Vehicle State Estimation Using Steering Torque

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***Abstract*— This paper presents a new approach to estimating vehicle sideslip using steering torque information. This method is especially suited to vehicles equipped with steer-by-wire systems since the steering torque can easily be determined from the current applied to the steering motor. By combining a linear vehicle model with the steering system model, a simple observer may be devised to estimate sideslip when yaw rate and steering angle are measured. The observer is validated on a test vehicle equipped with a steer-by-wire system.**

I. Introduction

The effectiveness of a vehicle dynamic stability control system relies on accurate knowledge of the vehicle states, particularly yaw rate and sideslip angle. While yaw rate is readily measured in production vehicles with inexpensive sensors, sideslip angle must be estimated by more sophisticated means. Stability systems currently available on production cars typically derive this value from integration of inertial sensors, but this estimation method is prone to uncertainty and errors [1], [2], [3]. For example, direct integration can accumulate sensor errors and unwanted measurements from road grade and bank angle.

An alternative estimation scheme overcomes some of these drawbacks by supplementing integration of inertial sensors with Global Positioning System (GPS) measurements [4]. Absolute GPS heading and velocity measurements eliminate the errors from inertial navigation system (INS) integration; conversely, INS sensors complement the GPS measurements by providing higher update rate estimates of the vehicle states. However, during periods of GPS signal loss, which frequently occur in urban driving environments, integration errors can still accumulate and lead to faulty estimates.

The growing presence of electric power steering systems in production vehicles introduces yet another absolute measurement—steering torque—from which vehicle sideslip angle may be estimated. Through the tire self-aligning moment, steering torque is directly related to the lateral front tire forces, which in turn relate to the tire slip angles and therefore the vehicle states. This paper develops two observer structures based on linear models of the vehicle and tire behavior to estimate the vehicle states from measurements of steering angle and yaw rate. Steering angle and yaw rate sensors are both inexpensive and common to vehicles already equipped with stability control systems. Steering torque sensors are intrinsic components of electric power steering systems [5]. Furthermore, as steer-by-wire technology approaches reality, complete knowledge of steering torque can be determined from the current applied to the electric steering actuator [6].

Fig. 1. Experimental steer-by-wire vehicle.

The first of the two observers combines the vehicle and steering system models into a single observer structure to estimate four states at once: sideslip angle, yaw rate, steering angle, and steering rate. The second observer incorporates an intermediate step. A disturbance observer based on the steering system model estimates the tire aligning moment; this estimate becomes the measurement part of a vehicle state observer for sideslip and yaw rate. The performance of the observers is verified on an experimental vehicle equipped with steer-by-wire capability.

II. Steer-by-Wire System

The vehicle considered in this study is a production model 1997 Chevrolet Corvette that has been converted to steer-by-wire (Fig. 1). The stock steering gear is a rack and pinion configuration with hydraulic power assist. The steer-by-wire conversion (Fig. 2) makes use of all the stock components except for the intermediate steering shaft, which is replaced by a brushless DC servomotor actuator to provide steering torque in place of the handwheel. Two rotary position sensors—one on the steering column and the other on the pinion—provide absolute measurements of both angles. The hydraulic power assist unit in the test vehicle is retained as part of the steer-by-wire system.

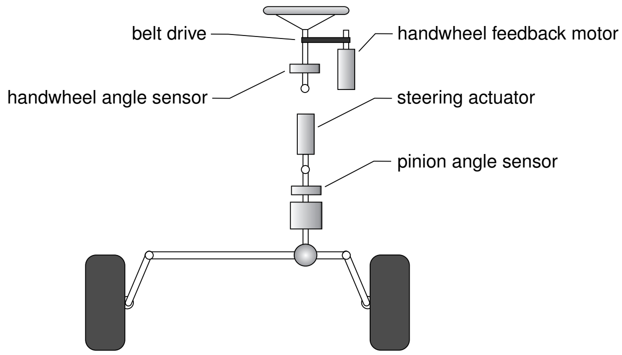

Fig. 2. Conventional steering system converted to steer-by-wire.

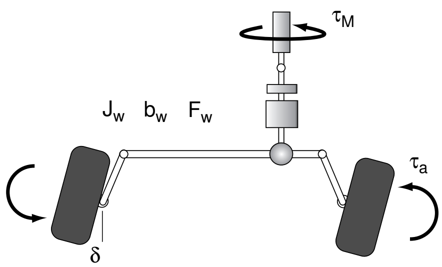

Fig. 3. Steering system dynamics.

