SPIN-DOWN POWER OF MAGNETARS

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There is growing evidence that soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs) are isolated neutron stars with superstrong magnetic fields, i.e., magnetars, marking them a distinguished species from the conventional species of spin-down-powered isolated neutron stars, i.e., radio pulsars. The current arguments in favor of the magnetar interpretation of SGR/AXP phenomenology will be reviewed. I will further discuss two major energy source powers in magnetars, i.e. a component due to magnetic dissipation and another component due to loss of the spin energy of the magnetar. The magnetic power is commonly invoked in the present magnetar models, while the spin-down power is currently ignored, mainly because the spin-down luminosity is about two orders of magnitude lower than the magnetic luminosity in the slowly-rotating magnetars. Nonetheless, the spin-down power, which usually manifests itself as a charge accelerator due to the unipolar effect, should be important in the earlier ages of magnetars, and may be still of interests in the slow magnetars. I will review some recent efforts in studying the spin-down-powered behaviors in magnetars and discuss some possible signatures of magnetar spin-down power. These include the quiescent low-frequency coherent emission, quiescent gamma-ray emission, as well as high energy neutrino emission from some of these objects. The detection of any of these signatures will provide solid proofs that magnetars are indeed isolated neutron stars.

1 Introduction

For a long time, radio pulsars have been regarded as the only manifestation of isolated neutron stars[a]. Recent observational developments indicate that isolated neutron stars also manifest themselves as other species[1], among which soft gamma-ray repeaters (SGRs) and anomalous X-ray repeaters (AXPs) have attracted growing attention in the neutron star community. These two types of objects originate, respectively, from the anomalous species of two distinct classes of phenomenon, i.e., gamma-ray bursts and accreting X-ray pulsars, but share many common features.

Recently, two observational facts finally connect a bridge between SGRs and AXPs. First, after being quiescent for more than twenty years, SGR

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[a] The internal compositions and equations-of-state of "neutron stars" are not well determined. These stars could be in principle more exotic, e.g., could be composed of pure strange quark matter. Here I refer to "neutron stars" as a broader class of objects that includes more exotic categories.

0526-66 is found to have a steep non-thermal spectrum in the quiescent state which is similar to the non-bursting AXPs[2]. Second, soft, repeating bursts were recently detected from two AXPs, 1E 1048-5937[3] and 1E 2259+586[4]. These suggest that SGRs/AXPs belong to a unified class of objects.

In the literature, there exist essentially four types of models to interpret SGR/AXP phenomenology. These are, according to the sequence of popularity, the magnetar model[5−9], the accretion model involving fossil disks[10,11,12], the models involving strange quark stars[13−16], and the models involving magnetic white dwarfs[17,18]. It is fair to say that at the current stage none of the models can interpret *all* SGR/AXP observations satisfactorily. Nonetheless, the magnetar model has its merit to interpret most observations under one single hypothesis, i.e., SGRs/AXPs are neutron stars with superstrong magnetic fields ($\sim 10^{14} - 10^{15}$ G at the surface). Other models either have troubles to interpret some observations (e.g. the accretion model fails to account for the super-Eddington SGR bursts, and is likely inconsistent with the optical/IR data) or have to introduce additional assumptions to account for data[19].

2 Evidence Supporting the Magnetar Hypothesis

Below I list the solid observational facts of SGR/AXPs and confront them with the magnetar model.

1. **Timing properties.** Known SGRs/AXPs exclusively have long periods [$P \sim (5 - 12)$ s] and large spin-down rates [$\dot{P} \sim 5 \times (10^{-13} - 10^{-10})$ s/s]. Assuming magnetic braking, this directly refers to a superstrong surface magnetic fields [$B_s \sim (10^{14} - 10^{15})$ G] if these objects are neutron stars. Irregular spin-down may be a common feature of these objects, and is not necessarily related to the bursting behavior. This could be accommodated in a magnetar model with twisted magnetosphere[9].

2. **Quiescent emission properties.** SGRs/AXPs all display a steady luminous X-ray emission with $L_x \sim (10^{35} - 10^{36})$ ergs/s, which could be explained in terms of magnetic dissipation (magnetic field decay[8], or magnetic enhanced cooling[20], or untwisting of a global current-carrying magnetosphere[9]). Optical/IR counterparts have been detected from three AXPs (4U 0142+61, 1E 2259+586, and 1E 1048.1-5937), but no promising interpretation within the magnetar model is proposed so far. No gamma-ray and radio emission has been firmly detected from the SGRs/AXPs.

