

# The DOT Calculus

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DOT:			
<b>Syntax</b>			
$t ::=$ $x$ $\{z \Rightarrow \bar{d}\}$ $t.m(t)$	<b>terms:</b> variable object method invocation	$S, T, U ::=$ $\top$ $\perp$ $T \wedge T$ $T \vee T$ $L : S..U$ $m(x : S) : U$ $p.L$ $\{z \Rightarrow T\}$	<b>types:</b> top bottom intersection union type member method member selection recursive self
$d ::=$ $L = T$ $m(x : T) = t$	<b>initialization:</b> type member method member	$\Gamma ::=$ $\emptyset \mid \Gamma, x : T$	<b>contexts:</b> variable bindings
$v ::=$ $\{z \Rightarrow \bar{d}\}$	<b>values:</b> object		
$p ::=$ $x$ $v$	<b>paths:</b> variable value		

Figure 1: DOT: Syntax

DOT:	
<b>Evaluation</b>	$t \longrightarrow t'$
$\frac{[z \mapsto \bar{d}]\bar{d} \ni m(x : T_{11}) = t_{12}}{\{z \Rightarrow \bar{d}\}.m(v_2) \longrightarrow [x \mapsto v_2]t_{12}}$	(E-APP)
$\frac{t_1 \longrightarrow t_1'}{t_1.m(t_2) \longrightarrow t_1'.m(t_2)}$	(E-APP1)
$\frac{t_2 \longrightarrow t_2'}{v_1.m(t_2) \longrightarrow v_1.m(t_2')}$	(E-APP2)

Figure 2: DOT: Small-Step Operational Semantics

DOT:			
<b>Subtyping</b>	$\Gamma \vdash S <: U$		
<b>Lattice structure</b>			
$\Gamma \vdash \perp <: T$	(BOT)	$\Gamma \vdash T <: \top$	(TOP)
$\frac{\Gamma \vdash T_1 <: T}{\Gamma \vdash T_1 \wedge T_2 <: T}$	(AND11)	$\frac{\Gamma \vdash T <: T_1}{\Gamma \vdash T <: T_1 \vee T_2}$	(OR21)
$\frac{\Gamma \vdash T_2 <: T}{\Gamma \vdash T_1 \wedge T_2 <: T}$	(AND12)	$\frac{\Gamma \vdash T <: T_2}{\Gamma \vdash T <: T_1 \vee T_2}$	(OR22)
$\frac{\Gamma \vdash T <: T_1, T <: T_2}{\Gamma \vdash T <: T_1 \wedge T_2}$	(AND2)	$\frac{\Gamma \vdash T_1 <: T, T_2 <: T}{\Gamma \vdash T_1 \vee T_2 <: T}$	(OR1)
<b>Type and method members</b>			
$\frac{\Gamma \vdash S_2 <: S_1, U_1 <: U_2}{\Gamma \vdash L : S_1..U_1 <: L : S_2..U_2}$	(TYP)	$\frac{\Gamma \vdash S_2 <: S_1, \Gamma, x : S_2 \vdash U_1 <: U_2}{\Gamma \vdash m(x : S_1) : U_1 <: m(x : S_2) : U_2}$	(FUN)
<b>Type selections</b>			
$\frac{\Gamma_{[x]} \vdash x :! (L : T..T)}{\Gamma \vdash T <: x.L}$	(SEL2)	$\frac{[z \mapsto \bar{d}]\bar{d} \ni L = T}{\Gamma \vdash T <: \{z \Rightarrow \bar{d}\}.L}$	(SSEL2)
$\frac{\Gamma_{[x]} \vdash x :! (L : \perp..T)}{\Gamma \vdash x.L <: T}$	(SEL1)	$\frac{[z \mapsto \bar{d}]\bar{d} \ni L = T}{\Gamma \vdash \{z \Rightarrow \bar{d}\}.L <: T}$	(SSEL1)
<b>Recursive self types</b>			
$\frac{\Gamma, z : T_1 \vdash T_1 <: T_2}{\Gamma \vdash \{z \Rightarrow T_1\} <: \{z \Rightarrow T_2\}}$	(BIND)	$\frac{\Gamma, z : T_1 \vdash T_1 <: T_2, z \notin \text{fv}(T_2)}{\Gamma \vdash \{z \Rightarrow T_1\} <: T_2}$	(BIND1)
<b>Properties</b>			
$\Gamma \vdash T <: T$	(REFL)	$\frac{\Gamma \vdash T_1 <: T_2, T_2 <: T_3}{\Gamma \vdash T_1 <: T_3}$	(TRANS)
Figure 3: DOT: Subtyping			

DOT:				
<b>Type assignment</b>	$\Gamma \vdash t :_{(!)} T$			
$\frac{\Gamma(x) = T}{\Gamma \vdash x :_{(!)} T}$	(VAR)	$\frac{\Gamma \vdash t :_{(!)} T_1, T_1 <: T_2}{\Gamma \vdash t :_{(!)} T_2}$	(SUB)	
$\frac{\Gamma \vdash p : [z \mapsto p]T}{\Gamma \vdash p : \{z \Rightarrow T\}}$	(PACK)	$\frac{\Gamma \vdash p :_{(!)} \{z \Rightarrow T\}}{\Gamma \vdash p :_{(!)} [z \mapsto p]T}$	(UNPACK)	
$\frac{\Gamma \vdash t : (m(x : T_1) : T_2), t_2 : T_1, x \notin \text{fv}(T_2)}{\Gamma \vdash t.m(t_2) : T_2}$	(TAPP)	$\frac{\Gamma \vdash t : (m(x : T_1) : T_2), p : T_1}{\Gamma \vdash t.m(p) : [x \mapsto p]T_2}$	(TAPPDEP)	
(labels disjoint)				
$\frac{\Gamma, x : T_1 \wedge \dots \wedge T_n \vdash d_i : T_i \quad \forall i, 1 \leq i \leq n}{\Gamma \vdash \{x \Rightarrow d_1 \dots d_n\} : [x \mapsto \{x \Rightarrow d_1 \dots d_n\}](T_1 \wedge \dots \wedge T_n)}$				(TOBJ)
<b>Member initialization</b>				
$\Gamma \vdash d : T$				
$\frac{\Gamma \vdash T <: T}{\Gamma \vdash (L = T) : (L : T..T)}$	(DTYP)	$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash (m(x) = t) : (m(x : T_1) : T_2)}$	(DFUN)	
Figure 4: DOT: Typing				