Comet 73P/Schwassmann-Wachmann: Nucleus Fragmentation, Its Light-Curve Signature, and Close Approach to Earth in 2006

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Abstract. *The history of observation of comet 73P is described, and the remarkable 1995 apparition (during which the nucleus split into a large number of fragments) is highlighted. The primary breakup event was accompanied by an enormous outburst at optical and radio wavelengths. The principal fragment and two surviving companions were observed as recently as 2000. The comet's very favorable return to the sun in 2006 offers an opportunity to search for these still-possibly-existing minor fragments of the original nucleus. One of this paper's objectives is to facilitate such an endeavor by providing a search ephemeris.*

1. Introduction

Cascading fragmentation is increasingly perceived as the dominant process of cometary extinction. This suggests that genuine disintegration of the original cometary nucleus, rather than its progressive deactivation and/or gradual sublimation, accounts in most cases for the object's end state. For comets that closely approach the sun, the fragmentation process is accompanied or followed by potentially significant, heliocentric-distance-dependent nucleus erosion. Although the mechanism is unknown, fragmentation appears to be essentially spontaneous, is usually nontidal, and could be facilitated by extremely low cohesion of cometary nuclei, with rotational and thermal stresses believed to play a role. Comets may and often do split more than once and over a number of revolutions about the sun. Unfortunately, little is known about the disintegration rate, the number of fragmentation steps and fragment generations, the size distribution of fragments as a function of time, and the temporal scales involved, which may vary significantly from case to case. As fragmentation products grow ever smaller and fainter with time, the flow of information is constrained by the detection threshold. Since this limit depends, besides instrumentation, on the observer's distance, great strides in the understanding of the process can be achieved during the earth's close encounters with comets that are known to have split.

An important property of split comets is brightness fluctuation of their fragments, which reflects irregular variations of their activity with time. It is not unusual for some of thee fragments to become temporarily undetected only to reappear later. A fragment's life span depends not only on its size, but also on its cohesion and physical behavior. Persistent fragments of periodic comets may survive for two or more revolutions about the sun, with the primary nucleus (the most massive fragment) often continuing to orbit the sun as if unaffected by the fragmentation events. On the other hand, in extreme cases all fragments may disintegrate catastrophically on a time scale of only a few weeks or so following a fatal fragmentation event, with the comet literally ceasing to exist. Investigations of the physical evolution of individual fragments of a split comet contribute significantly to our understanding of the fragmentation process.

Because of brightness fluctuations and gaps in observing a split comet (due to unfavorable observing conditions resulting from a changing projection geometry), it may become very difficult or impossible to identify the fragments over long periods of time without applying a sophisticated model that is capable of determining the most probable scenario for the comet's fragmentation sequence and hierarchy.

There are numerous documented cases of a close temporal relationship between a fragmentation event experienced by a comet and its outburst or flare-up. Both phenomena are likely to be inextricable products of suddenly increased activity, with the companion nucleus representing in fact the largest "particle" in the cloud of emerging dust ejecta.

In this paper, I apply the concept of cascading fragmentation to investigate the orbital evolution of the nucleus of comet 73P/Schwassmann-Wachmann, which split into a number of pieces in 1995, which has a very favorable return to the sun in 2006, and which was very recently recovered (Green 2005). As a necessary preparatory step for establishing the fragmentation sequence and hierarchy of companion nuclei, I first derive the comet's composite light curve by exploring all information available on its brightness since discovery. I then focus on the more extensively observed nucleus fragments, present a set of their most probable birth scenarios, and examine their potential relationship to the enormous outburst that the comet is known to have experienced in 1995. Finally, I provide search ephemerides for several potentially surviving nucleus fragments during this return, thus assisting observers in their efforts to recover as many nucleus fragments as possible. These predictions should also benefit a wide range of other comet science endeavors, including activities aimed at radar detection and scrutiny of the nucleus fragments and, more generally, offer information critical to future robotic exploration of comets and their nucleus environment.

2. The Observation History of Comet 73P

Comet 73P is a member of the Jupiter family of short-period comets, making one revolution about the sun in 5.4 years and currently reaching 0.94 AU from the sun at perihelion. This is the comet's sixth observed return. Its history makes 73P one of the best candidates for studies of cascading fragmentation.

Discovered in 1930, when it approached Earth to 0.062 AU on May 31, the comet was observed fairly extensively for nearly four months. Yet it was missed at the subsequent returns to the sun and eventually lost. It remained unobserved until 1979, when it arrived at perihelion five weeks later than predicted (the orbital error apparently amplified by a close approach to Jupiter in 1965), was picked up as a new comet by J. Johnston and M. Buhagiar at Perth (Candy 1979; Marsden 1984), and remained under observation for three months. Missed again during the unfavorable return of 1985, it was followed extensively in 1990 and especially in 1995. More recently, the comet was detected beyond 3 AU from the sun in March-April 2000 (Boehnhardt et al. 2002) and, remarkably, at elongations smaller than 27° from the sun in November and December 2000 (Marsden 2000, 2001) during the utterly unfavorable return to perihelion in early 2001. Before its 2005 recovery (Green 2005), the comet had last been seen in mid-December 2001.

The comet's physical aspect during the discovery apparition was of major interest, because the object was widely observed to have a double-tail appearance in May 1930, reminiscent of a spindle or a spiral nebula seen edgewise (*e.g.*, Van Biesbroeck 1930, Beyer 1931). Sekanina (1989) showed that the extension pointing away from the sun (which was not seen in June and July 1930) was a regular tail, while the broader and usually shorter appendage — reported also after perihelion (Dartayet 1931, Hartmann 1931) — was a sunward emission fan, providing information on the surface location of an active region responsible for the dust-ejecta anisotropy and on the nucleus spin-vector position. Sekanina's modeling of the fan-orientation variations with time led him to conclude that the nucleus was precessing, its rotation axis describing an angle of $\sim 90^\circ$ over a period of three months. The active region extended up to about 20° from the rotation pole and its area was estimated at 0.8 km^2.

The truly exciting apparition was that of 1995, when the comet underwent a huge outburst (Sec. 3) and, several months later, a multiple nucleus was observed for the first time (Sec. 4). Astrometric observations of two or more nucleus fragments were made during much of 1996, interrupted only by the comet's conjunction with the sun, and again in 2000 and 2001. No comprehensive investigation of this comet's fragmentation has ever been published.

The 2006 return to the sun offers an exceptional opportunity to search for the nucleus fragments observed in the past as well as for products of possible additional, more-recent fragmentation events that we are as yet unaware of. The return is almost as favorable as that of 1930, with the main comet predicted to approach Earth to 0.0787 AU, or 11.8 million km, on 2006 May 12.4 TT. This close encounter will allow observers with big telescopes to detect inert fragments as small as 80 meters across — and even smaller ones if they still show signs of activity. However, such detections — of apparent magnitude, say, 21-22 — will only be possible if a search ephemeris pinpointing their locations is available. In 2001, the differences in perihelion times among the nuclei reached up to more than $\sim$ 0.7 day (*e.g.*, Nakano 2000), which by 2006 are expected to increase to much more than 1 day (*e.g.*, Nakano and Marsden 2003a, 2003b), equivalent to separations of up to at least $\sim$ 4 million km along the orbit. At the earth's distance of $\sim$ 12 million km, such separations will project as more than 20° in the sky. A dependable ephemeris will indeed be absolutely indispensable.

3. The Composite Light Curve

No comprehensive study of the history of the light curve for comet 73P has ever been published, although the huge 1995 outburst would itself seem to justify such an effort. The highly favorable 2006 return to the sun adds more urgency to it.

Data on the integrated brightness of comet 73P have been reported from each of the observed returns to the sun. Previously I analyzed the comet's light curve from 1930 (Sekanina 1989) by examining a total of 44 visual magnitude estimates made by 10 observers (or observer groups), mostly around the time of closest approach to Earth. (The paper lists all references to the original sources.) When normalized to 1 AU from Earth by an inverse-square power law, the estimates appeared utterly discordant in spite of an introduction of personal/instrument corrections (see below). It appeared that some 1930 observers saw the comet brightening on its way to perihelion, while others fading. The culprit was obviously the "delta effect" brought about by the human eye's inability to detect faint outer fringes of a very extended object of an exceptionally low surface-brightness gradient. More recently I re-examined an augmented 1930 set of 63 mostly visual magnitudes using an inverse-first-power law, as proposed long ago by Öpik (1963), and was surprised to find that 80 percent of 55 data points by 15 observers with two or more published observations now became consistent with one another. In addition, the resulting light curve conformed to the light curves from the 1979 and 1990 apparitions and to the pre-outburst light curve from 1995, even though the comet's perihelion distance in 1930 was 0.07-0.08 AU greater. This is shown in Figure 1 by the solid curve attaining normalized magnitude H_Δ (at 1 AU from Earth) of 10.0 at perihelion and marked 1930-1995 prior to perihelion and 1930-1990 after perihelion.