3. **Burst properties.** SGR bursts are soft and repeating, with luminosity ranging from 10^{38} ergs/s all the way up to $\sim 10^{45}$ ergs/s (usually super-Eddington, and two most luminous bursts, namely giant flares, have been detected from SGR 0526-66 on March 5, 1979; and from SGR 1900+14 on August 27, 1998). A strength of the magnetar model is that it can interpret the bursting phenomenology successfully in terms of the magnetic cataclysmic dissipation events in superstrong magnetic fields. Super-Eddington bursts are natural in strong fields in which the Thomson cross section is suppressed.

4. **Environmental effects.** Most SGRs/AXPs are located close to supernova remnants (SNRs) in projection. Solid associations with the SNRs are yet firmly established. Real associations are consistent with the magnetar theory which predicts that these objects are young neutron stars, but the SNR ages are not fully consistent with the spin-down age of these objects. Assuming associations, SGRs have larger proper motions than AXPs. That one AXP with SNR association, 1E 2259+586, recently displayed hundreds of repeating bursts[4] make the issue more complicated. The claim that SGRs/AXPs are born in dense environments[12] is not confirmed[21].

5. **Cyclotron features**. Cyclotron features have been detected in SGR outbursts[22,23], which is consistent with the magnetar model if the features are of proton-origin, but refers to a much lower magnetic field if the features are of electron-origin.

In summary, though not fully unquestionable, the magnetar model is successful in many respects in interpreting the data. Notice that there is the issue whether a normal neutron star is stable under superstrong magnetic fields[24], but magnetars composed of strange quark matter could nonetheless be formed[25,19].

However, there is hitherto no definite proof that SGRs/AXPs are isolated neutron stars. The pulsar-like behavior, which invokes particle acceleration and emission due to the unipolar effect, is not detected in any of the proposed magnetars. This is an observational missing link between magnetars and pulsars.

3 Two Energy Sources in Magnetars, and a Missing Link Between Magnetars and Pulsars

If SGRs/AXPs are magnetars, there should be two independent energy sources in these objects, i.e., the magnetic energy and the spin energy of a neutron star. Assuming a dipole geometry, the total magnetic energy in a magnetar magnetosphere is $E_B \simeq (1/12)B_p^2R^3$. Taking $B_p = 6.4 \times 10^{19}$ G $\sqrt{P\dot{P}}$, and $R = 10^6$ cm R_6, the magnetic energy can be estimated

$$E_B = 1.7 \times 10^{46} \text{ ergs } (P/5 \text{ s})\dot{P}_{-11}R_6^3, \tag{1}$$

where $\dot{P}_{-11} = \dot{P}/(10^{-11})$. The rotation energy of the magnetar is

$$E_R = (1/2)I\Omega^2 = 7.9 \times 10^{44} \text{ erg } I_{45}(P/5 \text{ s})^{-2}, \tag{2}$$

where $I = 10^{45}$ g cm^3 I_{45} is the typical momentum of inertia of the magnetar. The critical line in the $P - \dot{P}$ diagram for the magnetic energy domination (i.e. $E_B > E_R$) is

$$\dot{P}_{-11} > 5.8P^{-3}I_{45}R_6^{-3}. \tag{3}$$

In reality, what is more relevant is to compare the energy release *rate* of the magnetic energy and the spin energy. The former could be in principle written as $L_B = dE_B/dt = -(1/6)(dB_p/dt)B_pR^3$. Theoretically, dB_p/dt is rather uncertain. It is more straightforward to take $L_B \sim 10^{35} - 10^{36}$ erg s^{-1} as directly inferred from the observations, e.g.

$$L_B = 10^{35} \text{ erg s}^{-1} \ L_{B,35}(B), \tag{4}$$

where $L_{B,35}(B)$ is an unknown function of B, but may be insensitive to B when $B_p \sim 10^{14} - 10^{15}$ G. The spin-down luminosity is

$$L_{sd} = -I\Omega\dot{\Omega} = 4\pi^2IP^{-3}\dot{P} = 3.2 \times 10^{33} \text{ erg s}^{-1} \ I_{45}(P/5 \text{ s})^{-3}\dot{P}_{-11}. \tag{5}$$

Let $L_B > L_{sd}$, the condition of magnetic luminosity domination is

$$P > 1.6 \text{ s } \dot{P}_{-11}^{1/3}I_{45}^{1/3}L_{B,35}^{-1/3}(B). \tag{6}$$

It is found that for the typical values of P and $\dot{P}$ of magnetars, these objects all lie in the magnetic-dominated regime (satisfying both conditions (3) and (6)). Nonetheless, they are not far from the transition boundary. More important, all magnetars ought to be born with millisecond initial period to ensure vigorous dynamo process to occur[26], which means that over the early lifetime of a magnetar, the spin-down energy should be the dominant energy source. Even at the present epoch (for typical P and $\dot{P}$ of magnetars), the spin-down luminosity (which marks the magnitudes of the pulsar behavior) is not too low. In fact, many pulsars with such similar L_{sd}'s are detected to be active.

