OSE SEMINAR 2013

Weather routing - using dynamic programming to win sailing races

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ÅBO NOVEMBER 15 2013

Table of Content

Weather routing

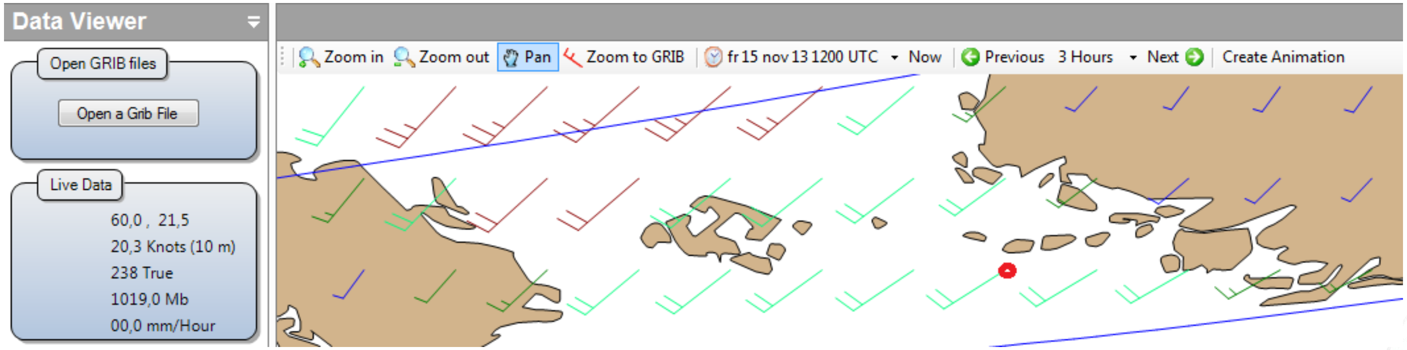
- The art of finding the fastest route between a given starting point A and given finishing point B given:
  - A weather forecast
    - Wind direction and speed
    - Optional: Wave direction, height and period
  - The boats performances data (called polar data)
    - Boat speed in given TWA (True Wind Angle) and TWS (True Wind Speed)
    - Optional: Sail and boat configuration
- The optimization problem is to find a route that:
  - Minimize the time to reach the destination
  - If finishing point B can’t be reached the distance left to B will be minimized
- Often solved using Dynamic Programming and a discrete time model

Dynamic Programming

- Simplifies a complex decision by breaking it up to smaller subproblems that can be solved recursively
  - Uses Bellman's "Principle of Optimality": *An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.*
- From a given **state** a given **action** moves us to a new **state** adding a given amount of **value**. When the destination **state** is reached all optimal **actions** can be found by backtracking to the initial starting **state**.

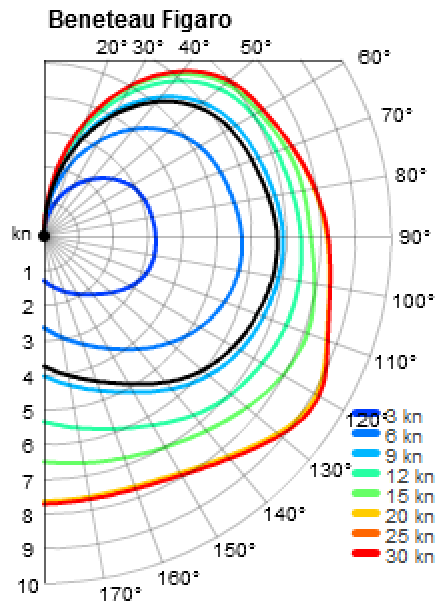
Weather models

- For useful and reliable routing good forecasts are needed
  - Todays' forecasts are reasonably good up to 7-10 days ahead
- Forecasts are distributed in GRIB-files (GRIdded Binary)
  - Discretization in space: 0.5-1°
  - Discretization in time: 1-12 hours
  - For each point in space and time:
    - TWS (True Wind Speed)
    - TWD (True Wind Direction)
    - Pressure
    - Rain
  - Forecast for the whole world for 7 days with maximum detail (0.5° and 3 hour resolution and all weather data) ~104 Mb
  - 7 day forecast for the whole world with good detail (1° and 12 hour resolution and TWS, TWD and pressure) ~5.2 Mb



