretical model +/- one standard deviation is indicated on Figure 1 as two dashed lines.

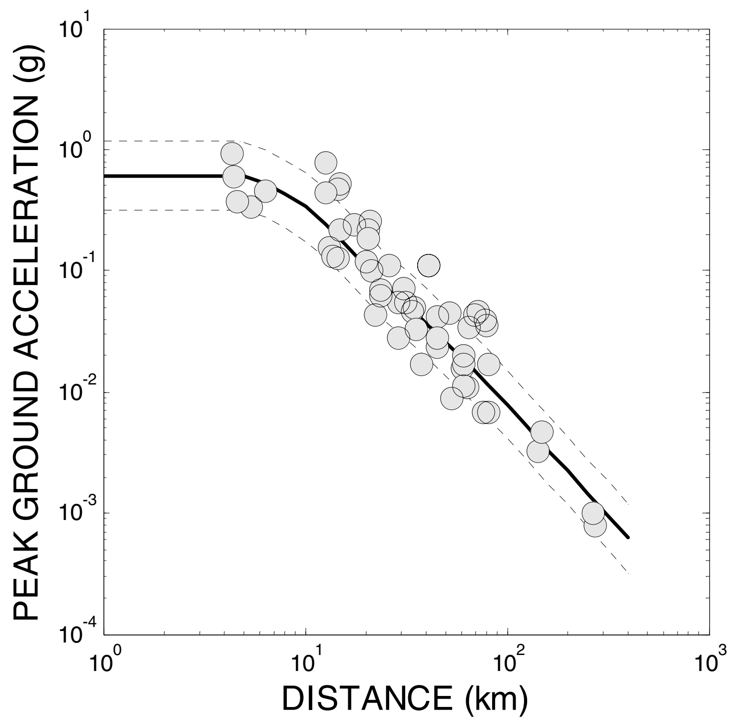

Figure 1: PGA values from 98 components of data from the June 2000 earthquakes in Iceland. The solid black curve represent the mean value of the theoretical attenuation model. The dashed lines represent +/- one standard deviation.

Strong-motion data obtained in shallow earthquakes in Europe and North-America

The theoretical model is applied to the data from the European and North-American earthquakes. The selected data includes 7 earthquakes and 100 records of horizontal acceleration (two components for each site). The result are seen is Figure 2. The standard deviation is $\sigma = 0.2920$ and is indicated with two dashed lines on Figure 2. It is seen that the rate of attenuation is similar as for the Icelandic earthquakes. The same model parameters are used as for the model except the spectral decay parameter of σ ?= 0.02 s instead of σ ?= 0.04 s. This is roughly speaking equivalent to lifting the curve in Figure 1 by a factor $\sqrt{2}$.

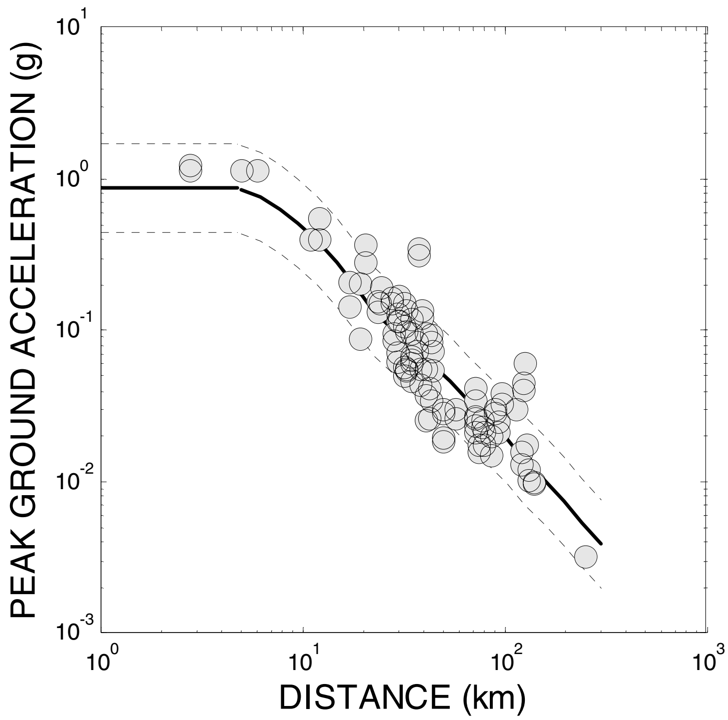

Figure 2: PGA values from 100 components of data from seven European and North-America earthquakes. The solid black curve represent the mean value of the theoretical attenuation model. The dashed lines represent +/- one standard deviation.

DISCUSSION AND CONCLUSION

The theoretical model is seen to fit the PGA data from the European and North-American data equally well as the data from the Icelandic earthquakes, as can see by comparing Figures 1 and 2. The parameters are the same as except $\kappa = 0.02$ instead of $\kappa = 0.04$ s as already mentioned. This is seen the lift the curve by a factor approximately equal to $\sqrt{2}$ compared with the curve seen in Figure 1. The same form of the attenuation curve is seen to fit both data sets equally well. There is however trade-off between the model parameters $\Delta\sigma$ and κ (see Boore et al. [18]). Different pair of $\Delta\sigma$ and κ can therefore give the same results. Instead of lowering the κ values the radius could be decreased, which increased the stress drop.

The source parameters have been estimated for the two Icelandic earthquakes using the acceleration data with an estimation procedure similar to the procedure described in Ólafsson et al. [5] and Ólafsson [11]. They agree well with the parameters used here as model parameters for the attenuation model. The parameters from the other data have not been computed by the same procedure. We know, however, that values of $\kappa = 0.03$ s to 0.04 s and $\Delta\sigma = 70 - 100$ bar are found to give a good fit to North-American data (see Boore [19]). The standard deviation was smaller for the Icelandic earthquake $\sigma = 0.2830$ compared with $\sigma = 0.2920$ for the North-American and European data-set. The non-Icelandic data is also seen to have a few outliers. The outliers are seen to bee around epicentral distances of 40 and 120 km. The outliers can possibly be due to "Moho bounce" (see Somerville et al. [20], Douglas [21])

Several of the empirical attenuation relations that have been presented in the literature have been applied to the Icelandic data from the June 2000 earthquakes, and have been found to give a poor fit to the data (see Ólafsson and Sigbjörnsson [10]). This has also been the observation for Icelandic earthquakes with lesser magnitudes (see Sigbjörnsson [22]). The empirical attenuation relations are also found to be dependent on geographical regions, where the data originates, which they are based on. In addition to this the empirical equations do often not fit well to specific earthquakes. This is not surprising considering the fact that the regression parameters are often estimated from earthquakes over a wide magnitude range.

More research is needed to determine if the theoretical modelling of attenuation as is presented here can be useful for hazard assessment. The model has to be applied to more accelerograms earthquakes which are grouped according to type of mechanism, soil conditions etc., in order to estimate model parameters and determine attenuation of ground motion.

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