Then, there comes a missing link between the magnetars and the radio pulsars. These two types of isolated neutron stars seem to solely manifest the two types of energy sources, respectively. The spin-down energy is clearly manifested in pulsars in terms of coherent radio emission, and non-thermal gamma-ray and X-ray emission; while in magnetars the magnetic dissipation energy is manifested in the form of luminous X-rays in the quiescent state and of soft gamma-rays in the burst state. Within the dominant energy output channel for the spin-down luminosity, i.e., the radio band and the gamma-ray band, magnetars are not firmly detected. If lack of magnetic-dominated behavior in normal pulsars is understandable because of their weak fields involved, non-detection of the spin-down-powered behavior in magnetars is in principle not justified. It is worth emphasizing that lack of radio and gamma-ray emission is the prediction of the accretion model for AXPs. Therefore studying the spin-down-powered behavior from magnetars is of great theoretical and observational interests. Only when any spin-down-powered behavior is firmly detected in SGRs/AXPs, could the accretion model be completely ruled out, and hence, presenting a final proof of the magnetar interpretation.

4 Spin-down-Powered Activity in Magnetars

The pulsar-like behavior is marked by particle-acceleration and pair-production in the magnetosphere. Particles are believed to be accelerated in gaps either in the polar cap region near the surface[27,28,29] or above the null charge surface[30]. Accelerated primary particles radiate through curvature radiation or inverse Compton scattering, and the resultant gamma-rays produce electron-positron pairs either through one photon ($\gamma(B) \rightarrow e^+e^-(B)$) or two photon ($\gamma\gamma \rightarrow e^+e^-$) processes. In the polar cap region, the secondary pairs also radiate via synchrotron radiation and inverse Compton scattering, leading to a photon-pair cascade[31,32]. The condition that pair production is prohibited defines radio pulsar death. Conventionally, this is defined through an energy budget criterion that requires a minimum potential to accelerate particles to a high enough energy in order to allow pair production to occur. This defines a pulsar death valley in the long P regime[33]. According to this criterion, the known magnetars are well above the death line, so that their spin-down-powered activity is in principle not prohibited.

In order to interpret the apparent radio quiescence of SGRs/AXPs, Baring & Harding[34,35] argued that pair production is suppressed in magnetars by another more exotic QED process, i.e., magnetic photon splitting. This interpretation relies on the assumption that all three photon splitting modes permitted by charge-parity invariance operate together due to (possible) strong vacuum dispersion effect in superstrong magnetic fields, so that photons with both $\perp$ and $\|$ polarization modes can split. In such a case, for a high enough magnetic field strength, photon splitting will overwhelm magnetic one photon pair production, so that gamma-rays essentially split to photons with lower energies before being materialized, and the magnetar magnetosphere is essentially pair free. Zhang[36] later found that even if one photon pair production can be completely suppressed by photon splitting (as conjectured by Baring & Harding), pairs may be formed via two-photon pair production, essentially because the magnetar near the surface region is a hot environment with a copious soft photon bath generated from magnetic dissipation. Another issue is that, as long as particles can keep being accelerated to higher altitudes where magnetic field strength is considerably degraded, one photon pair production will overtake photon splitting. This operates for the case of an inner gap type invoking space-charge-limited flow[37]. Both arguments suggest that a magnetar magnetosphere may not be pair free.

Now that the magnetar magnetospheric activity does not differ from that of radio pulsars intrinsically, there are good reasons to expect pulsar-like spin-powered activities from magnetars.

4.1 Coherent Emission from Magnetars?

If pairs are not prohibited in the magnetar magnetosphere, why SGRs/AXPs are silent in the conventional radio band? There could be several possible reasons. The most straightforward possibility is that they are actually radio loud, but the survey is not deep enough, or the radio beams do not sweep towards us due to a very narrow beaming angle of a slow rotator[21]. Other possibilities include that the typical coherent emission frequency is not in the conventional radio band[36,38], or that the coherent condition is fragile and is destroyed in the hot and twisted magnetospheric environment. Current radio emission search has not been conducted deep enough to draw firm conclusions that SGRs/AXPs are indeed dormant in the conventional radio band. Furthermore, searches on other related bands (e.g. even lower or higher frequencies) have not been systematically carried out. It remains interesting to keep eyes open on the forthcoming observational progresses. Two pieces of observational evidence are relevant. First, low frequency (below 100 MHz) pulsed emission has been reported to be detected from SGR 1900+14[39] and 1E 2259+586[40], but both objects have much stringent flux upper limits in the higher frequency bands (e.g. GHz), so that if the low-frequency detection is real, the spectral indices must be very steep[41]. Second, the IR/optical emission has been detected in several SGRs/AXPs. Attempts have been made to relate the emission to pulsar-like coherent emission[38]. However, the difficulty is that the IR/optical emission power exceeds the spin-down power, so that one needs an ultra-efficient coherent mechanism (which is currently lacking) to interpret the observation. Searching emission in even lower frequency domain may shed light on the physical process in the magnetar magnetospheres[36].