Polars

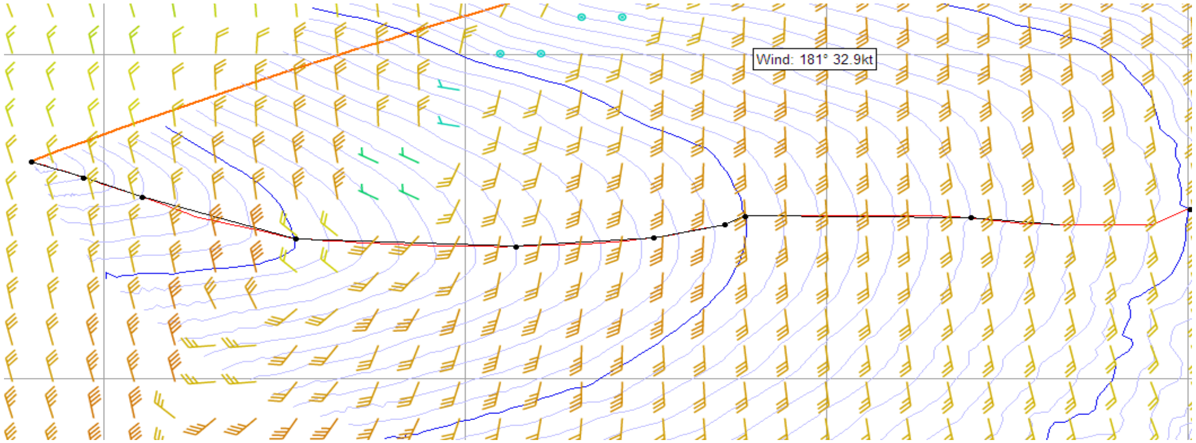
- The other data set needed for weather routing is boat performance data - the polars
  - Gives target boat speeds for given wind conditions (TWS, TWA)
    - TWA (True Wind Angle) is the angle difference between the boat's heading and the TWD (True Wind Direction)
  - Often includes optimal sail plan and boat configuration to achieve the target speeds
- For human use the data is often display as a polar graph



Polar graph for a Beneteau Figaro

Weather routing optimization

- Assumptions
  - Discrete time
    - time step 10min - 60 min
      - Depending on optimization horizon
    - Constant heading and velocity during a time step
  - Weather data 100% correct
- Weather data
  - Interpolated in both space and time
    - Quadratic interpolation
- Boat polars
  - Quadratic interpolation
- Optimized using Dynamic Programming
  - The result is the optimal route
  - A set of isochrones



Mathematical model

t – time step
x_t – position at time t
a_t – action at time t (heading)
$\Gamma(x_t)$ – set of available actions
$F(x,a)$ – payoff of move a from position x (distance travelled from x)
$V(x)$ – Value function

$$V(x_0) = \max_{\{a_t\}_{t=0}^{T}} \sum_{t=0}^{T} F(x_t, a_t)$$

subject to

$$\left.\begin{aligned} a_t &\in \Gamma(x_t) \\ x_{t+1} &= T(x_t, a_t) \end{aligned}\right\} \forall t$$