The magnitude observations reported since 1979, nearly all of which were taken from the *International Comet Quarterly (ICQ)*, were normalized to 1 AU from Earth with an inverse-square power law. The data reduction then followed a standard procedure, which, to the extent possible, corrected for personal and instrumental effects of observers. Their temporally overlapping individual light curves were visually compared and the scatter among them minimized by shifting them along the magnitude axis until the best match was in each case achieved. Time gaps between any two

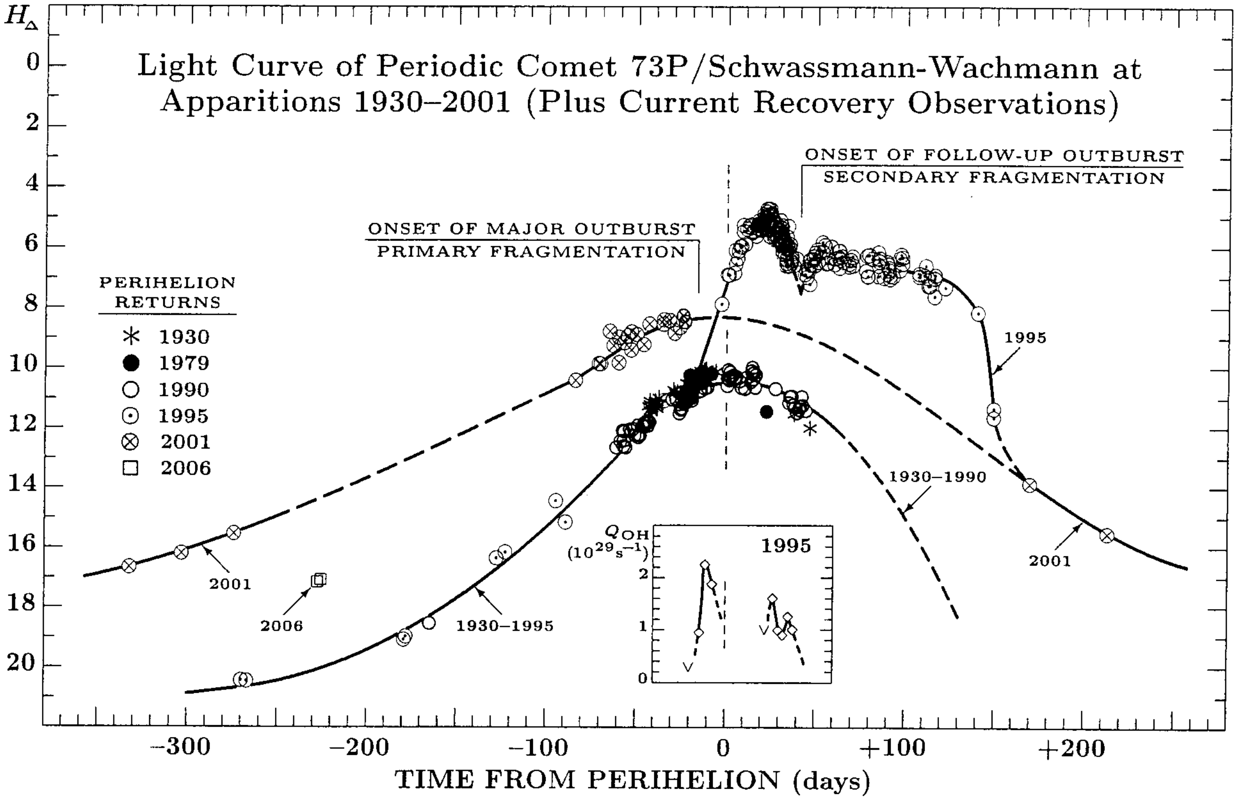

Figure 1. Visual light curve of comet 73P at the apparitions of 1930, 1979, 1990, 1995, and 2001. In 1930, the comet's perihelion distance was $q = 1.01$ AU; between 1979 and 2001, $q = 0.93$-0.94 AU. The onset times of the two outbursts in 1995 and their apparent coincidence with the times of primary- and secondary-nucleus fragmentation are marked. The 1995 perihelion occurred on September 22.9 TT. The inset depicts the parallel temporal variations in the hydroxyl production rate, measured by Crovisier et al. (1996).

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such light curves were spanned by additional data points provided by other observers. In this trial-and-error fashion, constant corrections were determined for the data sets that were reasonably uniform and the normalized magnitudes then converted to a common visual photometric system. The CCD magnitudes were corrected for a color index, when not measured in the visual system. The same observers with the same instruments were assigned the same corrections at all apparitions, which were thus dealt with independent of one another. The total number of data points employed in the light curve in Figure 1 amounts to 44 from 1930, 8 from 1979, 107 from 1990, 210 from 1995, and 26 from 2001.

The astonishing 1995 outburst began some 16 days before perihelion, on September 6-7. It was first detected with the Nançay Radio Telescope by Crovisier *et al.* (1996), whose results are shown in the inset of Figure 1. The comet's integrated signal in OH at 18 cm was below the detection limit in their run from September 1 to 5 (21 to 17 days before perihelion), but was clearly present in runs during September 8-10 (14 to 12 days before perihelion), 11-13, and 14-18. The peak OH production rate, apparently occurring on September 13 (Crovisier *et al.* 1995), was at least 10σ. The comet was next observed in the second half of October, when the signal was somewhat variable, corresponding on the average to a production rate of about half the peak September value.

Optically, the outburst was first detected on September 17-21 (Green 1996a), when the comet was at least 4 magnitudes more luminous than a month earlier; by October 9-10, the comet was brighter than apparent magnitude 6 (Green 1996b). As large amounts of dust ejecta continued to accumulate in the growing coma, the comet's brightness kept increasing for as long as 36 days, until October 12-13 or so. In an early phase of the outburst, the rate of brightening was approximately constant on the magnitude scale [and therefore exponential(!) on the brightness scale], amounting to ~ 0.2 mag/day, so that the comet was 1.2 times as bright at the end of the day as it had been at its beginning. The amplitude, measured as a difference between the normalized magnitudes at the onset and the peak, was fully 5 magnitudes. In mid-October the brightness leveled off and then started to fall, reaching apparent magnitude 8 in late October and early November, when a new upturn occurred about 41 days after perihelion. Calling it a follow-up outburst in Figure 1, I found that the rise time of this event was about two weeks and the amplitude some 1.4 magnitudes. As a result, the apparent visual magnitude was back to 7 in mid-November and the subsequent descent was very slow, at a rate of approximately 0.01 mag/day, a remarkably gradual development continuing for at least 10 weeks. An accelerated

TABLE 1
OUTBURSTS OF COMET 73P/SCHWASSMANN-WACHMANN IN 1995.

Outburst	Time t_0 of onset[a] 1995 (UT)	$t_0 - T$ (d)	Rise time (d)	Brightness amplitude (mag)	Normalized magnitude at onset	at peak
Major	Sept. 6.9	−16 ± 3	36 ± 4	5.0 ± 0.5	10.2 ± 0.4	5.2 ± 0.3
Follow-up	Nov. 2.9	+41 ± 1	14 ± 5	1.4 ± 0.3	7.5 ± 0.2	6.1 ± 0.2

[a] Date and time from perihelion passage T (minus sign = preperihelion, plus sign = postperihelion).

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drop in brightness did not commence until 18-19 weeks after perihelion and is rather poorly documented by very few observations. The parameters of the two outbursts are summarized in Table 1.

There are no total-magnitude data available from the end of February 1996 on, when the comet headed into a conjunction with the sun. C. Hergenrother's post-conjunction observations from 1996 September 20 and 21 (Green 1997) showed the comet at heliocentric distance $r = 3.6$ AU to be, on the average, only 0.3 magnitude brighter intrinsically at the same phase angle than Boehnhardt *et al.*'s (1999) pre-perihelion observations during 1994 December 27-30 at $r = 3.0$ AU, when compared using an inverse-square law. (Hergenrother's data points are way outside the margins of Figure 1, at a time of more than 360 days after perihelion.) It was therefore unclear, at that point, whether the comet was still in an excited state following the 1995 outbursts. This question was answered four years later when, some 300 days before perihelion, the object was detected independently by A. Nakamura, T. Oribe, and Hergenrother (Green 2000a) about 4 magnitudes brighter than during the previous return to the sun and brightening with decreasing heliocentric distance more rapidly than predicted by the inverse-square power law. Later, several weeks before perihelion, the normalized brightness was still about 2 magnitudes above the pre-outburst level of 1995. Unfortunately, during the exceedingly unfavorable 2001 apparition, the comet's brightness was estimated over a period of only 55 days before perihelion. Two additional observations made, respectively, by K. Kadota and by Nakamura (Green 2001) about 200 days after perihelion showed it to be intrinsically fainter than its interpolated brightness at the same heliocentric distance before perihelion, but still much brighter than on its approach to the sun in 1995. Hergenrother's 2005 recovery data (Green 2005), converted to visual magnitudes in Figure 1, suggest a further drop since the 2001 return to only a moderately elevated level relative to the pre-outburst light curve. A bare principal-nucleus fragment, presumably < 2 km in diameter (based on Boehnhardt *et al.*'s 1999 result that the parent nucleus was < 2.2 km across), should in late October 2005 be fainter than apparent magnitude 22.