The incorporation of power assist eliminates the need for extensive modifications to the existing steering system and allows the use of a much smaller actuator since the assist unit provides a majority of the steering effort.

The steering actuator, which consists of a motor and gearhead combination controlled by a servo amplifier, was selected based on the maximum torque and speed necessary to steer the vehicle under typical driving conditions including moderate emergency maneuvers. The steer-by-wire control system, developed in [7], determines the current, i_M, required by the steering servomotor to follow the driver's steering commands.

III. Steering System Model

The steering system shown in Fig. 3 is described by the following differential equation:

$$J_w\ddot{\delta} + b_w\dot{\delta} + \tau_f + \tau_a = r_s r_p \tau_M \tag{1}$$

where J_w and b_w are the moment of inertia and damping of the steering system at the road wheels and τ_f represents Coulomb friction. Furthermore, r_s is the steering ratio, and r_p is the torque magnification factor of the power steering system, here approximated by a constant. τ_M is the steering actuator torque, which can be written in terms of motor constant, k_M, motor current, i_M, motor efficiency, η, and gearhead ratio, r_g:

$$\tau_M = k_M i_M r_g \eta \tag{2}$$

The tire self-aligning moment, τ_a, is a function of the steering geometry, particularly caster angle, and the manner

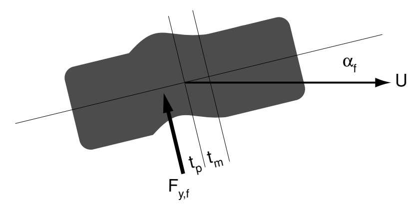

Fig. 4. Generation of aligning moment.

in which the tire deforms to generate lateral forces. In Fig. 4, $F_{y,f}$ is the lateral force acting on the tire, α_f is the tire slip angle, t_p is the pneumatic trail, the distance between the resultant point of application of lateral force and the center of the tire, t_m is the mechanical trail, the distance between the tire center and the steering axis, and U is the velocity of the tire at its center. The total aligning moment is given by

$$\tau_a = F_{y,f}(t_p + t_m) \tag{3}$$

where t_p and t_m are approximately known. Rewriting (1) in state space form yields:

$$\dot{x}_1 = A_1 x_1 + B_{1,1}u_1 + B_{1,2}\tau_a \tag{4}$$

where

$$\begin{aligned} x_1 &= \begin{bmatrix} \delta & \dot{\delta} \end{bmatrix}^T \\ A_1 &= \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b_w}{J_w} \end{bmatrix} \\ B_{1,1} &= \begin{bmatrix} 0 & 0 \\ \frac{r_s r_p}{J_w} & -\frac{1}{J_w} \end{bmatrix} \\ u_1 &= \begin{bmatrix} \tau_M & \tau_f \end{bmatrix}^T \\ B_{1,2} &= \begin{bmatrix} 0 \\ -\frac{1}{J_w} \end{bmatrix} \end{aligned}$$

and the aligning moment, τ_a, is treated as an external input to the steering system. The resisting torque, τ_f, due to friction is treated as an input:

$$\tau_f = F_w sgn(\dot{\delta}) \tag{5}$$

where the Coulomb friction constant, F_w, has been identified along with the inertia and damping constants.

IV. Linear Vehicle Model

A vehicle's handling dynamics in the horizontal plane are represented here by the single track, or bicycle model with states of sideslip angle, β, at the center of gravity (CG) and yaw rate, r. In Fig. 5, δ is the steering angle, u_x and u_y are the longitudinal and lateral components of the CG velocity, $F_{y,f}$ and $F_{y,r}$ are the lateral tire forces front and rear, respectively, and α_f and α_r are the tire slip angles. Derivation of the equations of motion for the bicycle model follows from the force and moment balance:

$$\begin{aligned} ma_y &= F_{y,f}\cos\delta + F_{y,r} \\ I_z\dot{r} &= aF_{y,f}\cos\delta - bF_{y,r} \end{aligned} \tag{6}$$

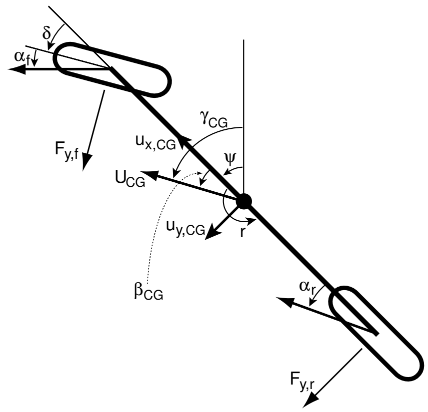

Fig. 5. Bicycle model.

