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COORDINATE REFERENCE SYSTEMS FOR HIGH PRECISION GEODESY

by

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CONTENTS

ABSTRACT

The advent of the Global Positioning System (GPS) meant that, for the first time, the geodetic and geophysical community has a tool for measurements on a global, continental and national scale. Global GPS networks are already competing with VLBI and SLR for the measurement of inter-continental baselines and earth rotation parameters. The development of the 'high accuracy fiducial GPS technique', as described in this thesis, has produced results comparable with mobile VLBI and SLR systems, but in shorter observational periods and at lower costs. Combined with global GPS networks, which have the potential to provide time-tagged fiducial station coordinates at the observational epochs, coordinates can be determined in a global reference frame. The results in this thesis, from a fiducial GPS campaign to monitor the vertical land movement at tide gauge sites in the UK, demonstrate that millimetric precisions and accuracies can be obtained in all three components over baselines of hundreds of kilometers.

The combination of GPS with existing 2-d classical triangulation networks for mapping, engineering surveying and navigation has caused many problems, since the GPS observations are 3-d and of a superior quality. In Europe these problems have been overcome by the establishment of a new high precision reference framework, EUREF, based on fiducial GPS carried out in 1989. This thesis also describes the determination of coordinates for the UK EUREF stations and their application for geodetic control in Great Britain.

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LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

Introduction

The word geodesy is derived from the Greek *geodaisia*, which translates as *to divide the Earth*. Today, geodesy is traditionally defined as the study of the size, shape and gravity field of the Earth, the positioning of points on its surface and the changes in these quantities with time. The first recorded attempt to measure the size of the Earth was by Eratosthenes in 200 BC, who noticed that during the summer solstice, when the sun was directly over the Egyptian town of Syene, it cast a shadow in the neighbouring town of Alexandria. By measuring the length of this shadow and the distance between the two towns he was able to calculate the diameter of the Earth. His value was to within 15% of today's accepted value. A truly remarkable feat, considering that the general opinion at the time was that the Earth was flat. Over the intervening 2000 years the techniques of geodesy slowly evolved with the first geodetic networks being established some 200 years ago.

In the early part of this century, geodetic networks were mainly developed to control mapping. These networks were based on pure triangulation, with scale provided by catenary baselines and the orientation from Laplace azimuths. One station was generally chosen as the origin, and the relationship of this point to the spheroid was defined, usually by setting the geodetic latitude and longitude at this point to be equal to the observed astronomic values and the spheroidal height to be equal to the orthometric height. These networks contained more observations than unknowns, therefore, they were adjusted using least squares techniques. However, the difficulties of handling large arrays meant that the networks had to be split up and adjusted as small blocks. Although this was the best that could be achieved at the time, it often resulted in large discrepancies across block boundaries. The Ordnance Survey of Great Britain 1936 (OSGB36) is an example of this kind of network, adjusted in seven blocks.

The Second World War highlighted the need for more homogeneous national networks for military mapping, and following the war, for civilian applications. This necessitated the need for the establishment of higher accuracy geodetic networks to control mapping. The development of Electromagnetic Distance Measurement (EDM) revolutionised the time taken and the accuracy achievable for the measurement of distances. Instead of just one or two catenary bases, it was now possible to measure numerous distances throughout the whole network. In addition, the development of digital computers enabled, for the first time, networks to be adjusted as a single block. In the UK this led to the Ordnance Survey of Great Britain 1970 (OSGB70), a re-adjustment of OSGB36 as a single block, including 180 EDM distances. OSGB70 was internally very consistent, but suffered from systematic biases in the distance and azimuth observations. This was attributed to poorly calibrated instruments, atmospheric effects and, of course, operator errors.

Satellite navigation started in the mid 1960's with the commissioning of the US Navy Navigation Satellite System otherwise known as Transit. Transit was made available to the civilian community in 1967. It enabled absolute point positioning to an accuracy of 1 metre, and relative positioning to an accuracy of 30 centimetres from observations made over only a few days. Transit provided observations which could be used to model the systematic biases in terrestrial observations and define the scale and orientation of geodetic control networks. In the UK this led to the Ordnance Survey (Scientific Network) 1980 OS(SN)80 which was a re-adjustment of OSGB70 including 11 Transit positions. This re-adjustment involved solving for extra unknowns in order to model for systematic biases in the distance and azimuth observations. The results from the OS(SN)80 adjustment found that the distances measured using a microwave EDM instrument were about 3 ppm too short, while the lightwave EDM instrument and Laplace azimuth observations had no significant systematic biases.

In the early 1970's, two positioning techniques evolved, namely Very Long Baseline Interferometry (VLBI) and Satellite Laser Ranging (SLR), which opened up a new era in geodesy. They were able to improve the measurement of geophysical phenomena on a global scale, such as polar motion, earth rotation and inter-continental plate motion. The introduction of mobile VLBI and SLR enabled measurements on a continental scale, such as the monitoring of crustal deformations in tectonically active regions. However, their application has been limited by their large costs, long observation periods (months) and restricted mobility.

In the late 1980's the Transit system was replaced by the US DoD's Global Positioning System (GPS). This can provide accuracies ranging from ±100 metres for absolute positioning, to a few millimetres over hundreds of kilometres for relative positioning using the fiducial GPS technique. The advent of GPS and the development of the highly accurate fiducial GPS technique, presented the geodetic and geophysical community with a new tool for measurements on a global, continental, or even national scale. The low cost and portability of GPS enables the measurement of dense networks using short observation periods (a few days), and even though GPS has yet to be declared fully operational (September 1994), it has already been used in a wide variety of geodetic and geophysical applications. On a global scale it is already competing with VLBI and SLR for determining polar motion and earth rotation parameters. In high accuracy deformation monitoring, it is replacing mobile VLBI and SLR on a continental scale, and it has also enabled new measurements to be made on a national scale, the most common applications being the monitoring of crustal deformations in areas prone to earthquakes, and vertical land movement at tide gauge sites for the determination of changes in mean sea level.

On a more local scale the use of GPS for mapping and engineering surveying has highlighted many problems with existing terrestrial geodetic networks. Firstly, the GPS observations are of a superior quality and, therefore, have to be forced to fit the existing network, and secondly, the transformation parameters between the GPS datum and the terrestrial datum are usually only known to a low level of accuracy. In Europe these problems have been addressed, through the establishment of a new datum (EUREF) based on VLBI, SLR and GPS. The aims of EUREF were to provide control for further national GPS measurements and allow the accurate determination of transformation parameters between National, Continental and Global datums.

GPS and the fiducial GPS technique are described in Chapter 2 along with a brief description of other positioning techniques mentioned in this thesis, namely Transit, VLBI and SLR. Chapter 3 describes the coordinate systems and reference datums available in the UK and the requirement of the new European Datum, EUREF. The establishment of EUREF, including details of the 1989 GPS campaign and the processing of this data set are described in Chapter 4. This campaign suffered from several problems leading to lower than expected accuracies for the UK stations. These problems were addressed by the UK Gauge Project, which re-observed the UK EUREF stations as part of a network for monitoring vertical land movement at tide gauge sites. The UK Gauge 1991 data set proved to be of very high quality, and therefore, suitable to perform a series of tests, the results of which have shown how the fiducial technique can produce accuracies of millimetres over hundreds of kilometres. The UK Gauge project and these tests are described in Chapter 5. The resulting coordinates from the UK Gauge 1991 Data Set are compared to the terrestrial horizontal and vertical datums of Great Britain in Chapter 6. Finally, conclusions and suggestions for further research are given in Chapter 7.

CHAPTER 2

Geodetic Space Techniques

This chapter primarily describes the Global Positioning System (GPS), and for completeness briefly reviews the other geodetic space techniques mentioned in this thesis. It is by no means comprehensive and aims to cover the terms and techniques used in subsequent chapters and should, therefore, only be treated as a reference.

Sections 2.1 and 2.2 describe GPS, this includes a description of the system, observables, the fiducial technique and the Nottingham in-house software, used by the author in Chapters 4 and 5. The following sections 2.3 to 2.7 describe the other geodetic space techniques, namely, Transit, Glonass, Very Long Baseline Interferometry, Satellite Laser Ranging and Lunar Laser Ranging, and gives details of their applications. The establishment of a global GPS network and results are described in section 2.8 and the future applications of GPS on a global, continental and national scale are discussed in section 2.9

2.1 The Global Positioning System

The NAVSTAR GPS (Navigation System using Time and Range / Global Positioning System) is a satellite-based radio navigation system that has been developed since 1972 by the United States Department of Defence. The system provides instantaneous, world-wide, three-dimensional navigation and positioning in any weather by one way microwave range measurements between GPS satellites and a GPS receiver. In addition it can be used to output instantaneous three-dimensional velocity, as well as enabling the transfer of absolute time. Although GPS was primarily developed for military purposes, it has been applied in many varied civilian applications ranging from navigation to surveying, with accuracies from 100 metres for absolute 'real-time' positioning to millimetres in relative static post processed positioning.