4. Discovery and Evolution of Nucleus Multiplicity

The multiplicity of the comet's nucleus was first detected by Boehnhardt and Käufl (1995) at the European Southern Observatory's (ESO) La Silla station in Chile during their observing run of 1995 December 12-14. The observations were made simultaneously with the 3.5-meter New Technology Telescope in the optical wavelength range and with the 3.6-meter telescope in the thermal infrared. The three optically detected fragments were aligned in a nearly rectilinear chain about 4″ long and oriented approximately along the projected direction of the sun. Based on the notation used by Marsden (1996), the westernmost of the three condensations became known as *A*, the easternmost as *C*, and the middle, initially the faintest one, as *B*. For clarity, I use italics to refer to the fragment designations in published accounts to distinguish them from the designations based on the results of this work, for which I will employ roman letters.

Next, the ESO images of the comet taken up to two weeks before the discovery of the nucleus multiplicity, by K. Reinsch on November 28 and by J. Storm on December 2, were processed and closely inspected by Boehnhardt *et al.* (1996), and the elongated central condensation was resolved into two components. The second component in these images was attributed to fragment *B*, but it could have been *A* as well (Sekanina *et al.* 1996).

Subsequent observations clearly indicated that *C* was the main, most-massive fragment. From 1995 December 23 on, the multiple nucleus was noticed at several observatories worldwide. Besides the three major condensations, additional companions were reported, but none of these was detected by more than one group and they all have remained unconfirmed. J. V. Scotti measured a condensation, officially designated *D* (Marsden 1996), less than 2″ to the east-northeast of *C* on December 27. Three more condensations detected by others between 1995 December 12 and 1996 January 21 have not received formal designations.

Nuclei *A*, *B*, and *C* were seen until mid-February 1996, after which time the comet was too close to the sun for observation. After conjunction with the sun, the comet was picked up in the second half of August 1996, when only two condensations were detected. Tentative identifications indicated that — besides the main nucleus *C* — the only companion visible was *B*. Both were observed by various observers until nearly the end of 1996.

When the comet was recovered on its way to the next perihelion passage, in the second half of November 2000, three widely separated condensations were observed; besides *C*, one of the companions was tentatively identified with *B*, while

Table 2
Astrometric Data Subsets for Companion Fragments of Comet 73P/Schwassmann-Wachmann.

Data subset	Time span (UT)	Separation distances from nucleus C[a]	Number of collected data points	Fragment identity		Companion relative to nucleus C[a]
				published	this work	
I	1995 Nov. 28–1996 Feb. 16	1–9″	45	*B*	B	closer
II	1995 Dec. 12–1996 Feb. 19	3–22″	67	*A*	A	more distant
III	1996 Aug. 22–1996 Dec. 14	17–25″	14	*B*	E[b]	
IV	2000 Nov. 19–2000 Dec. 29	468–651″	23	*B*	F	closer
V	2000 Nov. 28–2000 Dec. 20	1409–1704″	34	*E*	E	more distant
VI	2001 Jun. 18–2001 Dec. 10	132–198″	15	*B*	F	

[a] C and *C* always referrring to the same fragment.
[b] Nucleus B nearly coinciding with E.

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the other, not fitting *A*, was officially designated *E*, as a new fragment (Green 2000b). After perihelion, which occurred near the end of January 2001, two condensations — *C* and what was considered a likely candidate for *B* (Boehnhardt *et al.* 2002) — were under observation at ESO during the second half of 2001 until December 10. No known additional images have been obtained since (again, as of mid-October 2005).

The astrometric positions of the companion fragments relative to the principal nucleus *C* (a separation distance and a position angle or offsets in right ascension and declination) that I collected for this investigation totaled 198 sets. Their summary is in Table 2: The entire data set is divided into six subsets by fragment and/or by major gaps in the temporal distribution of observations. The columns are self-explanatory, except for the difference between columns 5 and 6. Column 5, with the fragment identifiers in italics, refers to the published designations. Column 6 uses roman letters and lists the fragment identifiers resulting from this investigation. For three of the six subsets, the identifiers differ.

5. Fragmentation Sequence and Hierarchy

Most astrometric observations summarized in Table 2 were made with large telescopes, some even with the ESO's Very Large Telescope. Except in cases when a fragment was only poorly condensed (and therefore hard to measure), the collected positions should be quite accurate, mostly better than ± 1″. A dependable model is thus expected to fit the observations, spanning 6 years, to better than this limit and to leave no systematic trends in the residuals. In addition, the model is also expected to provide a useful ephemeris for 2006 — that is, at a time almost 5 years after the last observation of any of the companion fragments.

5.1. The Fragmentation Model. The only computer code for modeling a sequence and hierarchy of a split comet that was extensively tested on a large number of cases is the author's multiparameter fragmentation model (Sekanina 1978, 1982). By fitting the motion of a companion fragment relative to the principal (the most massive and persistent) nucleus, the model allows the user to determine, by an iterative, least-squares, differential-correction procedure, up to five parameters: the time of fragmentation (or separation); the companion's differential nongravitational deceleration (which, expressed in units of 10^{-5} of the sun's gravitational acceleration, is assumed to act continuously between the times of separation and observation and to vary as the inverse square of heliocentric distance); and three components of the companion's separation velocity, which point along the cardinal directions defined by the right-handed 'RTN' coordinate system of the heliocentric orbit of the parent comet: the radial axis (away from the sun), the transverse axis (in the orbital plane ahead of the comet), and the normal axis (to the orbital pole from which the comet is seen to orbit the sun counterclockwise). The mutual gravitational attraction of fragments was neglected.

When the identity of the primary fragment is not in doubt, such as in the case of comet 73P, meaningful solutions for companion fragments are expected to yield positive decelerations. Of considerable assistance is an option provided by the employed model to solve for any combination of fewer than the five parameters, so that a total of 31 different versions of the code are available. This option proves most beneficial in the early phases of the iterative process, before the solution settles around the optimum parametric values, or when the convergence is slow. The differential planetary perturbations and the relativistic effect acting on the fragments' motions are accounted for in the applied-code version, which was more recently developed in a joint effort by the author and P. W. Chodas and for the first time used in analysis of comet D/1993 F2 (Shoemaker-Levy), which split and later collided with Jupiter (Sekanina *et al.* 1998).

Since the fragmentation model provided an optimized fit to astrometric offsets of companion nuclei from the principal nucleus C = *C*, a set of orbital elements for this reference object was required as input. Although fragmentation solutions are generally not very sensitive to the orbit's accuracy, I carefully selected the set of elements for these model calculations.

TABLE 3

PREDICTED ORBITS FOR THE PRINCIPAL NUCLEUS OF COMET 73P/SCHWASSMANN-WACHMANN AT ITS 2006 RETURN TO THE SUN (OSCULATION EPOCH 2006 MAY 25.0 TT; EQUINOX J2000.0)

Orbital element	Orbit NEW	Orbit JPL	Orbit NAK	Orbit MUR
Perihelion time T (2006 TT)	June 6.9497	June 7.1718	June 7.3766	June 6.9225
Argument of perihelion ω	198°.8039	198°.8052	198°.8088	198°.8083
Longitude of ascending node Ω	69°.8955	69°.8958	69°.8941	69°.8959
Orbital inclination i	11°.3960	11°.3963	11°.3970	11°.3957
Perihelion distance q (AU)	0.939135	0.939141	0.939164	0.939121
Orbital eccentricity e	0.693192	0.693232	0.693257	0.693214
Orbital period P (yr)	5.36	5.36	5.36	5.36
Nongravitational parameters:				
A_1 (10^{-8} AU/day^2)	+1.33	+0.9848	+0.831	+0.65
A_2 (10^{-8} AU/day^2)	−0.0520	+0.0692	+0.1791	−0.0681
A_3 (10^{-8} AU/day^2)	......	−0.0721	−0.19	......
Closest approach to Earth:				
Predicted time (2006 TT)	May 12.4	May 12.8	May 13.2	May 12.4
Predicted distance (AU)	0.0787	0.0760	0.0735	0.0791
Predicted distance (mil. km)	11.8	11.4	11.0	11.8
Number of observations used	224	358	343	226
Observations linked	1995–2005	1994–2001	1989–2001	1994–2000
RMS residual	±0″.7	±0″.84	±0″.95	±0″.83
Orbital elements by	B. G. Marsden[a]	M. S. W. Keesey[b]	S. Nakano[c]	K. Muraoka[d]

[a] See Green (2005); only post-outburst 1995 observations of nucleus C included (Marsden 2005, personal communication).