4.2 Non-thermal high energy emission from magnetars?

Non-thermal high energy emission is expected from both polar cap cascades and/or from outer gaps in magnetars. In the outer gap scenario, the gamma-ray luminosities of the magnetars have been recently predicted[42,43], which are consistent with the current upper limits on these objects. According to these predictions, some SGRs/AXPs should be detectable by the next generation gamma-ray detector, GLAST. In the polar cap scenario, high energy emission is also expected, but the typical spectrum would be considerably shifted to the softer regime due to the large opacities of the gamma-rays (due to one-photon, two-photon pair production and photon splitting). Also the beaming angle is correspondingly smaller. Another gamma-ray emission component is from the photo-meson interactions which might be important in some magnetars as discussed next. More work in this direction needs to be carried out.

4.3 High energy neutrinos from magnetars?

Zhang et al.[44] recently discussed another possible consequence of the magnetar spin-down-powered activity. The discussion is relevant to one half of the magnetar population, i.e., those with favorable geometry such that positive ions (likely protons or light nuclei) are accelerated from the polar cap region. The dominant photo-meson interaction leading to neutrinos occurs through the Δ-resonance,

$$p\gamma \rightarrow \Delta \rightarrow n\pi^+ \rightarrow n\nu_\mu\mu^+ \rightarrow n\nu_\mu e^+\nu_e\bar{\nu}_\mu . \tag{7}$$

Here the proton component is the manifestation of the spin-down energy, while the photon component is the manifestation of the magnetic energy. The maximum potential drop of a magnetar with the rotation frequency, $\Omega = 2\pi/P$ (where P is the period), and surface magnetic field at the pole, $B_p = 10^{15}\mathrm{G}B_{p,15}$, is

$$\Phi = \frac{\Omega^2 B_p R^3}{2c^2} \simeq 6.6\times 10^{15}\ \mathrm{V}\ B_{p,15}R_6^3P^{-2}\ , \tag{8}$$

where $R = 10^6 \mathrm{cm} R_6$ is the stellar radius. In order to achieve the photo-meson threshold condition

$$\epsilon_p \epsilon_\gamma \gtrsim 0.3\ (\mathrm{GeV})^2 f_g, \tag{9}$$

where

$$f_g \equiv (1 - \cos\theta_{p\gamma})^{-1} \tag{10}$$

is a geometric factor and $\theta_{p\gamma}$ is the maximum lab-frame incidence angle between protons and photons, a magnetar has to spin fast enough to produce neutrinos. The condition is

$$P < (2.4 - 6.8)\ \mathrm{s}\ B_{p,15}^{1/2} R_6^{3/2} f_g^{-1/2}\ , \tag{11}$$

which defines a "death valley" for photo-meson interaction and neutrino emission. This is indicated in Figure 1.

The geometric parameter f_g is uncertain. For the simplest case, thermal photons are expected to be emitted from the surface semi-isotropically, so that $\theta_{p\gamma} \leq 90^\mathrm{o}$ and $f_g \geq 1$. In recent magnetar models[9] the magnetosphere is assumed to be globally twisted and current-carrying. The non-relativistic charges in the closed field line region form a resonant cyclotron screen at a high altitude (about 10 stellar radii) with an optical depth higher than unity. The emergent X-ray photons would endure multiple Comptonization before escaping, and the mechanism is used to interpret the observed hard X-ray non-thermal tail in the SGR/AXP spectrum[9]. In such a picture, it is natural to expect some downward X-ray photons (with a luminosity comparable to what is observed) reflected from the resonant cyclotron screen into the open field line region. In such a case, $\theta_{p\gamma} \lesssim 180^\mathrm{o}$ could be achieved so that $f_g \gtrsim 1/2$. This is the most optimistic case for neutrino production. If such a pair reflection screen is ineffective, f_g should be estimated more conservatively by taking into account detailed geometrical effects. This gives $f_g \sim 2$. In Figure 1, death valleys for $f_g = 1/2, 1, 2$ are plotted against the known SGRs/AXPs in the $B_p - P$ diagram.