The design and development of GPS was organised in three phases. Firstly, the 'concept validation' phase which was mainly concerned with the design and development of prototype satellites and receivers, and the installation of the control segment. Secondly, the 'engineering test' phase which involved the deployment of Block I satellites and operational verification of the system. Finally, the 'fully operational' phase involving the deployment of Block II satellites to produce the 'full' constellation. At the time of writing (September 1994), GPS has nearly reached the fully operational phase.

2.1.1 System Organisation

The Global Positioning System consists of three segments: the Control Segment, the Space Segment and the User Segment.

2.1.1.1 The Control Segment

The Control Segment as its name suggests, maintains and supports GPS. It consists of a single Master Control Station (MCS), five monitoring stations (MS) and three ground antennas (GA). These perform three main tasks namely, to monitor the satellites, predict the satellite orbits and clock offsets, and up-load the navigation message containing the satellite orbit, health and clock information for transmission by the satellites to the user.

Each MS consists of a dual frequency GPS receiver, an external caesium atomic clock and a communications link with the MCS. They take a pseudo-range measurement every 1.5 seconds and filter these to produce a smoothed measurement every 15 minutes. This data is then transmitted to the MCS at Falcon Air Force Base, Colorado Springs. The MCS uses this data to predict a broadcast ephemeris and corrections to the satellite clock. This information is then up-loaded every hour to the satellite using the GA's. The MS's are located such that each Block II satellite will be tracked for over 90% of its orbit, and were coordinated using Transit Doppler in the World Geodetic System 1984. The caesium clocks of the MS's are used to define the GPS time frame *[Wells, 1986]*.

2.1.1.2 The Space Segment

The Space Segment, when fully operational, by early 1994, will consist of twenty-one Block II satellites, plus three active spares. The satellites are arranged in six orbital planes which are inclined at fifty-five degrees to the equatorial plane. The orbit has approximately a 12 hour period and an altitude of approximately 20000 kilometres. This high altitude enables simultaneous visibility to at least four satellites, with the necessary geometric strength, at any time of day anywhere in the world. Currently (September 1994) there is one Block I satellite still in operation (PRN 12) and twenty-four Block II satellites have been successfully launched (PRN's 1, 2, 4, 5, 6, 7, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29 & 31). This constellation is capable of providing a minimum of four satellites for 24 hours a day in the UK.

The signals transmitted by the satellites consist of two carrier waves (L1 and L2) which are modulated with timing codes (Figure 2.1). Superimposed on these codes is the navigation message. The high precision capabilities of GPS are achieved through the use of very stable atomic clocks used to generate the carrier waves and timing codes. Each satellite contains two highly stable rubidium and two caesium atomic clocks which generate the fundamental GPS satellite frequency of nominally 10.23 MHz, offset by 0.0045 Hz to compensate for relativistic effects. All the other frequencies are derived from this basic frequency. The L1 carrier frequency (1575.42 MHz) is obtained by multiplying the fundamental frequency by 154, whereas L2 is given by 120 times the fundamental frequency (1227.60 MHz) *[Bingley, 1993]*. The L1 and L2 carriers have wavelengths of 19 and 24 cm respectively. The reason for the use of two carrier frequencies is to enable a first order correction for the ionospheric delay by combining the frequencies in a pre-determined ratio. The L3 carrier frequency, which is classified, is intended to aid positioning in times of 'high ionospheric activity' *[Ffoulkes-Jones, 1990]*.

Two timing codes are frequency modulated onto these carrier frequencies: the Precise code (P code) with a repeat period of 38 weeks and wavelength of 30 metres, and the Coarse Acquisition code (C/A code) with a repeat period of one-millisecond and wavelength of 300 metres. The L1 frequency has both the P and the C/A codes modulated onto it, while the L2 has only the P code. These are used in the navigation mode to provide the Precise Positioning Service (PPS) and the Standard Positioning Service (SPS) respectively. The SPS is the lower accuracy positioning service designed for the civilian

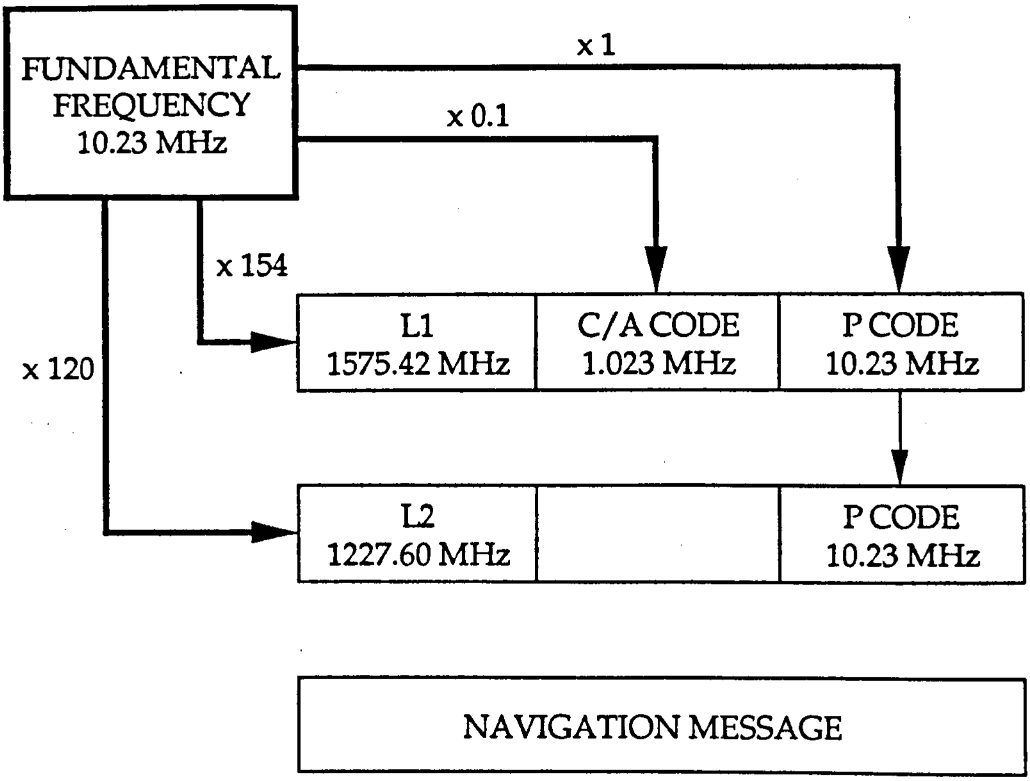


Figure 2.1 The Satellite Signal Structure

community providing a navigation accuracy of 100 metres. The user of this service will only have access to L1 frequency C/A code signals. The PPS is the higher accuracy service designed for military and some authorised civilian users. This service uses the P code signals on both frequencies yielding a navigation accuracy of 10 metres *[FRP, 1992]*.

Although both the P and C/A codes have the characteristic of random noise, they are generated by mathematical algorithms, and hence are referred to as pseudo-random noise (PRN). Each satellite has its own unique PRN code (one week portion of the P code), and this is used to unambiguously identify the satellite. When the system is fully operational, it is intended that the P code will be modulated with a classified W-code. This combination of P code and W code is known as the Y code, an encrypted and thus restricted version of the P code. This encryption is known as 'Anti- Spoofing' (A-S) and provides protection against hostile imitation of the P code signal.

The Broadcast Ephemeris

The Broadcast Ephemeris (BE) which is contained in the Navigation message describes the satellite orbits. For each satellite it contains six Keplerian parameters which define the mean satellite orbit and a further ten parameters describing small perturbations from this mean orbit. The BE is a predicted orbit, computed by extrapolating the orbit determined from the MS tracking data (pseudo-ranges). The BE is currently assessed to be accurate to 20 - 30 metres for Block I satellites and 5 - 10 metres for Block II satellites, although, the implementation of 'Selective Availability' (SA) degrades the accuracy for the Block II satellites. The BE is valid for up to two hours from the time of the ephemeris *[Ochieng, 1993]*.