[b] See http://ssd.jpl.nasa.gov/cgi-bin/da_shm?rec=900445; Keesey (2005, personal communication). Motion integrated from osculation epoch 2001 Nov. 27.

[c] See Marsden (2003a); Nakano and Green (2004).

[d] See http://www.aerth.net/comet/catalog/0073P/2001.html. Motion integrated from osculation epoch 2001 Jan. 11.

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Three of the options that I had are listed as orbits 'JPL', 'NAK', and 'MUR' in Table 3, in which the motion of the principal fragment was in each case integrated to a common near-perihelion osculation epoch in 2006 to allow comparison of the available orbital sets with Marsden's NEW set, which employs the 2006 recovery observations (Green 2005) but which did not exist at the time of my model calculations. Two of the three available sets are based on astrometric observations from the apparitions 1995 and 2001 (orbits denoted 'JPL' and 'MUR'), while the third was obtained by linking the data from the apparitions 1990, 1995, and 2001 (orbit denoted 'NAK'). Under ordinary circumstances, the three-apparition solution would clearly be preferable, but the point of much concern with this run was the linkage of the motion of the parent nucleus between 1989 and September 1995 with the motion of the principal fragment during the subsequent revolution about the sun. Momentum changes that this fragment was likely to experience during the 1995 fragmentation events could significantly affect any hybrid solution based on approximately equal contributions from the parent nucleus and the fragment. These concerns reached alarming proportions when I learnt of M. S. W. Keesey's (2005, personal communication) experience with a similar solution, which included the observations from 2000, but not 2001. Keesey says that this solution left systematic residuals of up to 2″.5 in 1989 and smaller ones in 1994-1995. However, when he added the 2001 data, no observations between February 1996 and April 2000 could be fitted, leaving residuals of up to 11″. Since this 1989-2001 orbit included the third nongravitational parameter (Keesey 2005, personal communication), the NAK solution (Nakano and Marsden 2003a, Nakano and Green 2004) must be subjected to the same, if not greater, difficulties. Comparison with the NEW set of elements shows that the 1994-2000/2001 solutions are indeed superior to the 1989-2001 solution.

As for the two-apparition runs (1994-2001), Keesey says that he began with a gravitational solution, which turned out to be utterly unacceptable, leaving systematic residuals of up to 16″ in 1994-1995 and up to 30″ in 2001. These findings justified his introduction of nongravitational parameters into the equations of motion. Keesey indeed found that the resulting solution (JPL) was then entirely satisfactory.

The two-apparition solutions in Table 3 appeared to represent preferable orbital determinations for the principal fragment because of their relatively minor contamination by observations of the parent nucleus. The orbital arc covered by the parent data was only 8.5 months, compared to nearly 6 years for the three-apparition runs. Since Muraoka's solution does not include the 2001 observations and is based on a substantially smaller data set, I decided to use the JPL solution in modeling the fragmentation process of 73P.

5.2. Fragment B. To start with, I assumed that B = *B*, implying that subsets I, III, IV, and VI in Table 2 all referred to the same companion fragment. Iteration of all five parameters failed to yield a converging solution. I then forced the time of fragmentation and solved for the remaining four parameters. When the breakup event was assumed to have coincided with the onset of the major outburst, the solution converged, but the results were unsatisfactory. The deceleration came out to be negative (−1.87 ± 0.46 units of 10^{-5} the sun's gravitational acceleration), the root-mean-square (RMS) residual was unacceptably large (± 1″.66), and the residuals displayed strong systematic trends of up to 1″ in 1995, up to 3″ in 1996 and 2000, and in excess of 3″ in 2001. When the fragmentation time was forced to coincide with the onset time of the follow-up outburst, the results were clearly worse, with an RMS residual of ± 2″.05 and systematic residuals in excess of 5″ in 2001.

Linking only subsets I, IV, and VI likewise failed to lead to an acceptable solution, with the deceleration again negative, the RMS residual ± 1″.07, and the systematic residuals now exceeding 1″ in 1995 and early 1996 and up to 4″ in 2001. Forcing the time of fragmentation did not improve the situation.

More experimentation with three subsets led to further disappointing solutions and to convergence problems. For example, linking only sets I, III, and VI and forcing the fragmentation time to coincide with the major outburst's onset time yielded an RMS residual ± 0″.84 and systematic residuals of up to 6″. Particularly disturbing was the inconsistency between the July and December 2001 positions, common to all described runs.

Linking only two subsets, I first chose I and VI. The best, although still rather unsatisfactory, solution was obtained by forcing the fragmentation time to coincide with the onset time of the follow-up outburst. The RMS residual was then ± 0″.74, the deceleration 3.45 ± 0.60 units, and the systematic residuals up to 2″. The five-parameter solution did not converge, and other solutions were less satisfactory than the described one.

Still-better solutions resulted from a linkage of subsets I and III. Even though the five-parameter version did not converge, it indicated an RMS residual near ± 0″.33 and very slight systematic residuals of < 1″ primarily in August-December 1996. When the fragmentation time was approximated by the onset time of the major outburst, the solution was better (though not perfect) than when the follow-up outburst was used instead.

Subset I alone left a very satisfactory RMS residual of ± 0″.20 with no systematic trends but a poorly defined deceleration of only 0.7 ± 1.3 units. The fragmentation time was found to be 1995 September 14 ± 16, deviating by only $\sim 0.5\sigma$ from the time of the major outburst. An assumption of no deceleration led to an equally good solution.

It appears rather unlikely that fragment B was detected after 1996. It unquestionably was observed as subset I and it may have contaminated the positions in subset III, although a preferred scenario is that this latter subset refers to another fragment.

5.3. Fragment E. Surprisingly, subset III could easily be linked with subset V, indicating that they both referred to fragment E. The five-parameter solution yielded 1995 September 11.0 ± 5.4 for the fragmentation time, deviating only 0.8σ from the onset time of the major outburst and suggesting that this fragment, too, was closely related to that event. Forcing the fragmentation time to coincide with the time of this outburst, I obtained an equally satisfactory solution, with an acceptable RMS residual of ± 0″.60 and no systematic trends. Interestingly, an ephemeris run back to 1995 indicated that, from its birth until the end of February 1996 (thus including the entire period of subsets I and II), fragment E was always less than 2″.2 from fragment C.

5.4. Fragment F. With the observations in subsets IV and VI as yet unaccounted for, I tried to link these two. This effort was most successful, yielding a solution with no systematic trends and with the July, September, and December 2001 positions mutually consistent. The resulting fragmentation time, 1995 October 28.3 ± 2.5, differed by 2.2σ from the onset time of the follow-up outburst. Forcing the fragmentation time to coincide with this outburst's time offered a solution that was about equally satisfactory. On the other hand, the assumption of coincidence with the major outburst led to an inferior solution with strong systematic residuals and a negative deceleration.

In the following, this fragment is called F. In August-December 1996, it should have been about 10″ farther from C than E, and in late 1995 and early 1996 its predicted location was between B and A (in early December 1995, very close to A). Its apparent absence implies that it took a few years before this fragment became active.

An alternative scenario, with fragment F sharing its direct parent with fragment B, was not contemplated because of uncertainties in the motion of B in 2000-2001. An unlikely common origin of F and B is suggested by their diverse birth-date preferences, the major outburst being favored by B, whereas the follow-up outburst is favored by F.

5.5. Fragment A. There appears to be no indication that observations other than subset II refer to this condensation. Its deceleration relative to the other fragments was fairly high, much more than 10 units. I investigated three possible birth scenarios based on direct parents common with C, B, or E, using offsets of the observed astrometric positions of A from predicted positions of the presumed parent successively approximated by each of the three fragments. The quality of fit was always very good and nearly the same in all three scenarios. However, the fragmentation time was poorly determined and therefore nondiscriminatory. I eventually solved the problem by requiring that the separation velocity be as low as possible. This condition led to B as the most likely fragment to share a common parent with A, implying a velocity of about 1.2 m/s when its breakup just preceded the follow-up outburst. A common parent with C would have implied ~ 2 m/s and a breakup at about the same time, while a common parent with E would have needed >3 m/s and a breakup soon after the separation of E from C.