Assuming above threshold, the neutrino fluxes for an individual magnetar is estimated as

$$\begin{aligned} L_\nu \sim\ & 5.8 \times 10^{32}\ \mathrm{erg/s} \left(\frac{\eta_p}{0.5}\right)^2 \\ & \times \left(\frac{f_c}{0.25}\right) B_{p,15}^3 R_6^{10} \left(\frac{P}{5\mathrm{s}}\right)^{-6} \left(\frac{T_\infty}{0.5\mathrm{keV}}\right)^4 \end{aligned} \tag{12}$$

with the typical neutrino energy

$$\epsilon_\nu \sim (1.4 - 2.2) f_g\ \mathrm{TeV}. \tag{13}$$

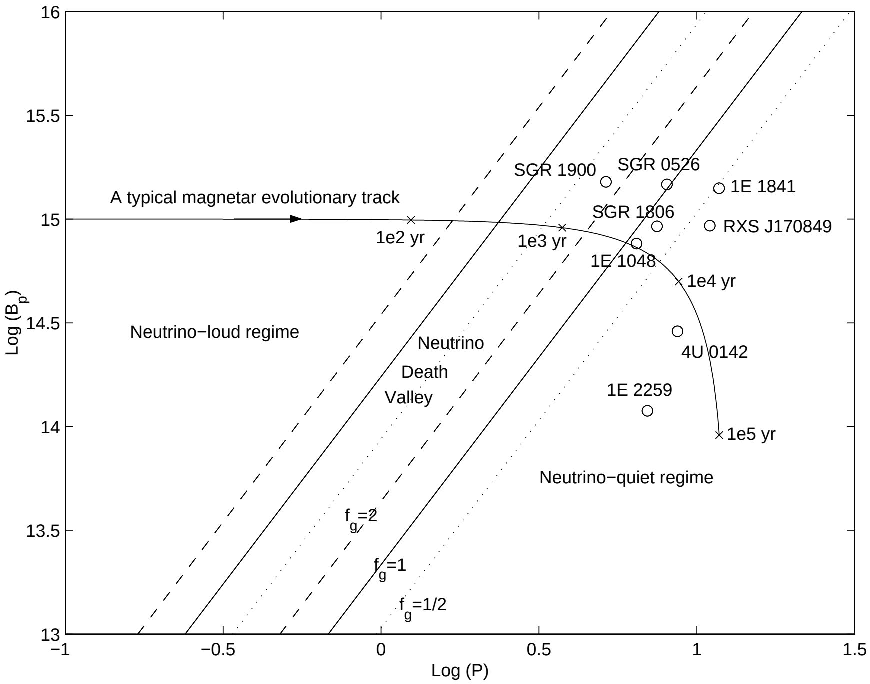

Figure 1. $P - B_p$ diagram of the known magnetars with P and $\dot{P}$ data available (data taken from http://www.atnf.csiro.au/people/pulsar/catalogue, maintained by R. N. Manchester) showing also the neutrino death valley between the two diagonal lines (solid lines for $f_g = 1$; dotted lines for $f_g = 1/2$; and dashed lines for $f_g = 2$, where f_g is the angular correction factor for the threshold condition), and a typical magnetar evolutionary track with $\tau_{mag} \sim 10^4$ yr (with typical ages marked along the track with crosses).

Table 1. Predicted on-beam neutrino-induced upward muon event rates for the four potential neutrino-emitting magnetars assuming they are above photo-meson threshold.

Name	P(s)	$\dot{P}(10^{-11}$s/s)	$B_p(10^{15}$G)	D(kpc)	$\frac{dN}{dAdt}$(km^{-2}yr^{-1})
SGR 1900+14	5.16	10.9	1.51	(3.0-9.0)	(1.5-13) $(\frac{0.1}{\Delta\Omega_\nu})$
SGR 0526−66	8.04	6.6	1.47	$\sim$ 50	$\sim$ 0.003 $(\frac{0.1}{\Delta\Omega_\nu})$
1E 1048−5937	6.45	2.2	0.761	(2.5-2.8)	(0.5-0.7) $(\frac{0.1}{\Delta\Omega_\nu})$
SGR 1806−20	7.48	2.8	0.924	(13.0-16.0)	(0.01-0.02) $(\frac{0.1}{\Delta\Omega_\nu})$