I_z is the moment of inertia of the vehicle about its yaw axis, m is the vehicle mass, a and b are distance of the front and rear axles from the CG, and a_y is lateral acceleration at the CG. In the linear region of tire operation—typically slip angles of four degrees or less—lateral force at the front and rear is related to slip angle by the total cornering stiffness coefficient of the front and rear tires:

$$\begin{aligned} F_{y,f} &= -C_{\alpha,f}\alpha_f \\ F_{y,r} &= -C_{\alpha,r}\alpha_r \end{aligned} \tag{7}$$

Taking small angle approximations, slip angle can be written in terms of u_x, u_y, and r:

$$\begin{aligned} \alpha_f &= \frac{u_y + ar}{u_x} - \delta \\ \alpha_r &= \frac{u_y - br}{u_x} \end{aligned} \tag{8}$$

Assuming constant longitudinal velocity $u_x = V$, the state equation for the bicycle model can be written as:

$$\dot{x}_2 = A_2 x_2 + B_2 \delta \tag{9}$$

where

$$\begin{aligned} x_2 &= \begin{bmatrix} \beta & r \end{bmatrix}^T \\ A_2 &= \begin{bmatrix} -\frac{C_{\alpha,0}}{mV} & -1 + \frac{C_{\alpha,1}}{mV^2} \\ \frac{C_{\alpha,1}}{I_z} & -\frac{C_{\alpha,2}}{I_z V} \end{bmatrix} \\ B_2 &= \begin{bmatrix} \frac{C_{\alpha,f}}{mV} \\ \frac{C_{\alpha,f}a}{I_z} \end{bmatrix} \end{aligned}$$

and to consolidate notation

$$\begin{aligned} C_{\alpha,0} &= C_{\alpha,f} + C_{\alpha,r} \\ C_{\alpha,1} &= C_{\alpha,r}b - C_{\alpha,f}a \\ C_{\alpha,2} &= C_{\alpha,f}a^2 + C_{\alpha,r}b^2 \end{aligned}$$

with states of the bicycle model defined by slip angle, β, at the CG and yaw rate, r. Given longitudinal and lateral velocity, u_x and u_y, at the CG, the sideslip angle is defined by:

$$\beta = \arctan\left(\frac{u_y}{u_x}\right) \tag{10}$$

Sideslip angle can also be defined by the difference between the vehicle's forward orientation, ψ, and the direction of the velocity, γ.

$$\beta = \gamma - \psi \tag{11}$$

V. Conventional Observer

When looking at the two state linear vehicle model described above, one might consider designing a simple state observer based on measurement of yaw rate alone. Unfortunately, there is one instance in which the sideslip angle is unobservable through yaw rate: the neutral steering case ($C_{\alpha,r}b - C_{\alpha,f}a$ equals zero). Therefore, an observer based on yaw rate alone is impractical as the vehicle handling characteristics approach the neutral steering configuration. One way to estimate sideslip in this situation is to combine the linear vehicle model with the steering system model. The aligning moment term in the steering system equation can be expressed as a function of the vehicle states and steering angle by substituting (8) and (9) into (3):

$$\tau_a = -C_{\alpha,f}(t_p + t_m)(\beta + \frac{a}{V}r - \delta) \tag{12}$$

Now combining (4) and (9) yields the following state space model:

$$\dot{x}_3 = A_3 x_3 + B_3 u_3 \tag{13}$$

where

$$\begin{aligned} x_3 &= \begin{bmatrix} \beta & r & \delta & \dot{\delta} \end{bmatrix}^T \\ A_3 &= \begin{bmatrix} -\frac{C_{\alpha,0}}{mV} & -1 + \frac{C_{\alpha,1}}{mV^2} & \frac{C_{\alpha,f}}{mV} & 0 \\ \frac{C_{\alpha,1}}{I_z} & -\frac{C_{\alpha,2}}{I_z V} & \frac{aC_{\alpha,f}}{I_z} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{C_{\alpha,3}}{J_w} & \frac{aC_{\alpha,3}}{J_w V} & -\frac{C_{\alpha,3}}{J_w} & -\frac{b_w}{J_w} \end{bmatrix} \\ B_3 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{r_s r_p k_M r_g \eta}{J_w} & -\frac{1}{J_w} \end{bmatrix} \\ u_3 &= \begin{bmatrix} i_M & \tau_f \end{bmatrix}^T \end{aligned}$$

