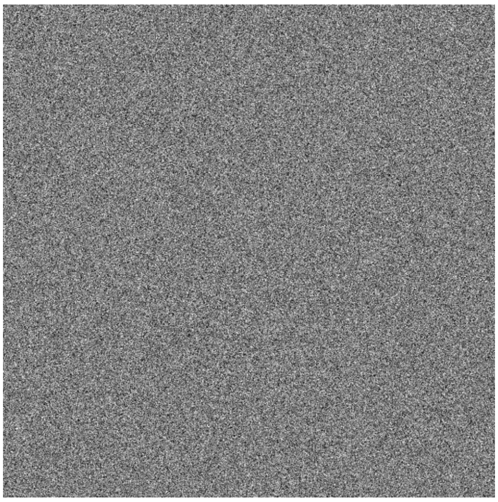
Colors of image noise

1 White Gaussian noise

Everybody knows about white Gaussian noise



White Gaussian noise is famous because it has very nice properties:

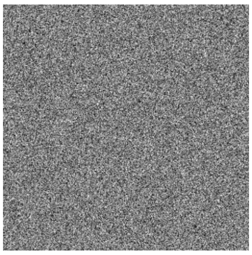
1. It is easy to generate using pseudorandom numbers
2. Each pixel is an independent, identically distributed Normal variable
3. The discrete Fourier, Hartley and Cosine transforms are also white Gaussian noise (except for the obvious symmetries)
4. In particular, the power spectrum is mostly flat
5. Applying a linear filter renders the pixel values non-independent, but they are still Normal and identically distributed.

Some properties of dubious convenience:

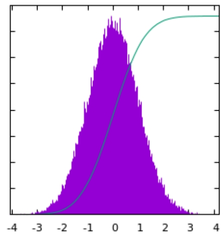
1. The mean is zero, thus it cannot be directly represented as a positive-valued image

2. Worse, the pixel values are not bounded, thus it has a-priori infinite dynamic range.
3. When you see it from far away (zooming-out), it disappears.

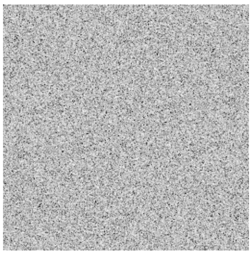
Statistics of white gaussian noise and its DFT:



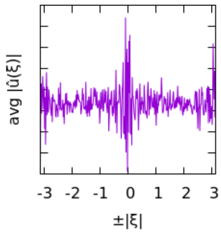
$u(x)$



histogram of u



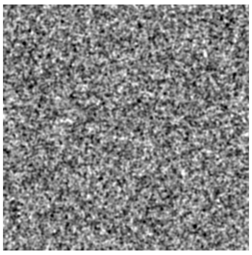
$\log|\hat{u}(\xi)|$



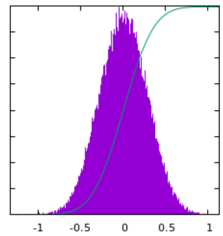
average spectral profile

2 Blurred white Gaussian noise

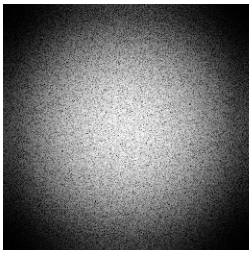
White gaussian noise blurred by a small gaussian kernel:



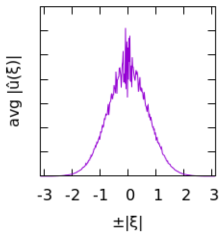
$u(x)$



histogram of u

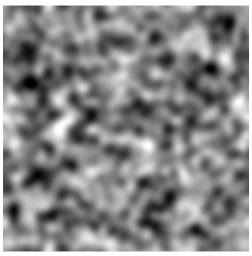


$\log|\hat{u}(\xi)|$

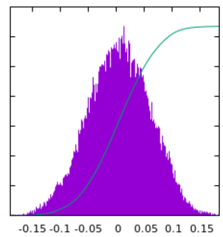


average spectral profile

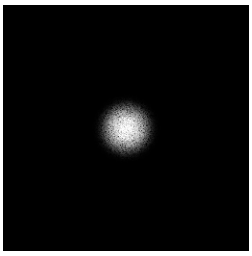
White gaussian noise blurred by a larger gaussian kernel:



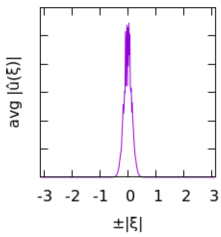
$u(x)$



histogram of u

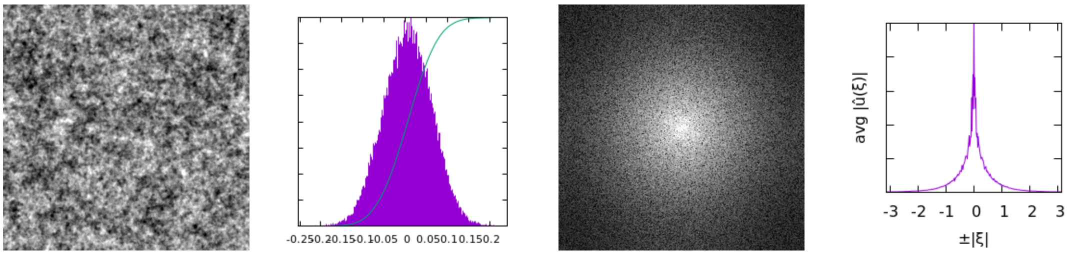


$\log|\hat{u}(\xi)|$



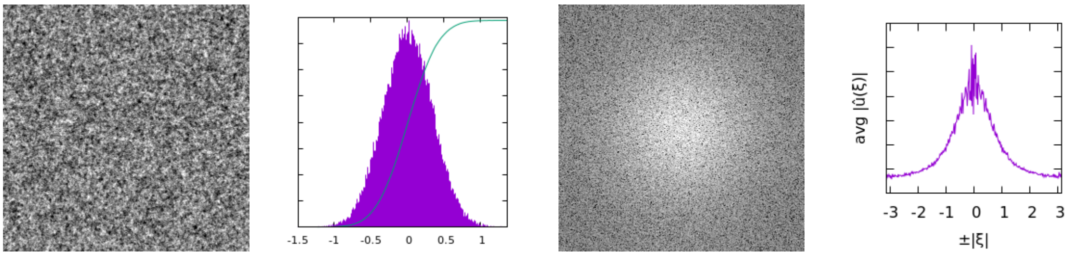
average spectral profile

White gaussian noise blurred by a Cauchy kernel:



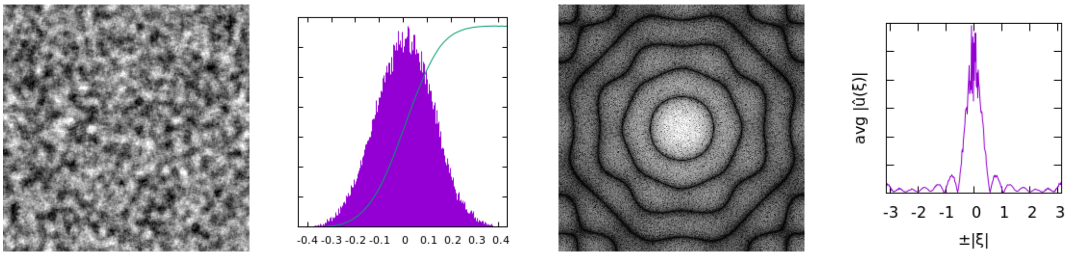
$u(x)$ | histogram of u | $\log|\hat{u}(\xi)|$ | average spectral profile

White gaussian noise blurred by a Laplace kernel:



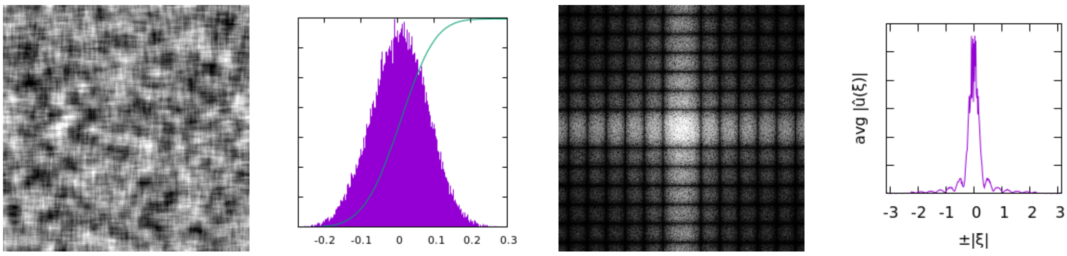
$u(x)$ | histogram of u | $\log|\hat{u}(\xi)|$ | average spectral profile

White gaussian noise blurred by a Disk kernel:



$u(x)$ | histogram of u | $\log|\hat{u}(\xi)|$ | average spectral profile

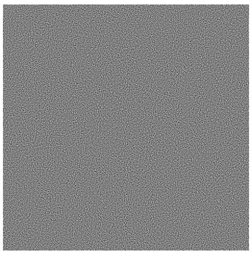
White gaussian noise blurred by a Square kernel:



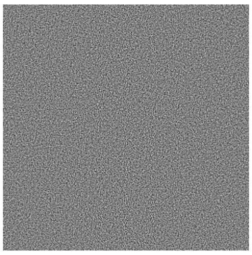
$u(x)$ | histogram of u | $\log|\hat{u}(\xi)|$ | average spectral profile

3 Colored gaussian noise

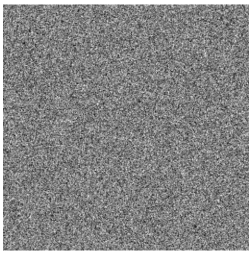
When the spectrum of noise decays as a power-law, we say that it is "colored" noise. The exponent α of the power law determines its color. The particular case of $\alpha = 0$ corresponds to white noise (a flat spectrum).



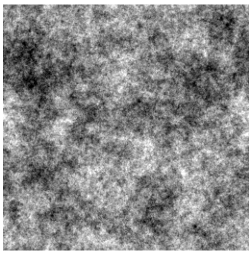
$\alpha = 2$ purple



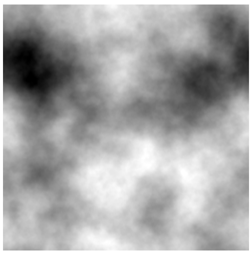
$\alpha = 1$ blue



$\alpha = 0$ white



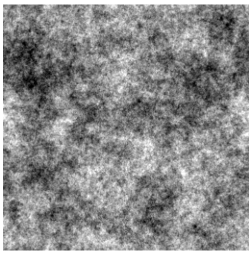
$\alpha = -1$ pink



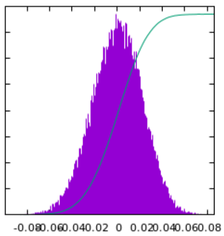
$\alpha = -2$ brown

$\alpha = -3$ smooth

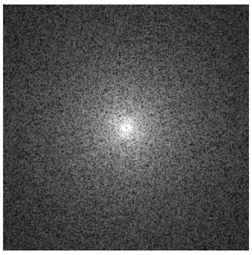
Statistics of Pink noise ($\alpha = -1$):