Here η_p is the efficiency factor for proton acceleration indicating the fraction of the proton energy achieved with respect to the maximum proton energy allowed; f_c is a cooling factor for pions after being generated; P is the rotation period of the magnetar; R_6 is the radius of the magnetar in unit of 10^6cm; $B_{p,15}$ is the surface polar cap magnetic field of the magnetar in unit of 10^{15}G; and T_∞ is the typical observed blackbody temperature of the magnetar quiescent X-ray emission (typically 0.5 keV). Assuming that this luminosity is beamed into a sweep-averaged solid angle $\Delta\Omega_\nu \sim 0.1$, which is typical for a polar cap angle ~ 0.01 and a moderate inclination angle of the rotator, an on-beam observer will detect a neutrino number flux at earth

$$\begin{aligned}\phi_\nu = \frac{L_\nu}{\Delta\Omega_\nu D^2 \epsilon_\nu} &\sim 2.1 \times 10^{-12}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1} \\ &\times \left(\frac{\Delta\Omega_\nu}{0.1}\right)^{-1} \left(\frac{\eta_p}{0.5}\right)^2 \left(\frac{f_c}{0.25}\right) B_{p,15}^3 R_6^{10} \\ &\times \left(\frac{P}{5\ \mathrm{s}}\right)^{-6} \left(\frac{T_\infty}{0.5\mathrm{keV}}\right)^4 \left(\frac{D}{5\ \mathrm{kpc}}\right)^{-2} \left(\frac{\epsilon_\nu}{2\ \mathrm{TeV}}\right)^{-1}, \end{aligned} \tag{14}$$

where D is the distance to the source. The probability of detecting a neutrino-induced upward muon with planned neutrino telescopes is $P_{\nu\rightarrow\mu} \simeq 1.3 \times 10^{-6}(\epsilon_\nu/\mathrm{TeV})$, so the on-beam upward muon event rate is

$$\begin{aligned}\frac{dN}{dAdt}(\mathrm{discrete}) &\simeq 1.7\ \mathrm{km}^{-2}\mathrm{yr}^{-1} \left(\frac{\Delta\Omega_\nu}{0.1}\right)^{-1} \left(\frac{\eta_p}{0.5}\right)^2 \\ &\times \left(\frac{f_c}{0.25}\right) B_{p,15}^3 R_6^{10} \left(\frac{P}{5\ \mathrm{s}}\right)^{-6} \left(\frac{T_\infty}{0.5\mathrm{keV}}\right)^4 \left(\frac{D}{5\ \mathrm{kpc}}\right)^{-2}. \end{aligned} \tag{15}$$

A smaller/larger $\Delta\Omega_\nu$ increases/decreases the on-beam neutrino flux, but decreases/increases the probability of on-beam detection. The chances for the observer to be in the neutrino beam are not large. Nonetheless, there is a small but finite probability for directly detecting some neutrinos from these objects. In Table 1, the predicted muon event rates for the four magnetar candidates are listed. The four magnetars are within the neutrino death valley under the most favorable conditions. We see that SGR 1900+14 and 1E 1048-5937 may be detected by km^3 telescopes (such as ICECUBE) with several years of operation, if they are above the photo-meson threshold and if their neutrino beams sweep the Earth.

A direct inference from the above proposal is that the entire population of young magnetars in the universe will contribute to a diffuse neutrino background, before crossing the neutrino death valley. The number flux of this background can be generally estimated as

$$\bar{\phi}_\nu \simeq \frac{0.5 f_{1/2}}{4\pi\bar{\epsilon}_\nu} \int_0^{D_H} \left[\int_0^{\tau_{mag,\nu}} \frac{L_\nu(t) f_b(t)}{4\pi f_b(t) D^2} dt\right] \mathcal{R}(D)(4\pi D^2) dD\ , \tag{16}$$

where $D_H \sim 10^{28}$ cm is the Hubble distance, and $\bar{\epsilon}_\nu$ is the typical energy of the neutrino background. The inner integral is the average total neutrino energy fluence per magnetar emitted towards earth during its neutrino-loud life time $\tau_{mag,\nu} \sim 5 \times 10^3$ yr, which is based on the known magnetars being marginal neutrino emitters. Since 9 magnetars have been discovered in the Galaxy with typical ages of 10^4 yr, the local (redshift $z = 0$) magnetar birth rate can be conservatively estimated

$$\mathcal{R}(0) \simeq 10^{-3}\ \mathrm{yr}^{-1}\mathrm{galaxy}^{-1}\mathcal{R}_{-3} \simeq 2 \times 10^{-5}\ \mathrm{yr}^{-1}\mathrm{Mpc}^{-3}\mathcal{R}_{-3}\,, \tag{17}$$

for a number density of galaxies $n_g = 0.02\ \mathrm{Mpc}^{-3}$. Assuming that the magnetar birth rate follows the star forming rate,

$$\mathcal{R}(z) \simeq \mathcal{R}(0)(1+z)^3 \tag{18}$$

for $z < 2$. The time-dependent beaming parameter $f_b(t)$ (which is the fraction of magnetars whose neutrino beams are directed towards us, so the sweep-averaged solid angle of the neutrino beam is $\Delta\Omega(t) = 4\pi f_b(t)$) cancels out. The outer integral is over the Hubble volume. For remote magnetars, the neutrino flux of an individual source drops as D^{-2} while the total number of magnetars increases as D^3 for $z \ll 1$. Therefore most of the diffuse neutrino emission comes from the farthest magnetars whose birth rate is the highest.