The Precise Ephemeris

Post-computed Precise Ephemerides (PE) are available from several sources. One such precise ephemeris is computed by the US Naval Surface Warfare Centre (NSWC) in cooperation with the DMA, and available from the US National Geodetic Survey after a period of eight weeks. It is computed using tracking data (pseudo-ranges) acquired from the five MS's plus an additional five stations. A total of eight days of data are smoothed, edited and used as input to orbit determination software. Unlike the BE no extrapolation is required and hence the precise ephemeris is of a higher accuracy. The NSWC PE is supplied in Earth Fixed cartesian coordinate positions and velocity vectors at 15 minute intervals. The overlap between successive weekly ephemerides is 5 to 10 metres, and gives an indication of its accuracy.

2.1.1.3 The User Segment

The User Segment consists of either military or civilian users, equipped with GPS receivers capable of tracking the signals transmitted by the satellites in order to determine position or time information. Since GPS receivers are passive, only receiving information, the number of users is unlimited. Receivers normally contain a quartz clock, which is approximately synchronised with GPS time and used to produce replica satellite signals for measurement purposes and time-tagging the observations.

There are two GPS observables, the pseudo-range and the carrier phase. Subsequently, there are two basic types of GPS receiver, namely navigation receivers, which measure pseudo-ranges to determine their position in 'real-time', and geodetic receivers, used for surveying and geodesy, which measure both pseudo-ranges and carrier phase and record this data for post-processing. Geodetic receivers use different techniques to access the carrier phase observables, this being dependent upon the status of Anti-Spoofing.

When A-S is off the code-correlation technique is used to access both the pseudo-range and carrier phase observables. Receivers are able to generate a replica of the codes using the receivers' internal clock. This replica signal is then correlated with the incoming signal to obtain the pseudo-range measurements and subtracted from the incoming signal to measure the carrier wave. Code-correlation necessitates the prior knowledge of both C/A and P-codes. The TI 4100 and WM 102 are examples of the original receivers that used the code-correlation technique. Today, with the threat of Anti-Spoofing (A-S) the latest receivers have to resort to other techniques for accessing the L2 observables when A-S is on.

The Trimble 4000 SLD and Minimac 2816 used the non-classified C/A code and code-correlation to access the L1 pseudo-range and L1 carrier phase. These receivers then employed a 'squaring technique' to access the L2 carrier phase. As the C/A and P codes consist of an 'apparently' random series of +1 and -1's modulations, these binary digits can be removed by squaring or multipling the incoming signal with itself in order to produce a constant stream of +1 modulations. The remaining signal which has twice the original carrier frequency, can now be measured. Squaring the signal halves the wavelength and decreases the signal to noise ratio. This makes it more difficult to correct cycle slips, which can be either full or half cycles, and hinders ambiguity resolution. The Trimble 4000 SST and Ashtech MDXII receivers used code-correlation when A-S is off to access all four observables, and resort to squaring the L2 carrier phase when A-S is on.

The latest generation of receivers uses code-correlation when A-S is off to access all four observables and use new techniques to measure P code L2 pseudo-ranges and L2 carrier phase. Cross-correlation receivers are a fairly new type that have only been on the market for the past few years. This technique was originally developed by JPL for the Turbo Rogue receiver and has since been sold to Trimble for their 4000 SSE receiver. When A-S is on they measure the L1 pseudo-ranges and carrier phase using the C/A code (code-correlation) and measure the difference between the L1 and L2 Y-codes. Since the P and W codes are the same on both frequencies this difference is equivalent to the P1-P2 pseudo-ranges. This can then be added to the L1 C/A code pseudo-range to produce an L2 pseudo-range, and a full wavelength L2 carrier phase measurement.

Code Correlation Squaring has been developed by Magnavox *[Hatch, 1992]* and has been used in the Leica 200 GPS receiver. It uses a narrow band width to compare the incoming Y-code with a receiver generated P code to produce a P code measurement. However, this technique results in a half wavelength L2 carrier phase measurement.

P-W tracking is the newest technique and has been developed by Ashtech. However, the receivers are not yet available and no results have been published. This technique claims to be able to produce a W code which is removed from the P+W code to leave a P code signal and full wavelength L2 carrier observable.

2.1.2 Using the Pseudo-Range Observable

Instantaneous navigation is accomplished by use of the timing codes (C/A or P) and the navigation message. GPS receivers compare the received timing codes with a replica code generated within the receiver. These two codes will be out of alignment, and the amount of movement required to align them (in seconds) is equivalent to the travel time between the satellite and the receiver. When this is scaled by the speed of light, it is effectively the range between the satellite and the receiver. However, neither the receiver or satellite clock are perfectly synchronised to the GPS time frame and the range is contaminated by the atmospheric and measurement errors. Therefore this range is known as a 'pseudo-range'. This is the fundamental measurement for GPS navigation and absolute positioning.

Pseudo-range measurements can be taken on either the C/A or P code signals. Due to the length of the PRN codes the P code pseudo-range will be an absolute measurement, whereas a C/A code pseudo-range will have a 300 km ambiguity. This ambiguity is not usually a problem as most users know their position to better than 300 km. In terms of distance the pseudo-range observable can be expressed as

$$R = \rho + (\Delta t^S - \Delta t_A)\, c + \text{Errors} \tag{2.1}$$

where

R	= pseudo-range between satellite s and receiver A (m)
ρ	= geometrical range between the satellite and receiver (m)
Δt^S	= satellite clock offset from GPS time frame (s) (contained in broadcast ephemeris as polynomial coefficients)
Δt_A	= receiver clock offset from GPS time frame (s)
c	= speed of light (m/s)
Errors	= atmospheric and measurement errors

Using three simultaneously observed pseudo-ranges, a basic trilateration calculation will yield a three-dimensional positional fix, provided the satellites and ground antenna are not co-planar. However, this neglects the receiver clock offset which can corrupt the solution by thousands of metres. To solve for the receiver clock offset a fourth pseudo-range is needed. If observations to more than four satellites are available, a least squares approach is used to include the redundant observations.

During the GPS engineering test phase, instantaneous absolute positions to an accuracy of approximately 15 m 2drms[1] could be obtained using the Standard Positioning Service (SPS). However, current US DoD policy has been to reduce the accuracy of the SPS to 100 m, whilst maintaining an accuracy of approximately 15 m 2drms[1] for the Precise Positioning Service (PPS), available for military navigation only *[FRP, 1992]*. This degradation and the encryption of the P code is known as Selective Availability (SA), a policy to deny unauthorised use of the full accuracy PPS.

[1] 2drms : twice the distance root mean square accuracy, with a 95% probability

SA is achieved by a combination of two effects. Firstly, *epsilon* which involves altering the Keplerian terms in the broadcast ephemeris and secondly, *dither* which involves upsetting the short term stability of the satellite oscillator. These have the combined effects of introducing rapidly changing errors into the observed pseudo-ranges. SA was first introduced in 1990 and since then the level of SA has been turned up and down intermitently.

In order to obtain higher accuracies for navigation, the 'differential GPS technique' (DGPS) is used (see Figure 2.2). In DGPS the coordinates of a reference station are known and the technique is based on the principle that any errors in the instantaneous absolute position of the reference station, due to errors in the broadcast satellite ephemerides, atmospheric effects and SA, will also be present as errors in the instantaneous absolute position of the user. The reference station, therefore, compares its known position with its instantaneous, computed position and transmits a series of corrections to the user. These corrections are in the form of corrections to pseudo-ranges and are transmitted via a real-time link, usually using a communications satellite such as those of INMARSAT. DGPS enables the user to determine an instantaneous relative position to an accuracy of approximately 5 - 7 m up to 500 kilometres *[Blanchard, 1990]*.