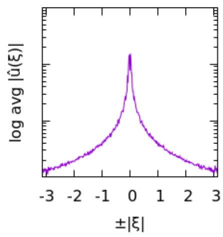
$u(x)$



histogram of u

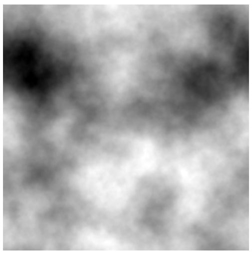


$\log |\hat{u}(\xi)|$

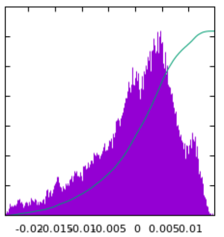


average spectral profile

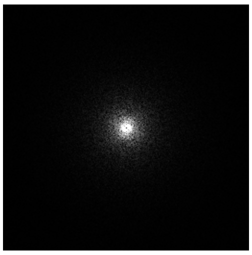
Statistics of Brown noise ($\alpha = -2$):



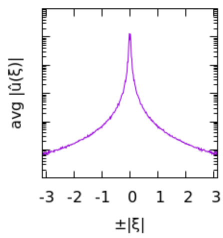
$u(x)$



histogram of u



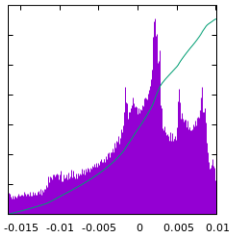
$\log |\hat{u}(\xi)|$



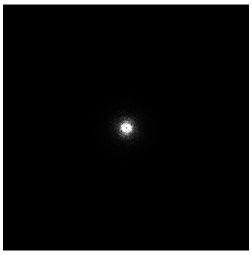
average spectral profile

Statistics of Smooth noise ($\alpha = -3$):

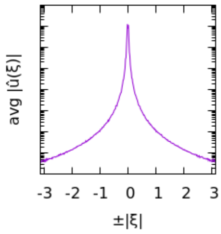
$u(x)$



histogram of u

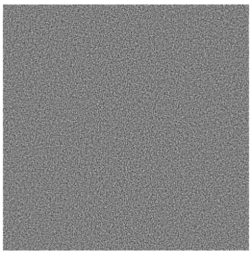


$\log|\hat{u}(\xi)|$

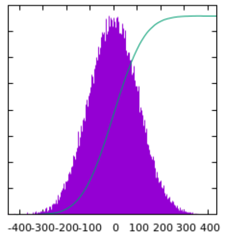


average spectral profile

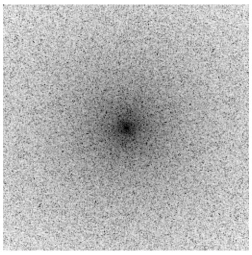
Statistics of Blue noise ($\alpha = 1$):



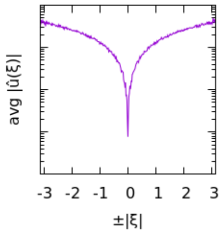
$u(x)$



histogram of u

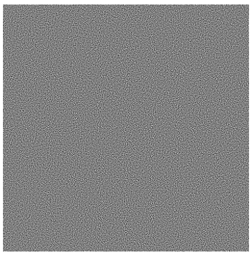


$\log|\hat{u}(\xi)|$

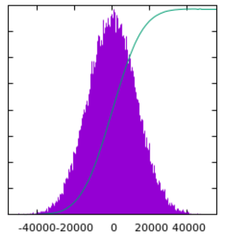


average spectral profile

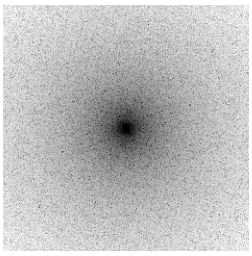
Statistics of Purple noise ($\alpha = 2$):



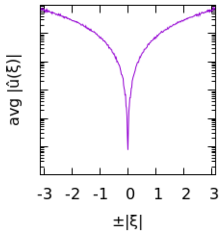
$u(x)$



histogram of u



$\log|\hat{u}(\xi)|$



average spectral profile