and

$$C_{\alpha,3} = (t_p + t_m)C_{\alpha,f}$$

with states of vehicle sideslip angle, β, yaw rate, r, steering angle, δ, and steering rate, $\dot{\delta}$. Note that after incorporating the steering system dynamics this system is now observable in the neutral steering case. The standard observer structure is given by:

$$\dot{\hat{x}}_3 = A_3\hat{x}_3 + B_3 u_3 + T_3(y_3 - C_3\hat{x}_3) \tag{14}$$

The vector, $\hat{x}_3$, contains the estimated states and y_3 is the vector of measurements—yaw rate and steering angle—directly available from vehicle sensors:

$$y_3 = \begin{bmatrix} r & \delta \end{bmatrix}^T = C_3 x_3 \tag{15}$$

where

$$C_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The observer in (14) can be rewritten as:

$$\dot{\hat{x}}_3 = (A_3 - T_3 C_3)\hat{x}_3 + B_3 u_3 + T_3 y_3 \qquad (16)$$

The estimator gain matrix, T_3, is chosen so that the matrix $A_3 - T_3 C_3$ has eigenvalues located at least ten times farther to the left than the system eigenvalues in the complex frequency plane. The sampling rate of the measurements should be yet another order of magnitude faster than the estimation error dynamics, which are given by:

$$\dot{\tilde{x}}_3 = (A_3 - T_3 C_3)\tilde{x}_3 \qquad (17)$$

where the estimation error is

$$\tilde{x}_3 = x_3 - \hat{x}_3$$

When T_3 is selected so that $A_3 - T_3 C_3$ has stable eigenvalues, the error dynamics approach zero exponentially.

VI. Disturbance Observer

As an alternative, one can first estimate the aligning moment by applying a disturbance observer to the steering system model described by (4). The aligning moment estimate then becomes a measurement for the state estimator based on the vehicle model given by (9). A disturbance observer structure for the steering system is simply constructed by appending the disturbance, τ_a, to the state vector, x_1, and augmenting the corresponding rows in the state matrices with zeroes:

$$\dot{z}_1 = F_1 z_1 + G_1 u_1 \qquad (18)$$

where

$$\begin{aligned} z_1 &= \begin{bmatrix} x_1{}^T & \tau_a \end{bmatrix}^T \\ F_1 &= \begin{bmatrix} A_1 & B_{1,2} \\ 0 & 0 \end{bmatrix} \\ G_1 &= \begin{bmatrix} B_{1,1} \\ 0 \end{bmatrix} \end{aligned}$$

The available measurement, y_1, is the steering angle, δ:

$$y_1 = \delta = C_1 z_1 \qquad (19)$$

where

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The disturbance observer is given by:

$$\dot{\hat{z}}_1 = (F_1 - L_1 C_1)\hat{z}_1 + G_1 u_1 + L_1 y_1 \qquad (20)$$

and the corresponding error dynamics are:

$$\dot{\tilde{z}}_1 = (F_1 - L_1 C_1)\tilde{z}_1 \qquad (21)$$

where the estimation error is

$$\tilde{z}_1 = z_1 - \hat{z}_1$$

This formulation of the disturbance observer is a technical simplification which assumes the derivative of disturbance torque, $\dot{\tau}_a$, is zero. In other words, it assumes the disturbance is varying slowly and independent of the steering system dynamics. In reality, as is evident from (3), the derivative of the disturbance does depend on the steering rate as well the dynamics of the vehicle. Making the assumption that $\dot{\tau}_a$ equals zero, however, results in a close approximation of disturbance torque and is similar to the approach taken in [8].