For young magnetars, the time-dependent neutrino luminosity may be also estimated as in (12). There are some noticeable differences, however. For example, due to radiation reaction and possible pair screening effect, $\eta_p \ll 1$. On the other hand, the pair screening altitude could be much higher than the altitude where pions are generated, so that pions could undergo substantial reacceleration before decaying. As a result f_c could be $\gg 1$. Notice that these uncertainties only influence the typical energy of the neutrino background, $\bar{\epsilon}_\nu$, but do not influence the number counts of the neutrino background (16), which can be estimated as follows. The time-dependent neutrino luminosity is

$$L_\nu(t) = A_{pc}(t) c n_{\pi^+}(t) \bar{\epsilon}_\nu\,, \tag{19}$$

where

$$A_{pc}(t) = \pi\Omega(t)R^3/c \tag{20}$$

is the time-dependent polar cap area,

$$\Omega(t) = \Omega_0(1 + t/t_c)^{-1/2} \tag{21}$$

is the time-dependent spin frequency of the magnetar since birth, and Ω_0 and t_c are constants dependent on the initial rotation period and polar magnetic field of the magnetar;

$$n_{\pi^+}(t) = \xi n_{\mathrm{GJ}}(t) = 10\xi_1 n_{\mathrm{GJ}}(t) \tag{22}$$

is the time-dependent number density of pions;

$$\xi \sim R/l_{p\gamma} \sim 10 \tag{23}$$

is the typical pion multiplicity;

$$n_{\rm GJ}(t) = \Omega(t) B_p / 2\pi c e\,; \tag{24}$$

$\bar{\epsilon}_\nu$ is the typical neutrino energy whose detailed value does not enter the problem (i.e. canceled out in eq.[16]). Averaging over the magnetar neutrino-loud lifetime $\tau_{mag,\nu}$, and properly taking into account the cosmological evolution, we estimate

$$\bar{\phi}_\nu \sim 10^{-13}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1} f_{1/2}\xi_1 \mathcal{R}_{-3}. \tag{25}$$

This background is insensitive to the location of the neutrino death valley ($\propto \ln \tau_{mag,\nu}$), because logarithmically the entire magnetar life-time essentially contribute to the final value equally. The detectability of this background, however, is sensitively dependent on the typical energy of the neutrinos, which in turn depends on whether the secondary pions undergo substantial re-acceleration before decaying to neutrinos. Detailed numerical simulations are needed to address this. Nonetheless, we can set lower and upper bounds for the typical neutrino energies. If pion re-acceleration is unimportant, the typical neutrino energy is bound from below to ~ 2 TeV due to the inverse Compton cooling, in which case the diffuse background is completely masked by the atmospheric background and non-detectable. If pion reacceleration is efficient, however, the typical neutrino energy is bound from above by the radiation reaction limit of the pions, and the typical neutrino energy could reach 1 PeV or even higher. Such a neutrino background would become observationally interesting for ICECUBE if $\bar{\epsilon}_\nu \geq 100$ TeV, and at such energies, the diffuse emission from other neutrino sources becomes weaker than this component.

4.4 Future directions

In the above discussions about magnetar particle accelerations, a dipole configuration for the magnetic component has been assumed, whilst a magnetar magnetosphere is certainly non-dipole. More specifically, Thompson et al[9] argue that the SGR/AXP phenomenology is consistent with the hypothesis that the magnetar magnetosphere is globally twisted. It would be interesting to study the charge-depleted acceleration regions in such a twisted magnetosphere, both near the polar cap region and in the "outer gap" region. A careful study in this direction is called for.

5 Concluding Remarks

Current data reveals an observational missing link between magnetars and pulsars. The pulsar-like behavior, powered by the spin-down energy of the neutron star, has not been firmly detected. Several theoretical efforts have been made to predict spin-down-powered activities in magnetars, including low-frequency coherent emission, gamma-ray emission and possible neutrino emission. These signals are expected to be faint, but are achievable by future facilities. Connecting this missing link with future observations would provide a solid proof that SGRs/AXPs are indeed isolated neutron stars with strong magnetic fields, i.e., magnetars.