TABLE 4
FRAGMENTATION MODEL SOLUTIONS FOR COMPANION NUCLEI OF COMET 73P/SCHWASSMANN-WACHMANN.

Fragmentation event's description parameter	Birth scenario for companion fragment			
	B	E	F	A
Direct parent shared with	C	C	C	B
Time of separation				
days from perihelion[a]	−16[b]	−16[b]	+41[c]	+33 ± 8[d]
date (1995 UT)	Sept. 6.9	Sept. 6.9	Nov. 2.9	Oct. 25.9
Separation velocity (m/s)				
Total	0.69 ± 0.01	1.07 ± 0.10	2.55 ± 0.08	1.19 ± 0.12
Radial	+0.52 ± 0.01	−0.91 ± 0.03	+1.74 ± 0.07	−0.27 ± 0.30
Transverse	+0.44 ± 0.01	−0.38 ± 0.25	−1.83 ± 0.09	−1.16 ± 0.10
Normal	+0.10 ± 0.01	+0.42 ± 0.06	+0.36 ± 0.03	+0.08 ± 0.01
Deceleration γ (units of 10^{-5} solar attraction)	0[e]	5.5 ± 1.2	7.48 ± 0.54	37.0 ± 2.9
Number of offset pairs used in the solution	30	27	30	42
Mean residual	±0″.20	±0″.60	±0″.71	±0″.30

[a] Minus sign = preperihelion, plus sign = postperihelion.
[b] Separation time assumed to coincide with the onset time of the major outburst.
[c] Separation time assumed to coincide with the onset time of the follow-up outburst.
[d] Determined by requiring a minimum separation velocity; error is 1σ.
[e] Deceleration assumed to be zero; when solved for, it came out to be +0.7 ± 1.3 units.

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5.6. The Proposed Fragmentation Scenario. Based on the performed calculations, I propose fragmentation solutions involving nuclei C, B, E, F, and A that are described by the optimized parameters presented in Table 4. The corresponding model for the fragmentation sequence and hierarchy of comet 73P is shown in Figure 2, which indicates that the products of the 1995 events represent two generations of fragments of the original parent nucleus, which itself was found by Boehnhardt et al. (1999) to have been less than 2.2 km across.

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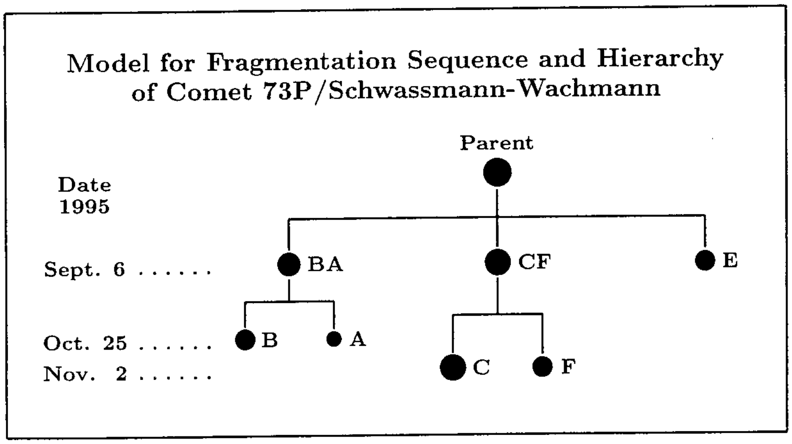

Figure 2. Proposed model for the 1995 fragmentation sequence and hierarchy of comet 73P, based on the analysis of motions of companion nuclei B, E, F, *and* A *relative to* C. *In this scheme,* BA, CF, *and* E *are the first-generation fragments of the parent nucleus, while* B, F, *and* A, *together with* C, *are the second-generation fragments. The dates of fragmentation are shown on the left.*

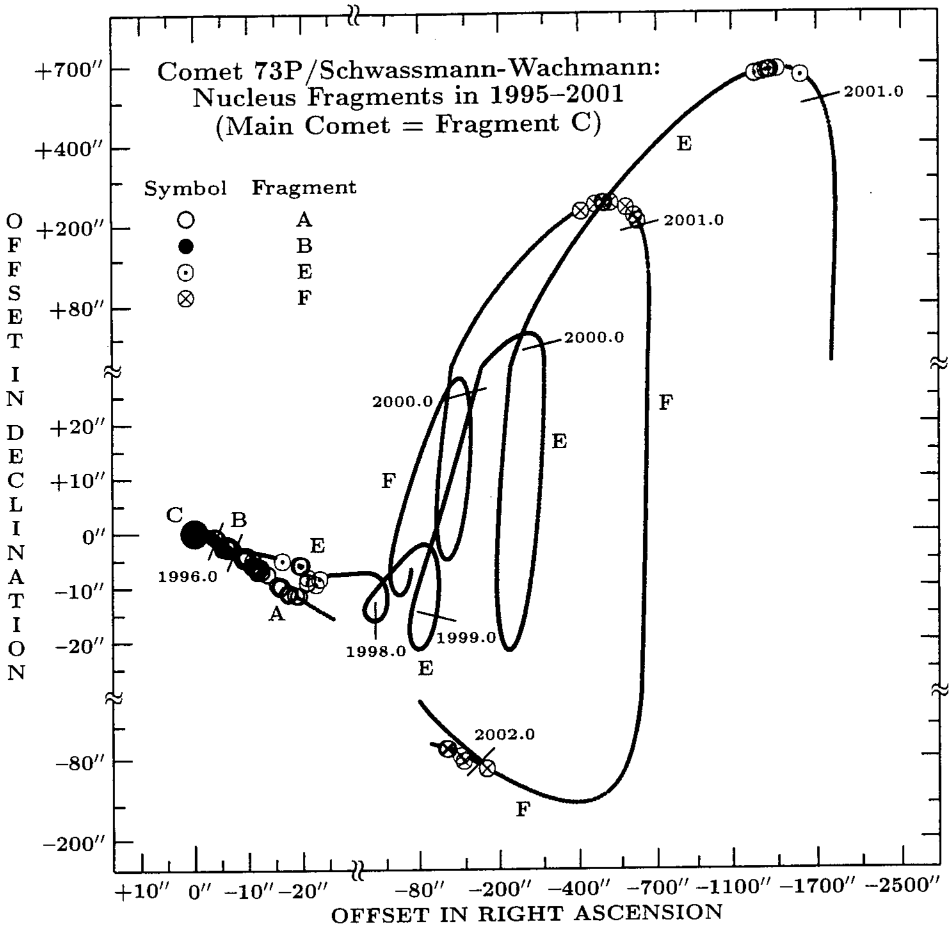

Figure 3. A fit to the observed motions of companion nuclei A, B, E, *and* F *relative to nucleus* C *between 1995 and the end of 2001. To increase clarity of the plot, the fit for fragment* F *is shown only from the beginning of 1999 on. The scale is linear within 30″ of* C, *but is proportional to an offset's cube root at larger distances.*

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Because of the strong preference of the separation times for fragments B, E, and F to coincide with the onset time of one of the two outbursts, the solutions shown in Table 4 were derived from four-parameter runs with the separation time forced accordingly. No significant error is thereby introduced, while the parameters are more robust. The results for nucleus A come also from a four-parameter run, with the fragmentation time determined by minimizing the separation velocity, which is presumably of rotational nature. Table 4 shows that this velocity is generally close to 1 m/s, with the exception of fragment F. It is possible that the first-generation fragment CF (Figure 2) was spun up during the fragmentation event of September 6-7 and that the second-generation fragment C was spun down during the event of November 2, thus regaining some inertial stability again. Calculations show that significant changes in the angular momentum of a splitting cometary nucleus can be expected (Sec. 6). Even with separation velocities as low as 1 m/s, the spin period of a nucleus 2 km in diameter comes out to be extremely short, less than 2 hours.

The observed and fitted motions of the four companion nuclei relative to nucleus C between 1995 and the end of 2001 are plotted in Figure 3 in projection onto the plane of the sky. The complex loops of the trajectories are effects of the earth's orbit about the sun. The plot shows an approximate alignment of the companions at any given time in a direction that, with time, approaches ever closer to the projected direction of the comet's orbit, a typical configuration conforming to the orbital angular-momentum law.