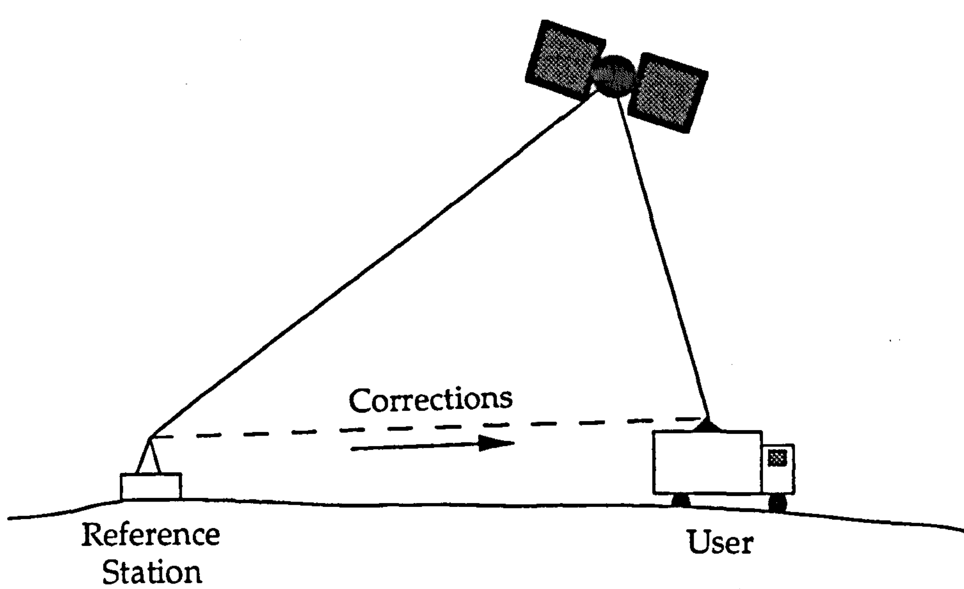


Figure 2.2 Differential GPS

The main limitation of DGPS is that as the user travels away from the reference station the corrections become less and less valid. A costly solution to this problem could be resolved by increasing the number of reference stations. However, recent research has led to the development of Wide Area Differential GPS (WADGPS) to overcome this limitation of DGPS. This involves separating the combined differential correction into its component parts (orbit, atmosphere etc) and dealing with these separately. Results have shown that fewer reference stations are required, and that there is no correlation between accuracies achieved and distance to the reference station *[Ashkenazi et al, 1992d]*. Tests at Nottingham have shown that accuracies of 2-5 m can be achieved using WADGPS *[Ochieng, 1993]*.

If the user is static and able to record data over several days the errors due to SA can be reduced by simply averaging to produce an accumulated single point pseudo-range position. Appendix A shows results using this technique to determine the WGS84 coordinates of several 'known' European SLR and VLBI stations. Accuracies of 1 to 2 metres were achieved after 2 to 3 days, which is quite remarkable considering WGS84 itself is only accurate to 1 to 2 metres. This provides a very cheap and computationally simple technique to determine WGS84 station coordinates in remote and unsurveyed areas of the world.

2.1.3 Using the Carrier Phase Observable

The pseudo-range observable has proven itself to be both robust and versatile, however, this is countered by its low observational accuracy, which is sufficient for navigation purposes. However, for geodetic and surveying applications positioning to much higher accuracies is required. In these cases 'relative positions' can be determined to a high accuracy by using two, or more, geodetic GPS receivers to make simultaneous measurements on the 'phase' of the carrier wave, rather than using the timing codes.

A geodetic GPS receiver measures the phase of the carrier wave using one of the techniques described earlier in section 2.1.1.3. Receivers are able to obtain a very precise measure of the fractional part of the phase, typically to a few millimetres and to keep a count of the integer number of cycles which have been added (or subtracted), as the satellite - receiver range changes, since it first 'locked' onto the signal. However, a receiver has no knowledge of the initial number of wavelengths between the satellite and the receiver. This is analogous to a one-armed clock, where it is possible to read the number of minutes, but not the number of hours. Consequently, all the phase measurements for a particular satellite / receiver combination include the same 'integer ambiguity', which corresponds to the unknown number of integer wavelengths at lock-on, and is usually solved for as one of the unknowns in a least squares adjustment.

Although the carrier phase gives a precise measure of the ranges from the satellites to the receiver, the accuracy of an absolute position computed from these ranges would be significantly degraded by errors in the satellite ephemerides, errors in the models used for atmospheric refraction, and errors in the receiver and satellite clocks. The effect of all these errors can be substantially reduced if the phase data is used to determine relative positions by using interferometric (differencing) techniques, cancelling errors that are common at the receiver and/or satellite. Although many 'differencing' combinations are possible, between receivers, satellites or epochs, forming a multitude of single, double and triple differences the preferred technique used at Nottingham is the 'double difference' observable. This reduces the effect of the aforementioned errors, as well as allowing the 'double difference' integer ambiguities to be resolved. However, before any differencing can be performed the satellite and receiver clocks must be corrected to be in the same time frame ie GPS. For the satellites this information is contained in the Broadcast Ephemeris and for the receivers the pseudoranges must be used to determine the receiver clock offsets *[Yau, 1986]*.

The observable output by most geodetic GPS receivers is the **Pure Phase Observable**. This is obtained by measuring the phase of the signal arriving from the satellite, relative to the phase of the replica signal generated by the receiver. This can be expressed as

$$\phi_A^S(\tau) = \phi^S(t) - \phi_A(\tau) + N_A^S \tag{2.2}$$

where $\phi_A^s(\tau)$ is the phase measurement on a carrier signal emitted from satellite s at receiver A at receipt time τ, $\phi^s(t)$ is the phase of the carrier signal leaving the satellite at time t, $\phi_A(\tau)$ is the receiver generated phase at time τ and N_A^s is the number of whole cycles between receiver *A* and satellite *s* at 'lock on'. The relationship between the satellite (t) and receiver (τ) time frames can be expressed in terms of the geometrical range (ρ_A^s)

$$\tau - t = \frac{\rho_A^s}{c} \tag{2.3}$$

where c is the speed of light in a vacuum.

Hence, the phase of the signal leaving the satellite at time τ can be expressed as

$$\phi^s(t) = \phi^s\left(\tau - \frac{\rho_A^s(t)}{c}\right) \tag{2.4}$$

This can be expanded using Taylor's theorm and ignoring any higher order terms gives

$$\phi^s(t) = \phi^s(\tau) - \frac{f}{c}\left(\rho_A^s(t)\right) \tag{2.5}$$

where f is the frequency of the observable.

The complete pure phase observable can, therefore, be expressed as

$$\phi_A^s(\tau) = \phi^s(\tau) - \frac{f}{c}\left(\rho_A^s(t)\right) - \phi_A(\tau) + N_A^s \tag{2.6}$$

The terms $\phi^s(\tau)$ and $\phi_A(\tau)$ are influenced by the behaviour of the satellite and receiver clocks respectively, and if not corrected can lead to errors of hundreds of kilometres in the station coordinates. These are usually eliminated using the following differences.

The Single Difference Observable is the difference between two pure carrier phase measurements. Conventionally this is performed between a common receiver or satellite but can be performed between two epochs. The single difference between two receivers A and B and a satellite s is expressed as

$$\phi^{s}_{AB}(\tau) = \phi^{s}_{B}(\tau) - \phi^{s}_{A}(\tau) = \phi_{AB}(\tau) - \frac{f}{c}\left(\rho^{s}_{AB}(t)\right) + N^{s}_{AB} \tag{2.7}$$

where $\phi_{AB} = \phi_B - \phi_A$, $\rho^{s}_{AB} = \rho^{s}_{B} - \rho^{s}_{A}$ and $N^{s}_{AB} = N^{s}_{B} - N^{s}_{A}$

This has eliminated the satellite clock induced errors as well as most of the effects of SA, and for short baselines the atmospheric delay errors. However, this observable is still affected by receiver clock errors.

The Double Difference Observable is the difference between two single difference phase observables. The double difference used at Nottingham is between two satellites s and t and two receivers A and B (see Figure 2.3), and is expressed as

$$\phi^{st}_{AB}(\tau) = \phi^{t}_{AB}(\tau) - \phi^{s}_{AB}(\tau) = -\frac{f}{c}\left(\rho^{st}_{AB}(t)\right) + N^{st}_{AB} \tag{2.8}$$

where $\rho^{st}_{AB} = \rho^{t}_{AB} - \rho^{s}_{AB}$ and $N^{st}_{AB} = N^{t}_{AB} + N^{s}_{AB}$

Since the observations to satellites s and t were taken simultaneously this difference totally eliminates the receiver clock terms. The ρ^{st}_{AB} term is related to the unknown station coordinates and the double difference integer ambiguity is determined as part of the least squares solution.

The Triple Difference Observable is between two double difference observables at two different epochs. This eliminates the carrier phase integer ambiguity leaving the station coordinates as the only unknowns. However, the increased noise of this observable from multiple differencing degrades the solution accuracy beyond any benefit gained from eliminating the integer ambiguity. This observable is commonly used for the detection and correction of cycle slips.

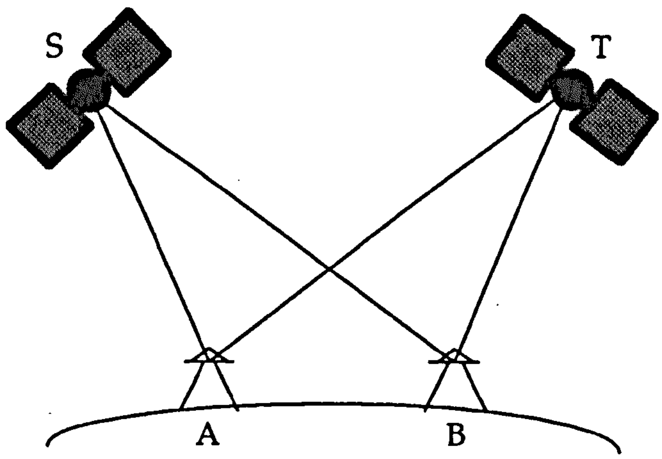


Figure 2.3 The Double Difference Observable

The L1 and L2 observables can be combined in many ways to produce observables of differing wavelengths. The most commonly used combinations are the Wide Lane (86cm wavelength) and the Narrow Lane (19cm). The use of the Wide Lane observable with the double difference algorithm leads to easier resolution of the integer ambiguities and can be used as a intermediate step towards the resolution of the L1 and L2 ambiguities (see section 2.2.5).