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References

1. G. G. Pavlov, V. E. Zavlin, & D. Sanwal, astro-ph/0206024 (2002).
2. S. R. Kulkarni et al., *Astrophys. J.*, **585**, 948 (2003).
3. F. P. Gavriil, V. M. Kaspi, & P. M. Woods, P. M. *Nature*, **419**, 142 (2002).
4. V. M. Kaspi et al., *Astrophys. J.*, **588**, L93 (2003).
5. R. C. Duncan, & C. Thompson, *Astrophys. J.*, **392**, L9 (1992).
6. B. Paczyński, *Acta Astronomica*, **42**, 145 (1992).
7. C. Thompson, & R. C. Duncan, *Mon. Not. R. Astro. Soc.*, **275**, 255 (1995).
8. C. Thompson, & R. C. Duncan, *Astrophys. J.*, **473**, 322 (1996).
9. C. Thompson, M. Lyutikov, & S. R. Kulkarni, *Astrophys. J.*, **574**, 322 (2002).
10. P. Chatterjee, L. Herquist, & R. Narayan, *Astrophys. J.*, **534**, 373 (2000).
11. M. A. Alpar, *Astrophys. J.*, **554**, 1245 (2001).
12. D. Marsden, et al., *Astrophys. J.*, **550**, 397 (2001).
13. C. Alcock, E. Farhi, & A. Olinto, *Phys. Rev. Lett.*, **57**, 2088 (1986).
14. K. S. Cheng, & Z. G. Dai, *Phys. Rev. Lett.*, **80**, 18 (1998).
15. B. Zhang, R. X. Xu, & G. J. Qiao, *Astrophys. J.*, **545**, L127 (2000).
16. V. V. Usov, *Phys. Rev. Lett.*, **87**, 021101 (2001).
17. B. Paczyński, *Astrophys. J.*, **365**, L9 (1990).
18. V. V. Usov, *Astrophys. J.*, **410**, 761 (1993).
19. B. Zhang, *Mem. S. A. It.*, **73**, 516 (2002) (astro-ph/0102098).

20. J. S. Heyl, & L. Hernquist, *Astrophys. J.*, **489**, L67 (1997).
21. B. M. Gaensler, P. O. Slane, E. V. Gotthelf, & G. Vasisht, *Astrophys. J.*, **559**, 963 (2001).
22. A. I. Ibrahim et al., *Astrophys. J.*, **574**, L51 (2002).
23. A. I. Ibrahim et al., *Astrophys. J.*, **584**, L17 (2002).
24. A. Pérez Martínez, H. Pérez Rojas, & H. Mosquera Cuesta, *Eur. Phys. J. C*, in press (2003) (astro-ph/0303213).
25. R. X. Xu, & F. H. Busse, *Astron. Astrophys.*, **371**, 963 (2001).
26. C. Thompson, & R. C. Duncan, *Astrophys. J.*, **408**, 194 (1993).
27. M. A. Ruderman, & P. G. Sutherland, *Astrophys. J.*, **196**, 51 (1975).
28. J. Arons, & E. T. Scharlemann, *Astrophys. J.*, **231**, 854 (1979).
29. A. K. Harding, & A. G. Muslimov, *Astrophys. J.*, **508**, 328 (1998).
30. K. S. Cheng, C. Ho, & M. A. Ruderman, *Astrophys. J.*, **300**, 500 (1986).
31. J. K. Daugherty, & A. K. Harding, *Astrophys. J.*, **458**, 278 (1996).
32. B. Zhang, & A. K. Harding, *Astrophys. J.*, **532**, 1150 (2000).
33. B. Zhang, In: J.L. Han & R. Wielebinski (eds) Proc. of Sino-German Radio Astronomy Conference on Radio Studies of Galactic Objects, Galaxies and AGNs, special issue of ChA&A (2002) (astro-ph/0209160).
34. M. G. Baring, & A. K. Harding, *Astrophys. J.*, **507**, L55 (1998).
35. M. G. Baring, & A. K. Harding, *Astrophys. J.*, **547**, 929 (2001).
36. B. Zhang, *Astrophys. J.*, **562**, L59 (2001).
37. B. Zhang, & A. K. Harding, *Astrophys. J.*, **535**, L51 (2000).
38. D. Eichler, M. Gedalin, & Y. Lyubarsky, *Astrophys. J.*, **578**, L121 (2002).
39. Yu. P. Shitov, *IAU Circ.*, 7110 (1999).
40. V. M. Malofeev, & O. I. Malov, in Sixth Conf. on Physics of Neutron Stars, ed. A. Koptsevich (St. Peterburg: Ioffe Inst.), 33 (2001).
41. D. R. Lorimer, & K. M. Xilouris, *Astrophys. J.*, **545**, L385 (2000).
42. K. S. Cheng, & L. Zhang, *Astrophys. J.*, **562**, 918 (2001).
43. L. Zhang, & K. S. Cheng, *Astrophys. J.*, **579**, 716 (2002).
44. B. Zhang, Z. G. Dai, P. Mészáros, E. Waxman, A. K. Harding, *Astrophys. J.*, in press (2003) (astro-ph/0210382).