In recursive form

$$V(x) = \max_{a_t \in \Gamma(x_t)} \{F(x,a) + V(T(x,a))\}$$

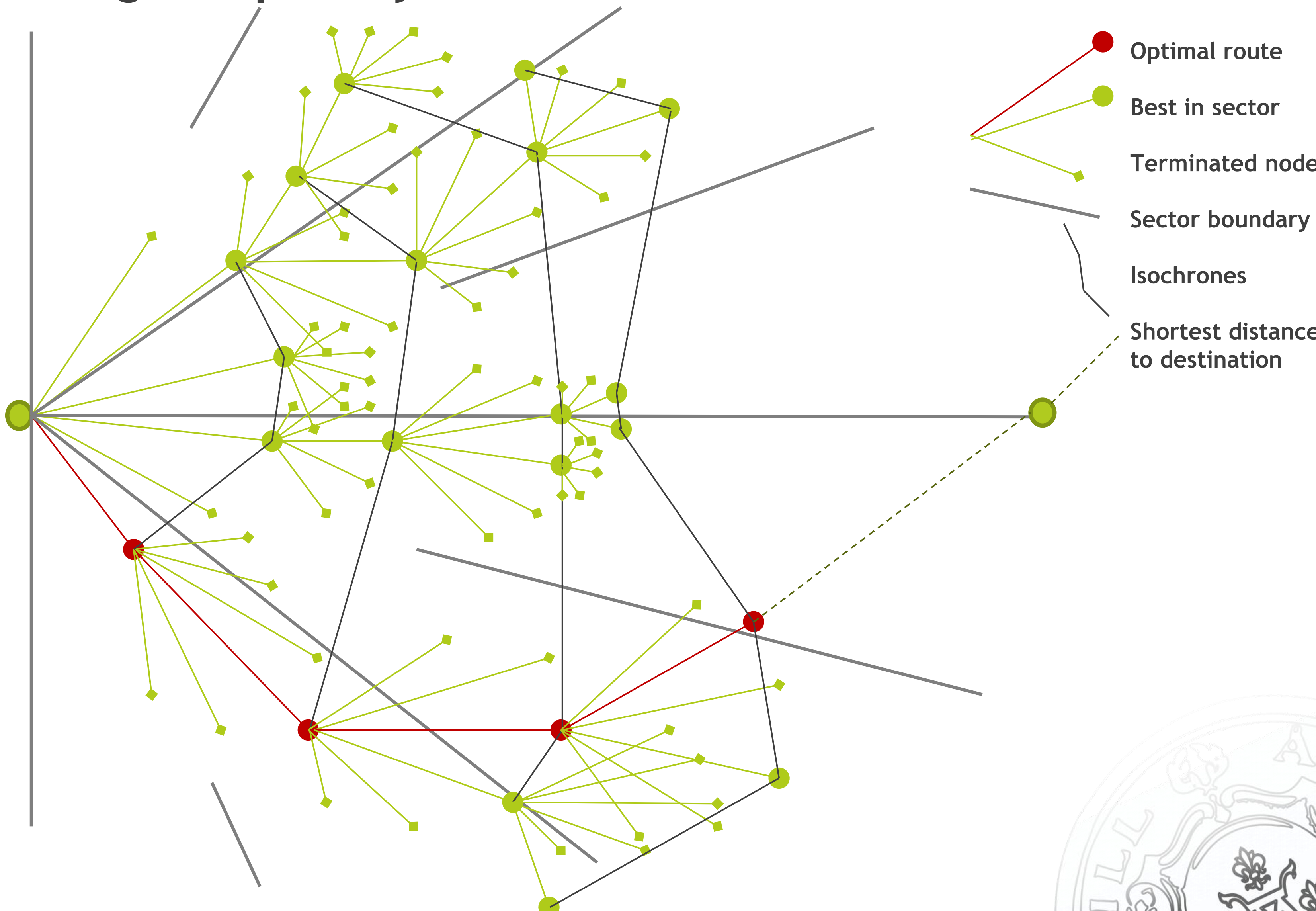
Termination criteria

- x_t is the destination position
- If iteration T is reached
  - Choose x_T that is closest to the destination

Complexity

- The number of points (**states**) to evaluate is huge
  - 68 discrete polar points
    - Can be reduced to 54 because sailing with TWA <36° or >155 ° is never optimal due to VMG (Velocity Made Good)
  - Reasonable assumptions about directions can be made
    - Never optimal to sail >75° off the direct course towards the finish
    - Reduces the number of points/time step down to 30 per time step
      - Using 10 minute discretization, a 6 hour route still consists of $1.5*10^{53}$ point evaluations
- To reduce the search space the space is divided into sectors and only the point furthest away from the start, in each sector, is used in the next iteration
  - By pruning the states and only leaving the ones furthest from the starting point reduces the number of points for a 24 hour route to around 5 000 000
    - This pruning creates isochrones for each time step
    - This reduces complexity from $O(N^2)$ to $O(N)$
      - The choice of number of sectors defines N
      - Sector size is variable with distance to starting point to reduce sector width far away from the start

Reducing complexity



“Optimal” routing

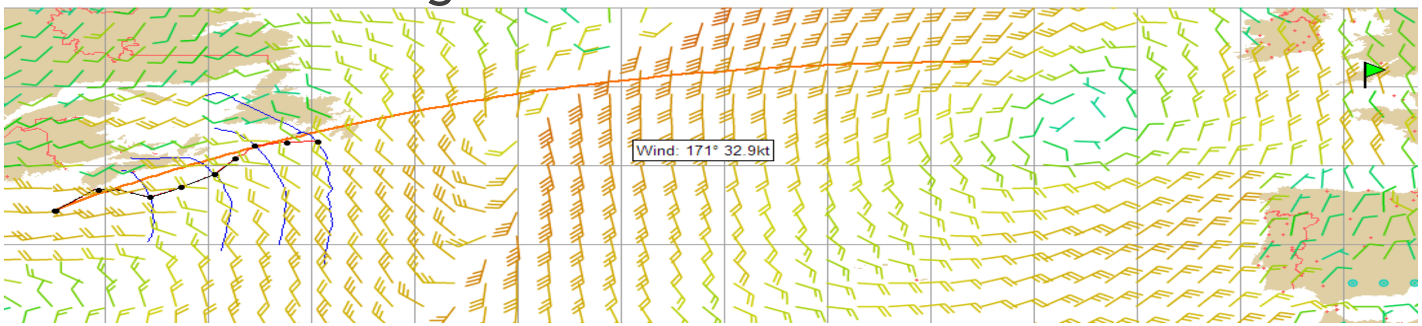
- The underlying mathematical model is a deterministic dynamic programming formulation
- To reduce computational complexity in order to reduce run time the following (heuristic) assumptions have been made:
  - Reducing the number of points using the sector method described
  - Assuming reasonable headings in relation to the destination
    - Valid if no obstacles are present between starting point and destination
  - Calculate optimal upwind/downwind VMG and only use TWAs in close proximity of optimal TWA when destination bearing is $<36^\circ$ or $>155^\circ$
- The assumptions make this approach heuristic because optimality can’t be guaranteed