2.1.4 Least Squares for GPS

The processing of GPS data generally involves many more observations than unknowns, ie it is over-determined. Since there is no reason to assume that any one observation is better or worse than any other, the process of least squares is used to estimate the unknown parameters which are the coordinates of the receivers, the integer ambiguities and other terms relating to the orbits and atmosphere. The least squares principle has been well documented and is therefore not explained here *[Cross, 1983]*.

2.1.5 Conventional GPS Surveying

For baselines of less than 10 km, it is sufficient to assume that the signal from a particular satellite to the two receivers is passing through the same atmosphere, and hence any errors in the atmospheric models are cancelled by the double difference technique. In this case the 'single frequency' L1 carrier phase observable can be used. For baselines greater than 10 km the effect of the upper atmosphere (ionosphere) has to be accounted for by using the 'dual frequency' L1/L2 observable, whereby a linear combination of L1 and L2 carrier phase is used to remove practically all ionospheric effects. Unfortunately, the same dual frequency approach cannot be used to remove the effects of the lower atmosphere (troposphere). This requires a model, either empirical or based on local meteorological conditions. These calculate a zenithal delay which is mapped down to the elevation angle required.

Two further potential sources of error in carrier phase measurements, which are not reduced by double differencing, are 'multipath' and 'cycle slips'. Multipath is caused by the satellite signal being reflected by a surface, such as the side of a building, before it reaches the antenna. This results in the signal, which has taken the direct path between the satellite and the receiver, being interfered with by the reflected signal. In some cases, the combined signal may not be decoded by the receiver. However, the usual effect is that the reflected signal appears as noise on the true signal, and degrades the accuracy of the resulting vector solution.

A cycle slip, or loss of lock, appears as a jump in the carrier phase data and occurs as a result of the receiver losing lock on the carrier wave. Cycle slips may be caused either by a physical barrier between the satellite and the receiver, excessive atmospheric disruption, interference from radio sources or by the receiver being used in a highly dynamic environment. The result of this is that the receiver loses its initial integer ambiguity and re-acquires a 'second integer ambiguity'. The difference between these two ambiguities is an arbitrary integer value which may be of the order of several millions of cycles, and the magnitude of the cycle slip is the number of integer cycles which must be added (or subtracted) in order to relate the second integer ambiguity back to the initial integer ambiguity. To produce precise relative vector solutions, it is essential that all cycle slips are detected and correctly 'repaired' before the data is processed.

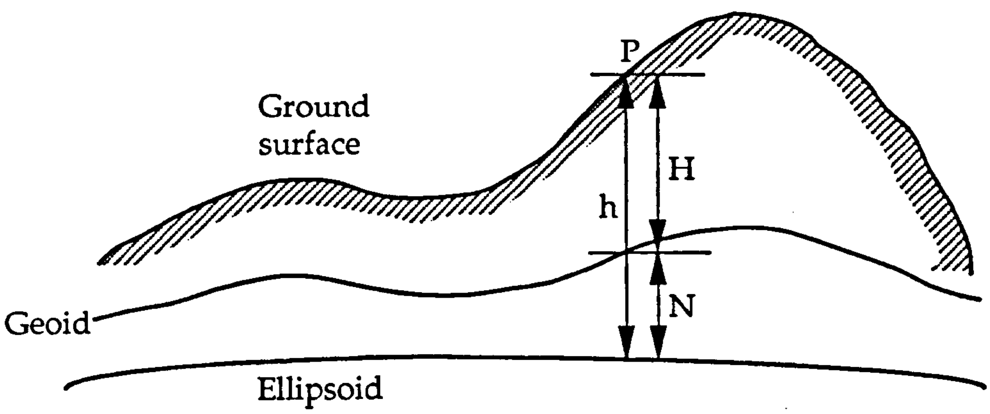
The 'conventional' use of GPS on short baselines (a few kilometres) requires at least two receivers to take data for at least 30 minutes, in order to resolve the integer ambiguities. This is generally known as 'static GPS surveying', and the amount (time) of data required to resolve the integer ambiguities is dependent on the length of the baseline and the number of satellites in the solution, because a change in their relative geometry is required. However, if these ambiguities can be determined in some other way, such as from a previously known baseline, then only a few phase measurements are required. This has led to 'kinematic GPS surveying', which involves a 'fixed receiver' over a reference station and a 'roving receiver', which takes measurements at several new stations, needing only to remain at each station for a short time.

Further improvements in ambiguity resolution algorithms and receiver hardware have led to the recent development of On-The-Fly (OTF) ambiguity resolution techniques which enable the acquisition of the integer ambiguities even while a receiver is moving. Research on OTF continues today, but initial results are very encouraging *[Walsh, 1993]*. Kinematic GPS surveying has opened an even wider range of commercial surveying applications, with the potential to compete against theodolite and level based traditional surveying techniques. The relative accuracies which can be obtained by 'conventional' GPS surveying techniques and commercial software packages are of the order of 5-10 mm $\pm$ 1 - 2 ppm *[Ashkenazi and Bingley, 1992]*.

2.1.6 The Datum Problem

GPS produces station coordinates and coordinate differences in a cartesian coordinate reference system, ie it is a complete three-dimensional surveying system. The GPS cartesian coordinates produced using the Broadcast Ephemeris are with respect to the World Geodetic System 1984 (WGS84). It is a relatively simple procedure to transform these cartesian (X, Y, Z) coordinates into latitude, longitude and height (Φ, λ, h) above the WGS84 reference ellipsoid. However, if positioning information is required in any other coordinate reference system then transformation parameters relating the two systems are required. Often these are only poorly known and may vary from one area to another. This has become one of the biggest problems of using GPS.

Another problem is the use of ellipsoidal heights or height differences. These do not correspond to changes in gravity potential, and therefore do not describe the direction of flow of water. Orthometric height differences, or as they are more commonly referred (but not entirely correctly) heights above mean sea level, which are related to changes in gravity potential, are more useful. The problem of converting ellipsoidal heights (h), or height differences, into orthometric heights (H) is the need to know the difference between the geoid and the ellipsoid (N), commonly known as the geoid ellipsoid separation (see Figure 2.4). Over small areas (10 x 10 kilometres) where the geoid ellipsoid separation does not change significantly this can be easily modelled, but over larger areas the separation has to be determined.



$$H = h - N$$

Figure 2.4 The Relationship Between the Geoid and the Ellipsoid

The value of N is not unique at any particular point on the Earth but varies for each reference ellipsoid. For example, N for Aberdeen is 0 metres for OSGB70(SN), 6 metres for ED 50 and 50 metres for WGS84 *[Cross, 1991]*. It is for this reason that several organisations have produced contour diagrams, formulae or computer algorithms for the determination of N. Currently, the WGS84 geoid can determine globally N with a standard deviation of 2 to 6 metres and is claimed to have an accuracy such that for 93% of the Earth's surface N will have a standard deviation of less than 4 metres *[DMA, 1987]*. The largest value of N for the WGS 84 geoid is over 100 metres and 30% of the

Earth's surface has an N over 30 metres. Therefore even for navigation N can not be ignored. In the UK, where there is an extensive network of gravity data several high precision relative geoids have been calculated with a quoted precision of 1-2 ppm *[Stewart, 1990]*. However, until the accuracy of N can be determined to the same accuracy as h, orthometric levelling over large areas remains impossible with GPS.

2.1.7 Limitations of Conventional GPS Surveying

As described above, in GPS surveying, carrier phase measurements are usually processed by using the 'double difference' phase observable. This assumes that errors in the satellite orbits and atmospheric models are cancelled by the double differencing process. As baseline lengths increase, this assumption becomes less and less valid and, for very large networks, these errors tend to become dominant and begin to significantly reduce the accuracies attainable to about 1 ppm or worse. However, there are several geophysical and engineering applications which require much higher accuracies than 1 ppm over baselines that may be several hundred kilometres long. These applications use the so-called 'fiducial GPS technique' which is now described in section 2.2.