Now the standard observer structure is applied to (9) only:

$$\dot{\hat{x}}_2 = A_2 \hat{x}_2 + B_2 u_2 + T_2 (y_2 - \hat{y}_2) \qquad (22)$$

The vector, $\hat{x}_2$, contains the states to be estimated and y_2 is the vector of "measurements"—in this case, yaw rate and the aligning moment estimate obtained from the disturbance observer. Recall that substituting (8), (9) and (2) into (1) expresses the aligning moment, τ_a, in terms of the vehicle states, β and r:

$$y_2 = \begin{bmatrix} r & \tau_a \end{bmatrix}^T = C_2 x_2 + D_2 \delta \qquad (23)$$

where

$$\begin{aligned} C_2 &= \begin{bmatrix} 0 & 1 \\ -(t_p + t_m)C_{\alpha,f} & -\frac{a(t_p+t_m)C_{\alpha,f}}{V} \end{bmatrix} \\ D_2 &= \begin{bmatrix} 0 \\ (t_p + t_m)C_{\alpha,f} \end{bmatrix} \end{aligned}$$

While (9) is unobservable in the neutral steering case when yaw rate, r, is the sole measurement, the addition of aligning moment, τ_a, to the measurement vector means that the system given by (9) and (23) will always be observable. The observer in (22) can be rewritten:

$$\dot{\hat{x}}_2 = (A_2 - T_2 C_2)\hat{x}_2 + (B_2 - T_2 D_2)\delta + T_2 y_2 \qquad (24)$$

As before, the estimator gain matrix, T_2, is chosen so that the matrix $A_2 - T_2 C_2$ has stable eigenvalues and the error dynamics are significantly faster than the system dynamics. The error dynamics here are given by:

$$\dot{\tilde{x}}_2 = (A_2 - T_2 C_2)\tilde{x}_2 \qquad (25)$$

where the estimation error is

$$\tilde{x}_2 = x_2 - \hat{x}_2$$

VII. Experimental Results

Both types of state observer have been implemented in real-time on the steer-by-wire test vehicle. The state estimates from the observers are simultaneously compared to results from a highly accurate sideslip estimation method based on measurements from a GPS/INS system installed in the test vehicle. Details and validation of the GPS-driven state estimation method can be found in [4]. As a reference, vehicle states calculated from the linear vehicle model—with parameters matched to the test vehicle—are also included in the comparison. The following figures correspond to the same test cycle during which the vehicle

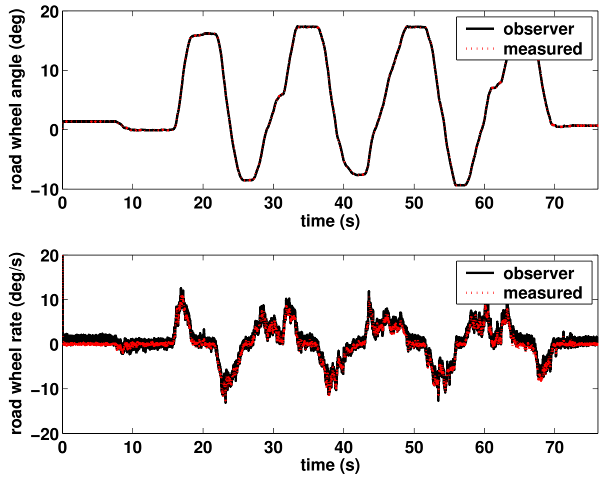

Fig. 6. Estimated steer angle and rate from conventional observer.

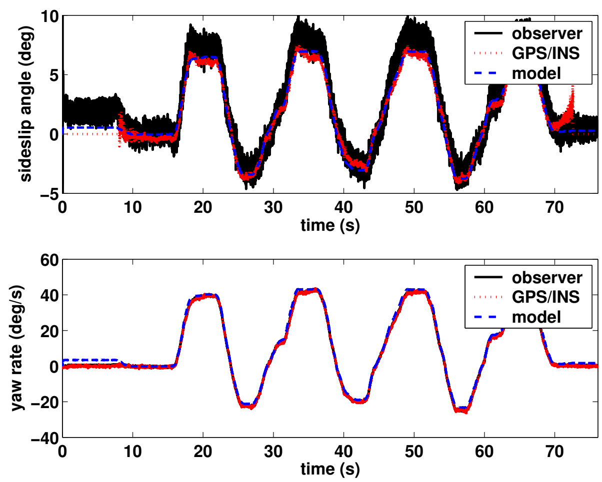

Fig. 7. Estimated sideslip and yaw rate from conventional observer.

is accelerated from a standing stop to a steady speed of $6.7m/s(15mi/hr)$ at which time the steering maneuver is initiated.