Limitations

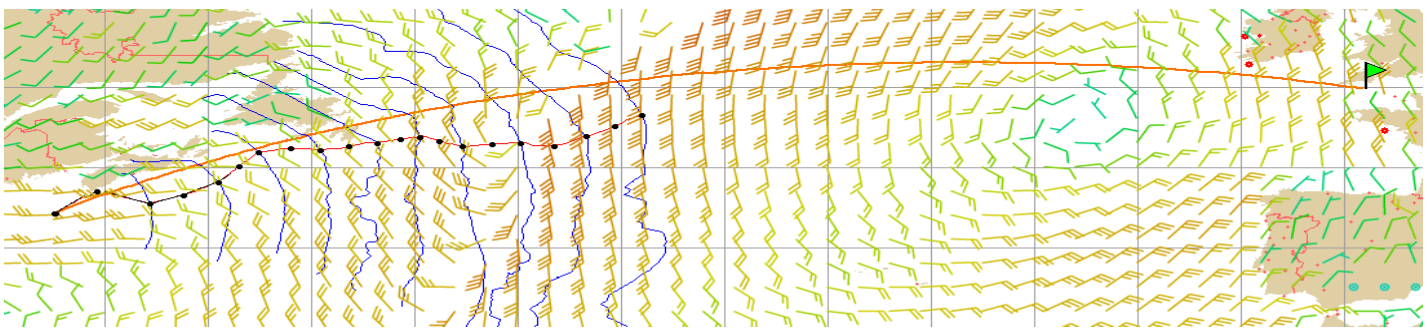
- The optimization assumes 100% correct weather forecasts
    - Stochastic weather routing models exist but are seldom used because stochastic weather forecasts are hard to come by
    - Uncertainties in forecasts can be included by running multiple optimizations with different parameter settings
- The assumptions made to reduce search space may cause weird behavior if land masses are situated along the route
- The optimization does not consider the ”riskiness” of a route
    - Human knowledge is needed for the risk assessment
- Optimization horizon length can greatly affect optimal routes
- If the destination is beyond the optimization horizon human knowledge is needed to analyze the results

Illustrative examples - riskiness and horizon length

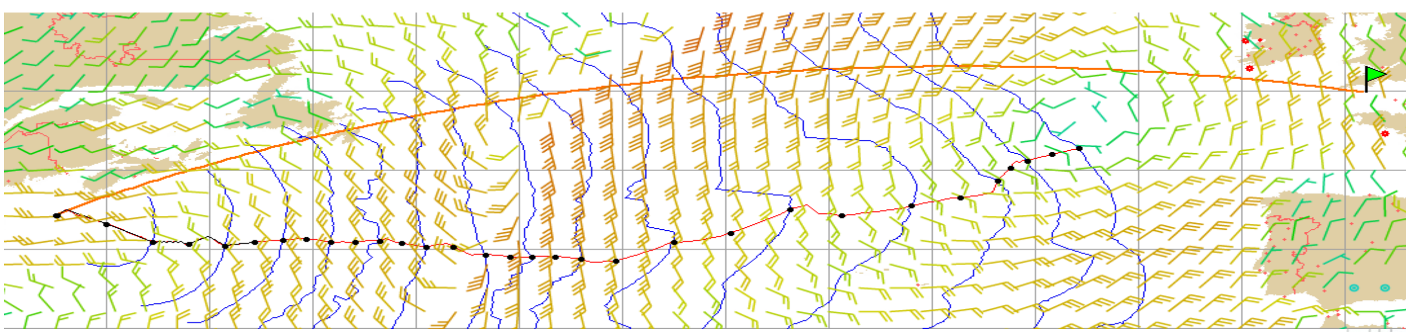
- Route: Ambrose Lighthouse – Lizard Point