2.2 Fiducial GPS

One of the major error sources in conventional GPS surveying is the time tagged satellite coordinates calculated using the Broadcast or Precise ephemeris. They are only accurate to about 20 m (Broadcast) and 10 m (Precise) in each component, corresponding to 1 ppm and 0.5 ppm respectively of the range from satellite to receiver. This in turn causes relative positions of the stations on the ground to be determined to a similar order of accuracy. The fiducial GPS technique was developed to overcome these limitations, and to achieve accuracies ranging from 0.1 to 0.01 ppm over baselines several hundred of kilometres long. This section reviews the principles of the fiducial GPS technique, including orbit improvement, multi-day orbits, atmospheric modelling, ambiguity resolution, reference frame definition, and discusses its applications. The section is concluded with a description of the IESSG software, data processing and analysis strategies employed by the author.

2.2.1 The Technique

The fiducial GPS technique involves the recording of simultaneous carrier phase measurements at a number of fiducial stations, whose coordinates are known to a very high order of accuracy, and at a number of new stations whose coordinates are required. The former may be stations located at or near to Very Long Baseline Interferometry (VLBI) or Satellite Laser Ranging (SLR) facilities, or stations whose coordinates have been determined by Mobile VLBI (MVLBI), Mobile SLR (MSLR) or as part of a global GPS network. A fiducial GPS network is made up of a minimum of three fiducial stations and any number of new stations, which may be points of interest, or other points included in order to strengthen the network and assist in the integer fixing process.

The fiducial GPS principle is based on a least squares adjustment, in which the complete set of carrier phase measurements from all stations are adjusted simultaneously, holding at least two and a third fiducial station coordinates fixed and solving not only for corrections to the three-dimensional coordinates of the new stations, but also for corrections to the satellite orbital parameters. The resulting adjusted network of ground stations and satellite orbits is positioned, scaled and orientated to the reference frame defined by the adopted coordinates of the fixed fiducial stations, thus transferring the high relative positional accuracies of the fiducial stations, via the adjusted satellite orbits, to the new stations. However, the accuracy of the coordinates of the new stations cannot be determined to better than those of the fiducial stations.

2.2.2 Orbit Improvement and Network Adjustment

Broadly speaking, there are two distinct stages in the fiducial GPS technique, namely orbit determination and network adjustment. The orbit determination process involves the computation of a theoretical orbit for each of the satellites, based on a 'force model'. This is done by modelling all of the forces acting on the satellite, and thus obtaining its time dependent acceleration vector. The force model includes gravitational attractions (eg earth, moon, sun and planets), surface forces (eg solar radiation, atmospheric drag) and other perturbing influences (eg thrusts). This acceleration vector is then numerically integrated twice with respect to time, once to obtain the velocity vector and again to obtain the position vector. The theoretical orbit for each satellite is computed by integrating the corresponding acceleration vector, starting from an initial state vector, usually obtained from the broadcast ephemeris, or preferably from a precise ephemeris. The theoretical orbits are then improved (ie re-positioned whilst keeping, in broad terms, their overall individual shapes) during the network adjustment process by introducing the carrier phase measurements made at the 'known' fiducial stations and the 'unknown' new stations. The network adjustment leads to corrections for the initial state vectors of the satellites, the coordinates of the new stations, and some of the force model components of the satellites.

2.2.3 Multi-Day Orbits

If data from several adjacent days are processed simultaneously in a single adjustment then the satellite orbits can be treated slightly differently. The 'theoretical' orbit is computed as a single arc which revolves around the earth twice for each day of data used, as opposed to a single pass overhead. This multi-day arc is then repositioned as a whole during the network adjustment and because substantially more data, with a greater observation span, is used the resulting adjusted orbits are more precise. This, in turn, leads to more precise ground coordinates. It is also worth mentioning that orbit precision can be further improved by using a larger network of fiducial stations, up to a global network, to constrain the satellite orbits while they are not over the area of interest.

2.2.4 Atmospheric Modelling

Measurements of the GPS carrier phase are significantly biased by atmospheric effects and these errors become more dominant as the baseline lengths increase. Such biases can be separated into those originating from the ionosphere and those from the troposphere. The ionosphere is a layer of charged particals or ions in the upper atmosphere, the density of which is non-uniform and related to the level of solar activity. For the GPS signals the ionospheric delay is inversely proportional to the squared frequency of the carrier wave. Therefore, due to the large baseline lengths present in a fiducial GPS network, dual frequency carrier phase measurements are essential, as the effect of the ionosphere is modelled by using a combination of the L1 and L2 carrier phase measurements. This correction accounts for about 98% of ionospheric delay errors *[Dodson et al, 1991]*.

Unfortunately, the delays occurring in the troposphere cannot be as easily removed, and any residual errors remaining will ultimately degrade the fiducial GPS network solution. The troposphere is the neutral part of the atmosphere between the Earth and Ionosphere containing the weather systems. The tropospheric delay depends upon the geometrical path length through the troposphere and is therefore a function of the elevation angle from receiver to satellite. The estimation of the delay at a given elevation is based on the value at the zenith combined with a simple mapping function. The tropospheric delay can be divided into two components, firstly the 'dry' component which is a function of the temperature and pressure and accounts for 80-90% of the total delay and, secondly the 'wet' component which is a function of the distribution of liquid water and water vapour in the atmosphere and therefore harder to model.

In conventional (non-fiducial) GPS surveying, tropospheric delay errors are modelled either by using a standard model for the atmosphere, or by using observed surface meteorological data. However, for a fiducial GPS network neither of these two methods is sufficiently accurate. Standard atmospheric models provide a broad approximation of expected tropospheric effects, but ignore the actual weather conditions. Added to this is the difficulty that observed surface meteorological data is plagued with calibration problems, and does not necessarily represent the conditions prevailing in the upper troposphere. These problems can be overcome by applying a standard atmospheric model, such as Saastamoinen's *[Saastamoinen, 1973]*, or Magnet *[Curley, 1988]* and then solving for 'scale factors' to account for the estimated 5 to 10 % error in the modelled tropospheric delay correction. These scale factors are solved as unknown parameters during the network adjustment, eg by modelling for a single (constant) or time varying (polynomial) scale factor for every station in the network. This has the effect of removing the average residual tropospheric error from the measurements, leading to significantly improved results *[Dodson et al, 1991]*.

2.2.5 Ambiguity Resolution

It is widely accepted that improved plan coordinates can be obtained by 'fine-tuning' the fiducial GPS network solution by 'fixing' the carrier phase integer ambiguities to their true integer values. However, the ionospherically free (L1/L2) observable, which is used to eliminate most of the effects of the ionospheric delay, does not have ambiguities that are integer in nature and cannot be resolved directly. Therefore, in order to obtain a solution which uses the ionospherically free observable and permits integer fixing a two stage process is employed. Firstly, a fiducial network solution is computed using the wide lane (L1-L2) observable which, since it has a wavelength of 86 centimetres, enables relatively easy resolution of the integer ambiguities. The second stage uses the wide lane integer ambiguities to enable resolution of both L1 and L2 integer ambiguities.

This is a sequential process, whereby the integers of the short baselines in the fiducial GPS network are resolved first. This leads to a strengthening of the network and an improvement in the satellite orbits which, in turn, allows the integers on successively longer baselines to be fixed as the orbits improve. In this context, however, it must be understood that fixing an integer to the wrong value is worse than not holding it fixed at all. This sequential process is often referred to as 'boot-strapping'. The effective resolution of the integer ambiguities depends greatly on the design of the fiducial GPS network, which must include a wide selection of baseline lengths, from about, say, 30 kilometres up to 2000 kilometres *[Blewitt, 1989; Dong and Bock, 1989]*.

2.2.6 Reference Frame Definition

The choice of reference framework and its treatment is crucial to the accuracies which can be achieved by a fiducial GPS network. In a free (no stations held fixed) fiducial adjustment the reference frame can be defined by the orbits, however, the resulting solution is ill-conditioned. Therefore, the reference frame needs to be defined by the adoption of fixed coordinate values for the fiducial stations. There are several global reference frameworks which can be used for this purpose. In particular, these include the various ITRF (International

Terrestrial Reference Frames) (see section 3.4.2), which are based on VLBI and SLR measurements and are defined by the adopted epochal coordinates of a network of stations from around the world. Theoretically, a reference frame need only be defined by a minimum of seven coordinate values, ie a minimum of three fiducial stations. It must be recognised, however, that any subset of three stations taken from a global reference framework will define a slightly different reference frame, from that of the whole reference framework.