Figs. 6 and 7 show the state estimates from the four-state observer. As expected, since steering angle and yaw rate are measurements, the estimates of steering angle and yaw rate tend to match the measured values almost exactly. The observed state of primary interest, sideslip angle, correlates well with the GPS/INS sideslip estimate, but the choice of observer gains found to produce a good steering rate estimate fails to filter noise in the sideslip estimate (Fig. 7). This problem illustrates the drawback of the four-state observer: the choice of a single observer gain matrix often compromises estimation performance due to the fact that the steering system dynamics are significantly faster than the vehicle dynamics.

By separating the state estimation into a disturbance

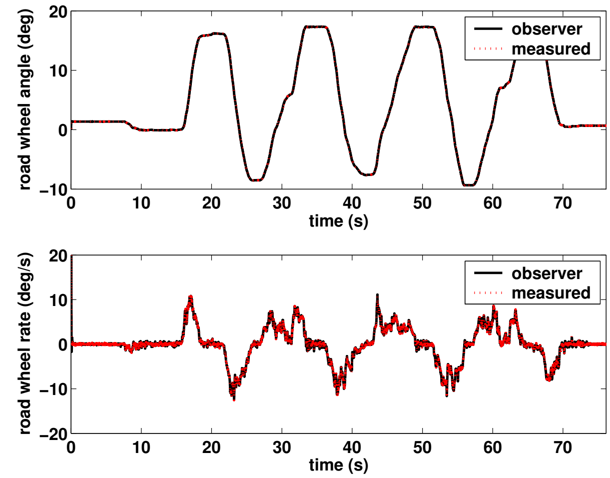

Fig. 8. Estimated steer angle and rate from disturbance observer.

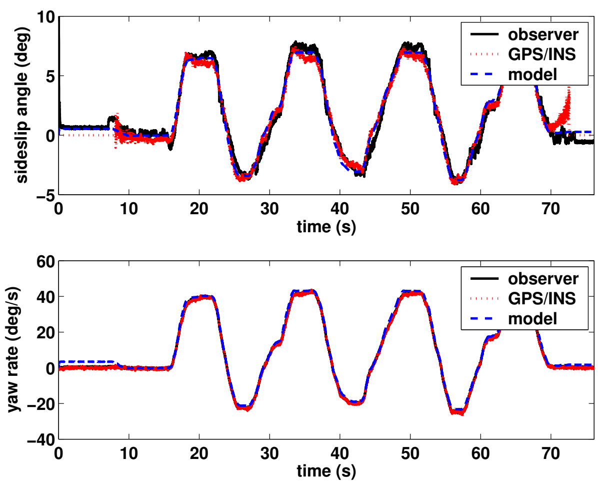

Fig. 9. Estimated sideslip and yaw rate from disturbance observer.

observer for the steering system and a two-state observer for the vehicle states, the alternative observer structure facilitates the selection of observer gains appropriate to either set of dynamics. The improvement is clear in the sideslip estimate of Fig. 9 and the steering rate estimate of Fig. 8.

Since the vehicle is operated well within the linear region of its dynamic behavior in these tests, the yaw rate and sideslip predicted by the linear vehicle model follow the estimated values closely. The estimated disturbance torque shown in Fig. 10, however, is not as well predicted by the aligning moment calculated from (3). In modeling the steering system of the test vehicle, several sources of uncertainty exist to cause such discrepancy, among them nonlinear hydraulic power steering characteristics and changes in suspension geometry (toe, camber, and caster angle) due to steering and suspension motions. Fortunately,

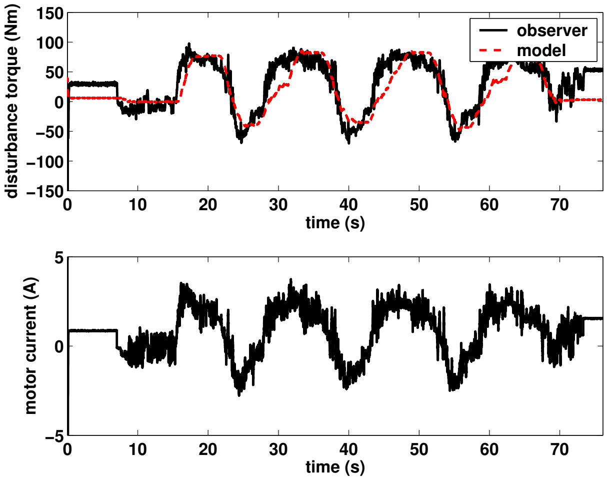

Fig. 10. Estimated disturbance torque.

many of the physical unknowns associated with test vehicle can be overcome in a clean-sheet steer-by-wire design. For example, if the suspension system were designed with large and invariant caster angle, the predictable effects of tire mechanical trail would dominate the nonlinear characteristics of the pneumatic trail.