2 days



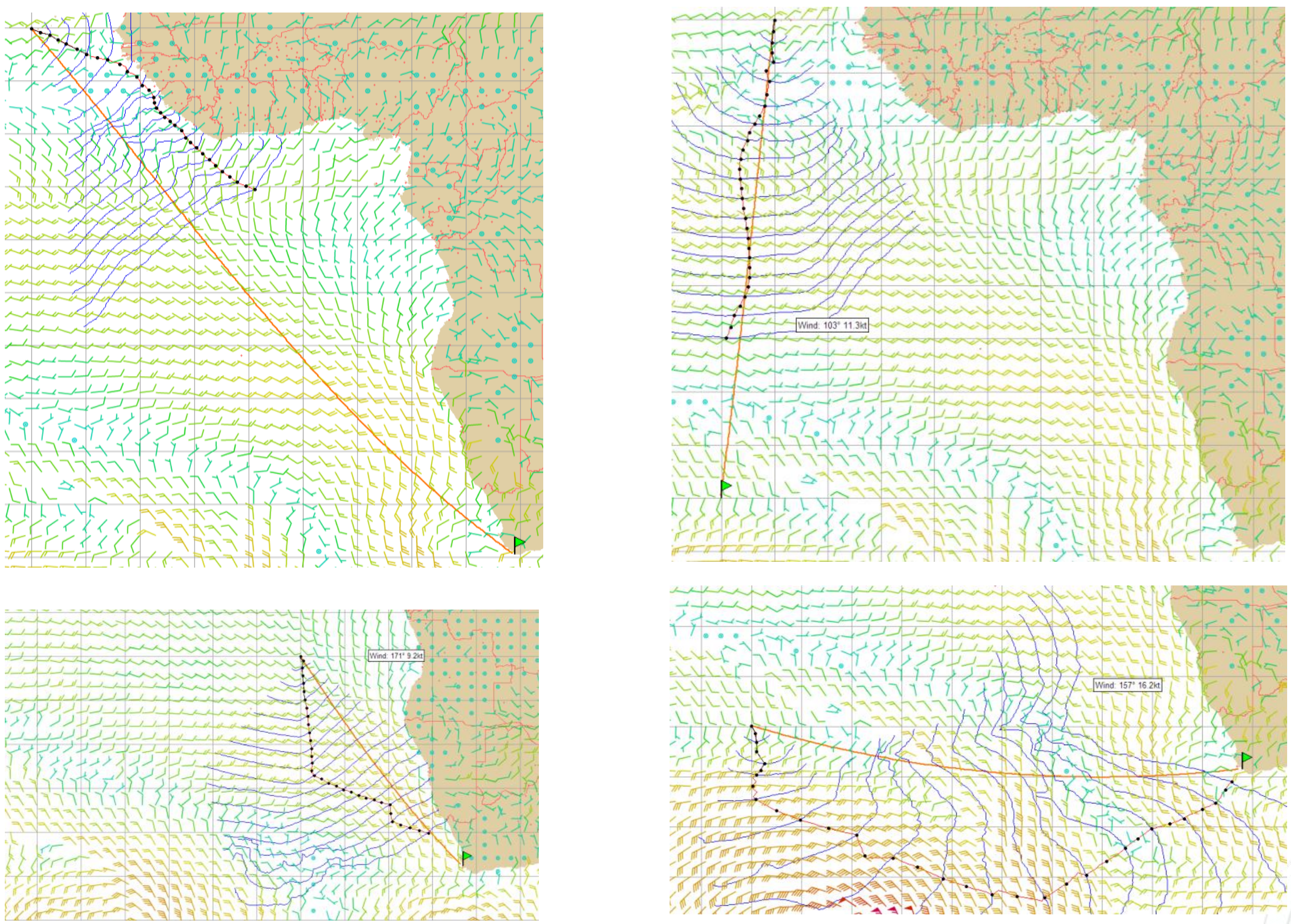
5 days



8 days

Illustrative examples - direct route vs. “right” route

- Route: Cape Verde – Cape Town



Conclusions

- The deterministic dynamic programming model is both efficient and accurate
  - For virtual sailing the model produces extremely good routing
    - 9 out of 10 leg winners in last Virtual Volvo Ocean Race used this routing software
    - Players also used expensive ”real” weather routing software for the virtual race
- It is easy to tailor the software for real sailing by including:
  - Wave and current models
  - Multirun capability with adjustable parameters
  - Improved sail management
  - Polar import/tuning
  - NMEA interface to get communicate with onboard instruments

Thank you!

Questions?