It is therefore crucial to treat with great care the decision of which fiducial stations to hold fixed, and which coordinate values to assign to them. Furthermore, for high accuracy deformation monitoring, it must also be recognised that the fiducial stations themselves are moving with time. The correct treatment of station velocities or plate motion models is, therefore, also essential in order to determine the correct time-tagged coordinates of the fiducial stations at the observation epoch.

2.2.7 Applications

The two main applications of fiducial GPS are the establishment of new geodetic networks, and the monitoring of small geophysical or engineering deformations. In both of these applications the transfer of a reference frame via the 'released' satellite orbits is crucial. In the first case, the new geodetic network is required in a particular reference frame, such as EUREF in ETRF 89. In the second case, the use of a reference frame is essential in order to ensure consistency between successive observation epochs.

In order to monitor small geophysical or engineering deformations, coordinates of a high order of accuracy are required so that a confident assessment of any deformations can be made. For small scale engineering structures, such as dams and reservoirs, monitoring can be performed 'relative' to an assumed stable point, close to the structure. On such a small scale the use of a precise ephemeris would provide the accuracy required. However, for larger scale engineering applications, such as offshore platforms, and geophysical applications, such as crustal deformations and tide gauge heights, measurements over much longer baselines are necessary. For 'short-term' deformation surveys, which may only be required over a duration of, say, one year, it may be sufficient to assume that the movement of fiducial stations within a consistent reference frame is negligible. For 'long-term' deformation surveys, which span several years, it is necessary to monitor the movement of fiducial stations themselves. This can be achieved by the adoption of coordinates from periodic global solutions by VLBI, SLR, or global GPS.

Fiducial GPS, therefore, enables the monitoring of 'absolute' deformations (ie in a global reference frame) to a very high order of accuracy. Furthermore, for 'long-term' deformation surveys the use of VLBI, SLR or Global GPS derived coordinates provides the '1st order deformation monitoring' required. The use of fiducial GPS for the establishment of a new geodetic network is described in chapter 4 and for 'long-term' deformation monitoring in chapter 5.

2.2.8 IESSG GPS Software and Data Processing

The IESSG is one of several geodetic research centres where a fiducial GPS software package has been developed. The IESSG package used by the author consisted of five main programs; FILTER-1, PSEUDO, ORBIT-1, PANIC-1 and CARNET, and several ancillary programs to enable 'cycle slip editing' and for post-adjustment analysis. All of this software had been written in-house by a succession of post-graduate research students over the ten year period 1980 to 1990. The author is indebted to all previous and present research students for their contribution to the IESSG software package and to the inherent bugs which have kept the author amused for many hours and in some cases weeks. The programs were written in FORTRAN 77 and were run either interactively or by batch processing on the University's ICL 3900 VME mainframe computer. Since 1991, many of these programs have been re-written and now form part of GAS (GPS Analysis Software) a commercially available package. The software and data processing described in this section is that used by the author during the past three years.

The main computational stages and programs involved in the processing of a Fiducial GPS network were

(1) Data filtering and Reformatting - FILTER-1
(2) Pseudo-range Point Positioning - PSEUDO

(3) Cycle Slip Correction - PANIC-1
(4) Data Selection - FILTER-1
(5) Orbit Determination - ORBIT-1
(6) Network Adjustment - PANIC-1
(7) Combination of Arc Solutions - CARNET
(8) Quality Assessment - SLRTRANS and COORDDIFF

2.2.8.1 Data Filtering and Reformatting - FILTER-1

Prior to data filtering the receiver specific binary data is decoded into the RINEX format *[Gurtner et al ,1989a]* using manufacturers software. FILTER-1 was used to convert the RINEX format data into NOTTM1 format, and filter out any unwanted data. The program was run from a control file which enables the filtering of a complete day of network observations, up to 20 stations, in a few minutes.

The complete list of operations performed by FILTER-1 are:

(i) Convert RINEX format data into NOTTM1 format.
(ii) Remove observations from unwanted satellites.
(iii) Re-order the observations at each epoch, such that the satellite IDs ascend numerically, to allow faster data access during processing.
(iv) Remove epochs containing fewer than a specified number of observations. A minimum of two observations are required at each epoch for double difference processing.
(v) Remove satellites below a specified elevation angle (usually 15 degrees).
(vi) Remove noisy pseudo-range observations.
(vii) 'Repair' large L1 cycle slips using the 'delta pseudo-range' technique, by comparing the difference in phase with the difference in pseudo-range at successive epochs.

2.2.8.2 Pseudo-range Point Positioning - PSEUDO

The purpose of PSEUDO was to provide the approximate coordinates required for use by the double difference adjustment program PANIC-1. PSEUDO used the pseudo-range observations to compute the receivers absolute XYZ cartesian coordinates, and as a by-product the receiver clock offset per epoch. Since the receivers were static, the data from every epoch was combined in a single accumulated least squares adjustment. The program iterates from the initial coordinates input until no further movement occurs.

2.2.8.3 Cycle Slip Correction - PANIC-1

Cycle slip correction is carried out in a double difference sense using PANIC-1 to perform a single baseline solution. A baseline was defined by a 'fixed' station, assumed to have 'clean' phase data, and a 'free' station, assumed to have phase data contaminated by cycle slips. The satellite positions were held fixed to the GPS broadcast ephemeris, or, for longer baselines, a precise ephemeris. The baselines used in cycle slip editing were the baselines which were defined in the network adjustment. For an n station network, (n - 1) baselines had to be defined. Baseline definition was performed by minimising the lengths of the baselines, whilst attempting to maximise the amount of double difference data available.

The strategy employed in cycle slip correction was to clean the free station with respect to the fixed station, and then the free station can become a fixed station in subsequent single baseline solutions. However, subsequent stations are not clean in an 'absolute' sense, but only relative to the first station. Therefore, if this station contains any cycle slips these will be propagated through the network. This double difference clean data is not clean in a single difference or pure phase sense and cannot be used in other software.

The cycle slip correction process consists of firstly correcting small L1 cycle slips, then correcting large L2 cycle slips, and finally correcting small L2 cycle slips. Small cycle slips (1 - 10 cycles) were detected as 'jumps' in the double difference residuals output in the PANIC-1 single baseline solution. In practice, these were corrected by an interactive program, XXSLIP, which corrected the free station phase data. The corrected free station phase data was then input into PANIC-1 and another single baseline solution performed. This procedure was repeated until the free station phase data was clean, when the residuals output from PANIC-1 were 'smooth'. When the L1 phase data was clean, large L2 cycle slips were repaired by the program L2SLIP, which compared the difference in L1 phase with the difference in L2 phase at successive epochs. Small L2 cycle slips were then repaired in the same manner as small L1 cycle slips. It is common for there to be more cycle slips on the L2 frequency due to its lower signal to noise ratio.

On baselines greater than 10 km, some slips present on the L1 or L2 may not be detectable because of the noise levels on these frequencies. Therefore, it is also necessary to detect and correct slips using the dual frequency ionospherically free (L1/L2) and the widelane (L1-L2) observables. These are more sensitive than the single frequency solutions over longer lines where atmospheric effects will become more significant. However, as the baseline length increases to hundreds of kilometres the noise level increases to produce rapidly changing residuals which can look like cycle slips. In this case the decision of whether or not a slip is present is left to the discretion, judgement and experience of the user.

Two further problems that hinder cycle slip correction are gaps and half cycle slips. A large gap in the data may cause an apparent slip due to the gradual drift of the residuals. These slips often disappear when using dual frequency combinations but if they do not their correction is again left to the judgement of the user. Half cycle slips occur when using receivers that square the L2 carrier phase. These slips are often indistinguishable from noise and can propagate through the network before they are detected. Again their correction is left to the discretion of the user.

When cycle slip correction is complete for a particular network it is useful to perform a 'preliminary network solution', where the fiducial stations are held fixed, and the satellite orbits are held fixed to the GPS broadcast ephemeris, or preferably a precise ephemeris. This solution will indicate the presence of any remaining cycle slips in the network, in order that they can be resolved prior to the fiducial GPS data processing stage.

Cycle slip correction is probably the most laborious and time consuming task involved in fiducial GPS data processing. It is essential that all cycle slips are detected and repaired before the data can be processed in a network adjustment.

2.2.8.4 Data Selection - FILTER-1

Fiducial data sets are usually observed with an epoch separation of 15 or 30 seconds. This is imperative for cycle slip correction, particularly during periods of rapidly varying atmospheric conditions. However, for the network adjustment process it is the span of data and corresponding change in the satellite - station geometry that are important, rather than the number of observations processed. Therefore, the data sets are 'thinned' to reduce the quantity of data and the computational requirements, whilst maintaining sufficient redundancy in the solution. This was done using the program FILTER-1 which thinned the data to a user defined epoch separation that was a multiple of the original epoch separation, usually 1 or 2 minutes.