It is important to note that although the imperfect disturbance torque estimate is used as a measurement in the vehicle state observer, the estimates of the vehicle states do not apparently suffer in the same way from these uncertainties. This suggests that two-tiered vehicle state estimation scheme can be made to be fairly robust to parameter uncertainty in the steering system model as long as there is a reliable vehicle model with accurate yaw rate measurement.

VIII. Conclusion

As steering torque information becomes more common in automotive steering systems—in the form of either electric power steering or steer-by-wire—a useful connection can be drawn between forces and vehicle motion: the knowledge of forces acting on the steering system through the tires in turn provides information on the motion of the vehicle itself. Like GPS-based estimation, vehicle state estimation using steering torque is not subject to the problems of error accumulation from inertial sensor integration. Unlike GPS, however, the signal is never lost, and no extra and expensive equipment is necessary if a vehicle is equipped with electric power steering or, in the near future, steer-by-wire technology.

Two observer structures based on linear models of the vehicle and steering system dynamics have been developed to take advantage of this additional measurement. As demonstrated in the experimental work, the combination of readily available measurements from steering torque, steering angle, and yaw rate sensors generates a sideslip angle estimate comparable to that obtained from highly accurate measurements by a sophisticated GPS/INS system. This has many practical implications for the next generation of fully integrated automotive stability control systems, since all of the measurement devices necessary for precise vehicle control already exist and have been inexpensively implemented on production cars.

Furthermore, in a carefully designed steering system, the aligning moment disturbance effectively communicates the tire forces acting on the vehicle—regardless of whether it is being operated in the linear or nonlinear handling region—and thus addresses the limitations of using a linear model-based observer structure. Future work will investigate how to apply this information to extend the ability of the observer to predict vehicle motion throughout the entire range of handling behavior.

IX. Acknowledgments

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References

[1] A. van Zanten, "Evolution of electronic control systems for improving the vehicle dynamic behavior," in *Proceedings of the International Symposium on Advanced Vehicle Control (AVEC), Hiroshima, Japan*, 2002.

[2] Y. Fukada, "Estimation of vehicle slip-angle with combination method of model observer and direct integration," in *Proceedings of the International Symposium on Advanced Vehicle Control (AVEC), Nagoya, Japan*, 1998.

[3] M. Abe, Y. Kano, and K. Suzuki, "An experimental validation of side-slip control to compensate vehicle lateral dynamics for a loss of stability due to nonlinear tire characteristics," in *Proceedings of the International Symposium on Advanced Vehicle Control (AVEC), Ann Arbor, MI*, 2000.

[4] J. Ryu, E. Rossetter, and J. C. Gerdes, "Vehicle sideslip and roll parameter estimation using GPS," in *Proceedings of the International Symposium on Advanced Vehicle Control (AVEC), Hiroshima, Japan*, 2002.

[5] D. Peter and R. Gerhard, "Electric power steering—the first step on the way to steer by wire," 1999, SAE Technical Paper No. 1999-01-0401.

[6] H. Lupker, J. Zuurbier, and R. Verschuren, "Steer-by-wire innovations and demonstrator," in *Proceedings of the International Symposium on Advanced Vehicle Control (AVEC), Hiroshima, Japan*, 2002.

[7] P. Yih, J. Ryu, and J. C. Gerdes, "Modification of vehicle handling characteristics via steer-by-wire," in *Proceedings of the 2003 American Control Conference, Denver, CO*, 2003, pp. 2578–2583.

[8] Y. Yasui, W. Tanaka, E. Ono, Y. Muragishi, K. Asano, M. Momiyama, S. Ogawa, K. Asano, Y. Imoto, and H. Kato, "Wheel grip factor estimation apparatus," 2004, United States Patent Application Pub. No. US 2004/0019417 A1.