6. Effects of Nucleus Fragmentation on the Spin Rate

Because there still is no consensus about the mechanism that makes cometary nuclei split far from the sun and the planets, quantitative investigations of the role of rotation (one of the candidate causes) are of much interest. Here I discuss a highly idealized case of a spherical parent nucleus of diameter D_0, uniform density ρ_0, spin vector $\vec{\omega}_0$ passing through the center of mass, and moment of inertia I_0. This object is assumed to split in a mode that can be approximated

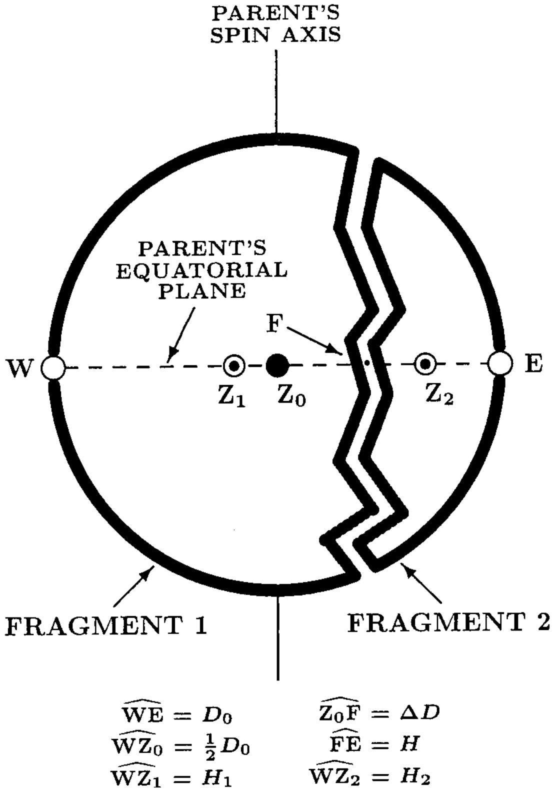

Figure 4. Splitting of a (parent) cometary nucleus (of spherical shape and uniform density and rotating about an inertially fixed spin axis) into two pieces, with a more-massive fragment 1 and a less-massive fragment 2. The mode of splitting can be approximated by slicing the nucleus along a plane parallel to its spin axis and passing through point F. *Z*$_0$, *Z*$_1$, *and Z*$_2$ *are, respectively, the centers of mass of the parent nucleus and the two fragments.* W *and* E *are points in the equatorial plane projected west and east.*

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by a slice along a plane parallel to the inertially fixed spin axis at a distance ΔD (Figure 4). The splitting generates two fragments shaped like spherical segments, whose dimensions along the line perpendicular to the slice in the parent's equatorial plane are $D_1 = \frac{1}{2}D_0 + \Delta D = D_0 - H$ (primary, more-massive fragment 1) and $D_2 = \frac{1}{2}D_0 - \Delta D = H$ (secondary, less massive fragment 2). Let their (unknown) initial spin vectors (both assumed to be parallel to $\vec{\omega}_0$) be $\vec{\omega}_1$ and $\vec{\omega}_2$ and their moments of inertia relative to the parent's center of mass I_1 and I_2, respectively. The conservation of momentum law and the conservation of energy law require that

$$I_0|\vec{\omega}_0| \;=\; I_1|\vec{\omega_1}| \;+\; I_2|\vec{\omega}_2|, \tag{1}$$

$$\frac{1}{2}I_0|\vec{\omega}_0|^2(1-\lambda) = \frac{1}{2}I_1|\vec{\omega}_1|^2 + \frac{1}{2}I_2|\vec{\omega}_2|^2, \tag{2}$$

where λ is the fraction of the rotational energy of the parent nucleus that is lost (heating, mechanical friction, etc.). Because of the symmetry, I will consider the primary fragment always on the left in Figure 4 and the secondary, whose height $H \leq D_0/2$, on the right.

The task is to find solutions for the spin rates of the fragments, $\omega_k = |\vec{\omega_k}|$ (k = 1,2), that satisfy the two equations for given values of the fragmentation parameter H, the parent's spin rate, $\omega_0 = |\vec{\omega_0}|$, and the lost energy, $0.5\lambda\, I_0\, \omega^2$. The moments of inertia are known, since for the parent nucleus $I_0 = \frac{1}{60}\pi\, \rho_0\, D_0^5$, while for the fragments they are calculated from the basic equation,

$$I_k = \int_{(V_k)} r_\perp^2 \; dm \qquad (k = 1,2), \tag{3}$$

where, for either fragment, $r_\perp$ is the distance of each mass element dm of the body from its new spin axis that passes through its center of mass, while (V_k) means that the expression is integrated over the fragment's whole volume. In

Figure 4 the center of mass is shown for both the parent (Z_0) and the fragments (Z_1 and Z_2). Referring each fragment's moment of inertia to the parent's center of mass, an additional term is to be added to (3), equal to a product of the fragment's mass and the square of the distance between its center of mass and that of the parent.

The problem can readily be solved in terms of dimensionless parameters $\Omega_k = \omega_k/\omega_0$ and $J_k = I_k/I_0$ ($k = 1, 2$), as is apparent from Eqs. (1) and (2). Instead of I_0, however, I express the dimensionless moments of inertia $\Im_k$ ($k = 1, 2$) in units of $\frac{\pi}{32}\,\rho_0\,D_0^5$, in which case

$$\Im_1 = \Im + \Im_1', \tag{4}$$

$$\Im_2 = \frac{8}{15} - \Im + \Im_2', \tag{5}$$

where

$$\Im = \frac{4}{15} - \frac{1}{2}(1-\Theta)\left[1 - \frac{2}{3}(1-\Theta)^2 + \frac{1}{5}(1-\Theta)^4\right], \tag{6}$$

$$\Im_1' = \Theta^2\left(1 - \frac{1}{3}\Theta\right)(1-\Theta_1)^2, \tag{7}$$

$$\Im_2' = \left[\frac{4}{3} - \Theta^2\left(1 - \frac{1}{3}\Theta\right)\right](1-\Theta_2)^2. \tag{8}$$

Here $\Theta = 2(1 - H/D_0)$ is a fragmentation parameter (Figure 4), whose range is $1 \leq \Theta < 2$; it determines the mass ratio (which is physically more important) of the fragments, $M_1/M_2 \geq 1$:

$$\frac{M_1}{M_2} = \frac{\Theta^2\,(1-\frac{1}{3}\Theta)}{\frac{4}{3} - \Theta^2\,(1-\frac{1}{3}\Theta)}. \tag{9}$$

The locations Z_1 and Z_2 of the centers of mass of the fragments are in Figure 4 described by H_1 and H_2, respectively. Defining $\Theta_k = 2H_k/D_0$ ($k = 1, 2$), one finds Θ_k from the following equations, best solved by rapidly converging iterations:

$$\Theta_1 = \Theta\sqrt{\frac{1-\frac{1}{3}\Theta}{2\,(1-\frac{1}{3}\Theta_1)}}, \tag{10}$$

$$\Theta_2 = \sqrt{\frac{\frac{4}{3} + \Theta^2\,(1-\frac{1}{3}\Theta)}{2\,(1-\frac{1}{3}\Theta_2)}}, \tag{11}$$

where $0.65 < \Theta_1 < 1$ and $1.35 < \Theta_2 < 2$. The dimensionless spin rates $\Omega_k = \omega_k/\omega_0$ are then

$$\Omega_k = \frac{8}{15}\left\{\frac{1}{\Im_1+\Im_2} \pm (-1)^{k+1}\sqrt{\left(\frac{1}{\Im_k} - \frac{1}{\Im_1+\Im_2}\right)\left[\frac{15}{8}(1-\lambda) - \frac{1}{\Im_1+\Im_2}\right]}\right\} \quad (k = 1, 2). \tag{12}$$

Equation (12) shows that there is a constraint on the lost energy. The first term of the square-root expression is always positive as $\Im_1$ and $\Im_2$ are positive. For the second term to be positive, λ must satisfy a condition

$$\lambda < 1 - \frac{8}{15}(\Im_1+\Im_2)^{-1}. \tag{13}$$

As the mass ratio M_1/M_2 increases, the losses measured by the total rotational energy decrease, as one expects. When the energy-loss factor λ reaches its maximum value, both fragments have the same spin rate equal to $(1-\lambda)\omega_0$.

The results of this model's application are listed in Table 5 as rotation periods of the fragments, $P_k = 2\,\pi/\omega_k$ ($k = 1, 2$), for a parent's rotation period $P_0 = 6$ hours. The table shows that, as suggested in Sec. 5.6, fragments can indeed be either spun up or spun down and that especially a secondary fragment much less massive than the primary can acquire a spin rate almost twice the parent's rate. When energy losses are trivial, one fragment is spun up, the other spun down. Of course, changes in the spin rate are much greater for the smaller of the two fragments.

While these results are encouraging, one should be aware of at least two problems. One is the various assumptions on which the model rests and which imply that the results should be taken with caution. The other is the fact that, as postulated, the rotation of the fragments is extremely unstable, because the axis differs dramatically from the axis of maximum moment of inertia. One can expect that especially smaller fragments should experience violent tumbling, also aggravated — as it appears to be the case — by torques due to activity of their own.

TABLE 5

EFFECT OF NUCLEUS FRAGMENTATION ON ROTATION PERIODS OF FRAGMENTS FOR ASSUMED PARENT ROTATION PERIOD P_0 = 6 HOURS.

Fragmentation parameter Θ	Fragments' mass ratio, M_1/M_2	Energy losses (percent) maximum, $\lambda_{\max}$	Energy losses (percent) adopted, λ	Rotation periods of fragments (hr) case $P_1 < P_2$ P_1	case $P_1 < P_2$ P_2	case $P_1 > P_2$ P_1	case $P_1 > P_2$ P_2
1.00	1.00	23	20	6.5	9.8	9.8	6.5
			10	5.5	13.3	13.3	5.5
			0	5.0	17.3	17.3	5.0
1.20	1.84	22	20	6.8	9.7	8.9	6.4
			10	5.8	15.0	11.3	5.2
			0	5.4	22.5	13.5	4.6
1.40	3.63	19	10	6.1	15.3	9.3	4.9
			0	5.7	30.5	10.6	4.2
1.60	9.03	13	10	6.4	11.4	7.5	4.9
			0	5.9	45.7	8.3	3.7
1.80	34.7	5	0	6.0	90.6	6.7	3.3
1.90	137	2	0	6.0	180	6.2	3.1
1.95	541	0.4	0	6.0	360	6.1	3.0

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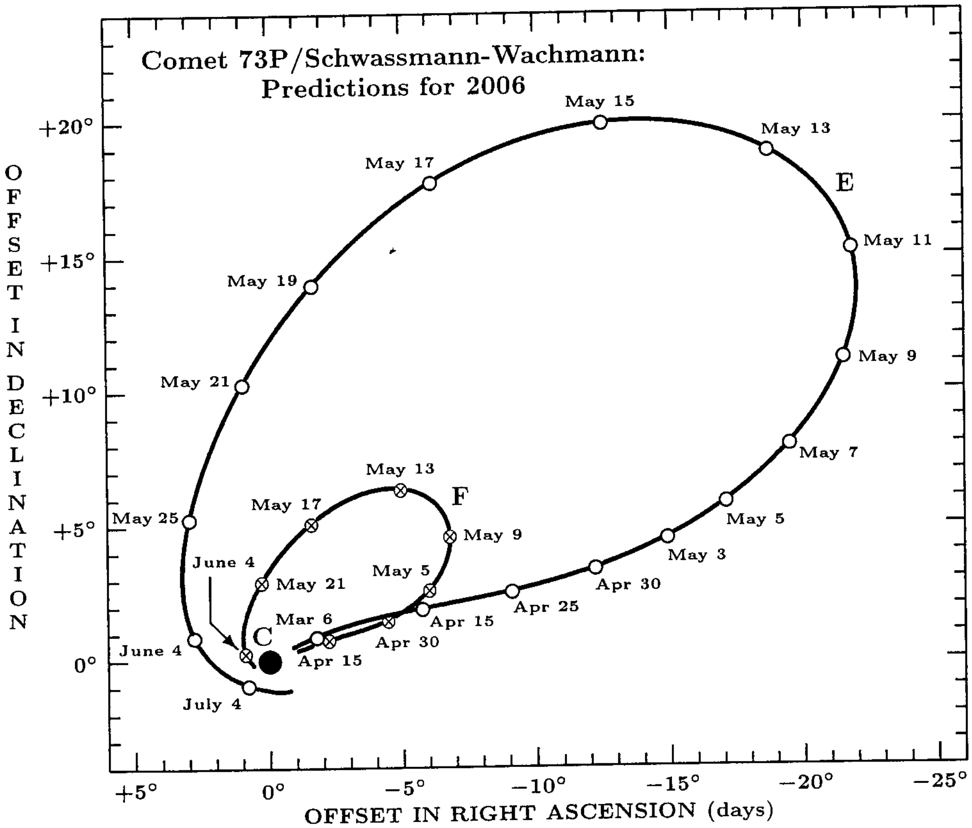

Figure 5. Predicted motion of fragments E *and* F *relative to the principal fragment* C *in projection onto the plane of the sky around the time of the close encounter of comet 73P with Earth. Unlike in Table 6, the JPL set of elements for* C *was used. The predicted distances for fragment* B *(not shown) are about 5-8 percent greater than those of* F*. The times are for* 0^h *TT.*

TABLE 6

EPHEMERIDES FOR FRAGMENTS E, F, AND B RELATIVE TO FRAGMENT C OF COMET 73P/SCHWASSMANN-WACHMANN IN 2005/2006[a].

		Astrometric offset from fragment C for					
		fragment E		fragment F		fragment B	
Date (0 TT)	Distance from Earth (AU)	separation distance	position angle	separation distance	position angle	separation distance	position angle
2005							
Oct. 27	3.153	15′.8	293°.8	6′.1	293°.8	6′.2	293°.8
Nov. 6	2.960	17.6	294.7	6.7	294.6	6.9	294.7
16	2.761	19.7	295.4	7.5	295.4	7.7	295.4
26	2.557	22.2	296.0	8.4	296.1	8.7	296.1
Dec. 6	2.351	25.1	296.6	9.5	296.7	9.8	296.6
16	2.144	28.6	297.0	10.7	297.1	11.1	297.1
26	1.938	32.9	297.4	12.3	297.5	12.8	297.5
2006							
Jan. 5	1.736	0°.64	297°.7	0°.24	297°.8	0°.25	297°.7
15	1.539	0.74	297.8	0.28	298.0	0.29	297.9
25	1.350	0.88	297.9	0.33	298.1	0.34	297.9
Feb. 4	1.170	1.05	297.8	0.39	298.0	0.41	297.8
14	1.002	1.28	297.6	0.47	297.8	0.49	297.6
24	0.845	1.57	297.1	0.58	297.4	0.61	297.2
Mar. 6	0.701	1.95	296.4	0.72	296.7	0.76	296.5
16	0.571	2.48	295.4	0.91	295.7	0.96	295.5
26	0.454	3.22	294.0	1.18	294.3	1.25	294.0
Apr. 5	0.349	4.30	292.1	1.59	292.3	1.68	292.1
15	0.255	6.06	289.9	2.25	290.2	2.38	289.9
25	0.170	9.48	288.9	3.55	289.1	3.76	288.8
30	0.133	12.63	290.7	4.73	290.8	5.02	290.5
May 3	0.113	15.37	293.7	5.72	293.8	6.08	293.5
5	0.101	17.63	297.1	6.49	297.3	6.90	296.9
7	0.0915	20.13	302.1	7.26	302.2	7.73	301.8
9	0.0840	22.53	308.6	7.85	308.7	8.38	308.3
11	0.0796	24.07	316.3	7.98	316.4	8.54	316.0
12	0.0788	24.20	320.5	7.78	320.6	8.34	320.2
13	0.0789	23.76	324.8	7.41	324.9	7.94	324.4
14	0.0799	22.70	329.2	6.87	329.3	7.37	328.8
15	0.0818	21.09	333.6	6.22	333.7	6.68	333.2
16	0.0845	19.09	338.3	5.52	338.3	5.93	337.7
17	0.0880	16.88	343.1	4.82	343.1	5.18	342.5
18	0.0921	14.68	348.1	4.17	348.2	4.48	347.5
19	0.0969	12.63	353.4	3.59	353.5	3.85	352.7
21	0.108	9.30	4.8	2.67	4.9	2.85	4.1
23	0.120	7.01	17.0	2.05	17.0	2.18	16.2
25	0.134	5.52	29.0	1.64	29.1	1.74	28.3
27	0.148	4.55	40.3	1.38	40.4	1.45	39.8
30	0.170	3.63	54.9	1.12	55.0	1.18	54.7
June 4	0.209	2.76	73.5	0.88	73.8	0.92	73.9

[a] Using Marsden's NEW set of orbital elements from Table 3.

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7. Predictions for the 2006 Return to Perihelion

The fragmentation sequence and hierarchy, determined by the results in Table 4 and Figure 2, allow one to provide ephemerides for the potentially surviving companion nuclei during the 2006 return of comet 73P to perihelion. Projected onto the plane of the sky, the motions of fragments E and F relative to nucleus C are plotted in Figure 5, based on the JPL orbital set from Table 3.

To avoid overcrowding of Figure 5, the ephemeris for fragment B (whose separation distances are more uncertain,

TABLE 7

EFFECT OF ORBITAL SET CHOICE ON EPHEMERIS OF FRAGMENT E RELATIVE TO FRAGMENT C NEAR CLOSEST APPROACH TO EARTH.

			Difference in astrometric offset of fragment E from fragment C					
	Orbit JPL		NEW minus JPL		NAK minus JPL		MUR minus JPL	
Date 2006 (0 TT)	separation distance	position angle	separation distance	position angle	separation distance	position angle	separation distance	position angle
Mar. 26	3°.20	293°.9	+0°.02	+0°.1	−0°.02	−0°.1	+0°.02	+0°.1
Apr. 5	4.27	291.9	+0.03	+0.2	−0.04	−0.2	+0.04	+0.2
15	5.99	289.6	+0.07	+0.3	−0.07	−0.3	+0.08	+0.4
25	9.31	288.2	+0.17	+0.7	−0.16	−0.6	+0.19	+0.8
30	12.39	289.6	+0.24	+1.1	−0.23	−1.0	+0.27	+1.2
May 3	15.11	292.4	+0.26	+1.3	−0.28	−1.3	+0.30	+1.5
5	17.39	295.6	+0.24	+1.5	−0.27	−1.5	+0.26	+1.7
7	20.02	300.4	+0.11	+1.7	−0.20	−1.6	+0.11	+1.9
9	22.71	307.0	−0.18	+1.6	+0.04	−1.6	−0.22	+1.7
11	24.78	315.1	−0.71	+1.2	+0.51	−1.3	−0.82	+1.4
12	25.24	319.6	−1.04	+0.9	+0.82	−1.1	−1.19	+1.0
13	25.12	324.1	−1.36	+0.7	+1.16	−0.8	−1.55	+0.8
14	24.33	328.7	−1.63	+0.5	+1.47	−0.5	−1.85	+0.5
15	22.90	333.4	−1.81	+0.2	+1.69	−0.3	−2.05	+0.3
16	20.93	338.2	−1.84	+0.1	+1.80	−0.2	−2.09	+0.1
17	18.65	343.1	−1.77	0.0	+1.78	0.0	−2.00	−0.1
18	16.29	348.3	−1.61	−0.2	+1.64	+0.1	−1.81	−0.2
19	14.04	353.7	−1.41	−0.3	+1.45	+0.1	−1.58	−0.3
21	10.31	5.2	−1.01	−0.4	+1.05	+0.2	−1.14	−0.4
23	7.72	17.3	−0.71	−0.3	+0.74	+0.3	−0.80	−0.4
25	6.04	29.3	−0.52	−0.3	+0.53	+0.3	−0.59	−0.3
27	4.94	40.5	−0.39	−0.2	+0.40	+0.2	−0.44	−0.2
30	3.91	54.9	−0.28	0.0	+0.29	+0.1	−0.31	0.0
June 4	2.94	73.4	−0.18	+0.1	+0.18	−0.2	−0.20	+0.2

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but expected to be 5-8 percent greater than those of fragment F) is not plotted. In tabular form, the ephemerides for all three fragments, calculated with the 1995-2005 set of elements for nucleus C (orbit 'NEW' in Table 3), are presented in Table 6. Ephemerides for fragments E and F starting in late September 2005 were published electronically (Sekanina 2005). No ephemeris is provided for fragment A, whose high deceleration strongly suggests that it has not survived.

An important issue is that of the accuracy of the ephemerides. Their intrinsic accuracy is determined by the fragmentation sequence and hierarchy of the comet. If the companion fragments were correctly identified, the uncertainties in their 2006 positions should be fairly small, perhaps on the order of 10′ or so. The uncertainty in the ephemeris for B is greater than this, because this fragment has not apparently been observed since 1996. The available 2.5-month arc is therefore extrapolated over a period of time about 50 times as long. In addition, its survival is statistically less likely than that of fragments E and F.

Apart from its intrinsic accuracy, an ephemeris for any companion fragment depends critically on the set of orbital elements used, as both the apparent separation distance and the position angle of companion fragments are very sensitive to the perihelion time. With the comet recovered, this has become only a minor issue. To illustrate it, I show in Table 7 the scatter among the four orbits from Table 3 in the separation distance and position angle of fragment E relative to the principal fragment in the period of time from 2006 March 26 to June 4. As the perihelion time will further be refined, one can conclude from Table 7 that the maximum effect on the separation distance of fragment E on May 16 is about 0′.5 for a change of 0.001 day in the perihelion time.

8. Conclusions

The collected astrometric data for comet 73P were found to refer mostly to primary fragment C or one of four companion fragments: subset I referring to component B, subset II to A, subsets III and V to E, and subsets IV and VI to F. Only a few data points referred to the other, fleeting fragments, which are ignored in this paper.

There is a strong correlation between the comet's fragmentation sequence and hierarchy, on the one hand, and its two outbursts in 1995 on the other hand. The first, major outburst, beginning around September 6 (more than two weeks before perihelion) had an amplitude of 5 magnitudes and a rise time of 5 weeks, and it accompanied the breakup of the parent nucleus into fragment E and two precursors to fragments A + B and C + F. The follow-up outburst, on November 2 or so (some 6 weeks after perihelion), had an amplitude of nearly 1.5 magnitudes and a rise time of two weeks; it accompanied the splitting of one of the precursors into fragments C and F. This evidence strongly supports a hypothesis proposed for the split comet C/2001 A2 (Sekanina *et al.* 2002), which says that the presence or absence of an outburst related to a fragmentation event depends on the steepness of the size distribution of the accompanying cloud of particulate debris.

The decelerations of fragments B, E, and F suggest that these are sizable bodies like fragments of other comets known to have survived for one or more revolutions about the sun (*e.g.*, Sekanina 1999). On the other hand, fragment A was much smaller and is not expected to have survived. Given the dimensions of the parent nucleus (Boehnhardt *et al.* 1999), and assuming rotational nature of the separation velocities, their derived range (mostly near 1 m/s, but 2.5 m/s for fragment F) suggest a rapid rotation with major fragmentation-driven spin-up and/or spin-down effects.

The problem of identifying the companion fragments can never be dismissed as one of no concern. Especially the similarities between nuclei B and F are most intriguing. The strongest argument against the identity of (or a very close relationship between) the two is based on fitting the astrometric observations in the second half of 2001. All investigated scenarios pointed to major discrepancies when these data were assumed to refer to fragment B, its birth coinciding essentially with the onset of the major outburst of 1995. The difficulties disappeared instantly, once the 2000 and 2001 observations were assigned to another fragment, F, with its origin linked to the follow-up outburst. Although one can argue that the inverse-square power law adopted for the variations in the nongravitational deceleration may not always approximate the observed motions of comet fragments satisfactorily enough, it is easy to counter by pointing out that the fitting obstacles involving observations at large heliocentric distances and spanning a period as short as a few months cannot be of this origin because any minor acceleration effect (such as these forces appear to be) is much too gentle to make so much difference so suddenly.

A prediction of the motions of companion fragments during the comet's close approach to Earth in mid-May 2006 shows that the separation distance from C should peak at more than 24° for fragment E, but near 8° to 8°.5 for B and F. The uncertainties of the prediction are difficult to estimate, confined perhaps to 10′ along the orbit, but they are negligibly small across the orbit. Since the rate of fragment disintegration is unknown, one of three possible recovery states can be expected at each predicted location: (i) no apparent decay since the previous observations, in which case the result should be a relatively easy detection of the fragment; (ii) some moderate crumbling, in which case there should be a number of fainter fragments distributed along the orbit at distances from C about equal to or somewhat greater than the predicted location; or (iii) advanced or complete disintegration, in which case there is a little or no chance of detecting any fragments at the location.

As a final remark, one should not ignore the remote possibility of unknown fragments released at any time after the 1995 perihelion (including far from the sun). For example, a fragment separating from C sixteen days before the 2001 perihelion with the same separation velocity and subjected to the same deceleration as fragment E would on 2006 May 11.0 TT be located 11°.3 from C at position angle 316°.8 — farther than some of the 1995 fragments!

In the short run, the presented results should benefit all observers who plan to participate in monitoring the comet's nuclei during its upcoming return to perihelion, whether optically, by radar, in the infrared, etc., especially during the close encounter with Earth in mid-May 2006. The major companion nuclei are thus ready for searches in a coordinated effort to observe fragments down to the least dimension that can possibly be detected.

More generally, this is a contribution in the quest to understand cascading fragmentation of comets by presenting a sequence and hierarchy of one of the most difficult multiply split comets. This work thus provides fundamental information on the disintegration processes in comets and on their physical evolution and demise, with broad applications to cometary science, including the exploration of comets by space missions.

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The CARA Project and the Af(rho) Approach to Cometary Photometry*

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Abstract. *The project named "Cometary Archive for Amateur Astronomers (CARA) was developed among a group of Italian comet observers and is devoted to CCD photometry of cometary comae for the derivation of the aperture-independent quantity Afρ. The main goal is to create a photometric numerical archive. In its current status, the project concerns mainly the dust component of cometary emission, but the possibility of getting data also for the gas component (with proper techniques) is under consideration. Filtered observations are highly encouraged (especially in the R and*

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