2.2.8.5 Orbit Determination Program - ORBIT-1

For fiducial GPS data processing, satellite orbits were predicted using the orbit determination program ORBIT-1, which is part of the in-house developed SODAPOP package (Satellite Orbit Determination and Analysis Package Of Programs) *[Hill, 1989]*. ORBIT was initially developed for determining the orbit of LAGEOS, the satellite used in SLR *[Moore, 1987]*, and has since been adapted for computing the orbits of GPS satellites *[Whalley, 1990]*.

ORBIT-1 produces individual satellite orbits for a specified time span, and also produces the partial derivatives which are required for solving the orbital unknowns in the network adjustment program PANIC-1. This is done by modelling all the forces which act on a satellite and using this acceleration model in a double integration procedure to calculate the position of the satellite as a function of time. ORBIT-1 required as input a satellite state vector, which consists of eight orbital parameters; three-dimensional position, three-dimensional velocity, a direct solar radiation pressure coefficient and a Y-bias acceleration.

To produce the 'initial integrated orbit' the satellite's position and velocity are taken from the GPS broadcast ephemeris, or preferably a precise ephemeris, and the solar radiation pressure coefficient and Y-bias are given approximate values of 1.5 and 0.0 respectively. To produce subsequent 'new integrated orbits', the satellite state vectors can be taken from the output of the network adjustment program PANIC-1.

2.2.8.6 Network Adjustment Program - PANIC-1

The IESSG network adjustment program, PANIC-1 (Program for the Adjustment of Networks using Interferometric Carrier phase), is based on explicit double difference carrier phase observations.

PANIC-1 can adjust a network of up to 20 stations and 9 satellites for a single day arc, or smaller networks for multi-day arcs, and is capable of estimating the following unknown parameters:

(i) three-dimensional coordinates of all the unknown stations,
(ii) up to 8 orbital parameters for each satellite,
(iii) tropospheric scale factors, and
(iv) carrier phase integer ambiguities.

PANIC is a highly flexible program containing a large number of options and models which may be selected by the operator. Some of the options, notably the choice of observables, which are combinations of the L1 and L2 phase measurements, are especially useful with fiducial GPS. Other models include, the correction of receiver time-tag errors, tropospheric delays with or without constant or time-varying scaling parameters, satellite clock drifts, the correction of antenna phase centre variations, and the effect of earth body tides on the coordinates of the ground stations. Full details of the options available in PANIC-1 can be found in *[Ffoulkes-Jones, 1990]*.

If the resulting ambiguities are integer by nature, then a sequential least squares algorithm may be invoked to resolve the ambiguities to their correct integer values using a 99.9% statistical confidence test *[Blewitt, 1989]*. Each ambiguity is scanned in turn and the real valued estimate nearest to an integer is fixed, provided that the statistical probability is sufficiently high. This is performed using a sequential least squares algorithm, which updates the remaining ambiguities and covariance matrix. This results in some of the unfixed ambiguities gaining values that can subsequently be fixed and this iteration continues until no more ambiguities can be fixed, whilst maintaining the confidence level for the statistical probability.

2.2.8.7 Fiducial GPS Data Processing Strategy

Once the data has been cleaned of cycle slips, the 'fiducial GPS data processing' can begin. Outlined below is the strategy used by the author for obtaining a fiducial network solution, for a particular multi-day arc:

(1) Generate an initial integrated orbit using ORBIT.
Input state vectors derived from the GPS broadcast ephemeris or NGS precise ephemeris.

(2) Perform an L1/L2, ionospherically free, network adjustment using PANIC.
Fix a minimum of three fiducial stations.
Input the initial integrated orbit, and solve for x orbital parameters.
Solve for a constant or time-varying tropospheric scale factor per station, per day.

(3) Generate a new integrated orbit using ORBIT.
Input state vectors output from PANIC (2).

(4) Perform an L1/L2, ionospherically free, network adjustment using PANIC.
Fix a minimum of three fiducial stations.
Input the new integrated orbit, and solve for x orbital parameters.
Solve for a constant or time-varying tropospheric scale factor per station, per day.

Numbers 5 and 6 are only carried out if an integer fixed solution is required.

(5) Perform an L1 - L2, wide lane, network adjustment using PANIC.
Fix all stations.
Input the new integrated orbit, and solve for no orbital parameters.

Solve for a constant or time-varying tropospheric scale factor per baseline, per day.
Solve for the wide lane integer ambiguities.

(6) Perform an L1/Widelane, ionospherically free, network adjustment using PANIC.
Fix a minimum of three fiducial stations.
Input the new integrated orbit, and solve for x orbital parameters.
Input the wide lane integer ambiguities output from PANIC (5).
Solve for a constant or time-varying tropospheric scale factor per station, per day.
Solve for the L1 integer ambiguities.

The number of orbital parameters (x) to solve for depends on the length of the solution. Typical values of x, *[Whalley, 1990]*, are; six (position and velocity) for a single day arc solution, seven (position, velocity and direct solar radiation pressure coefficient) for a 2 day arc solution, and eight (position, velocity, direct solar radiation pressure coefficient and Y-bias acceleration) for a 3, or more, day arc solution.

2.2.8.8 Auxiliary Processing and Analysis

CARNET

CARNET (Cartesian Network) is a 3-d network adjustment program used to combine survey observations (baselines, positions, distance, horizontal angles, azimuths) using variance-covariance analysis *[Lowe, 1993]*. It was used by the author to combine network solutions output from PANIC-1, with full covariance, into a weighted mean coordinate set. In addition its post-adjustment analysis was used to detect problems with particular stations, such as incorrect antenna height on one or more days.

SLRTRANS

This program was originally taken from the SODAPOP suite *[Hill, 1989]*. It was used to compute both cartesian and geodetic coordinate differences between two coordinate sets before and after any systematic biases (Helmert Transformations) had been removed.

COORDDIFF

This program was originally written by Ffoulkes-Jones *[1990]* and modified by the author to calculate session to session differences in baseline components from a weighted mean.

2.3 Transit Doppler

The Transit Doppler Satellite System, also known as the US Navy Navigation Satellite System (NNSS), uses as the name implies, a position determination process based on the Doppler shift principle. It was developed for the US Navy to update Polaris submarine inertial systems, the idea being that when a satellite was expected a submarine could rise to just below the surface, so that the antenna was just above water, and track the satellite to determine its position. Transit Doppler became fully operational in 1964, and was made available to civilian users in 1967. It has since been used very successfully by both sea navigators, surveyors and geodesists. Geodetic applications include the control of systematic biases in geodetic networks, such as OS(SN)80 in the UK (see section 3.2.1.4), and the realisation of the WGS84 (see section 3.4.1.2). In addition Transit has been used as part of an international service to monitor Earth rotation parameters (see section 3.4.2).

The system consists of satellites in a circular polar 'birdcage' orbit at an altitude of 1100 km and with a period of 106 minutes. At present (September 1993) there is a constellation of 7 operational and 3 spare satellites, of which four are the original OSCAR type and six are the newer NOVA type. Each satellite contains a highly stable oscillator which is used to transmit two continuous signals, one nominally at 399.968 MHz and the other exactly 3/8 of this. These two frequencies are modulated with time markers every even minute of Universal Time (UT), according to the satellite clock, and the satellite navigation message. This message, consists of the broadcast ephemeris and corrections for each satellite clock's drift from UT, and is up-loaded every 12 or 24 hours for the OSCAR or NOVA satellites respectively.

Two types of ephemerides are available, the broadcast and the precise. The broadcast ephemeris consists of predicted values of the orbital parameters derived by using a geopotential model and the satellite tracking data obtained from the four Operational Network Stations (OPNET). It has a positional standard error of 20 metres and from 1975 has used the WGS72 reference frame and since 1989 the WGS84 reference frame. The precise ephemeris is post-computed using the data from 13 Tracking Network stations (TRANET) in the NSWC 9Z2 reference frame and has a positional standard error of 1 metre. The precise ephemeris is only available to military and authorised civilian users.

The Transit Doppler instrument receives the continuously transmitted signals as the satellite pass over the ground station and measures the integrated Doppler count over fixed time intervals. These Doppler measurements are then used to determine range rate observation equations and combined with the satellite positional information from the ephemeris to produce the absolute position of the receiver with respect to the coordinate system of the ephemeris.

Single point positioning using Transit is capable of achieving accuracies of 1 to 1.5 metres in each direction using Precise Ephemeris and 3 to 5 metres using the Broadcast Ephemeris. This would require 30 to 50 good satellite passes taken over 2 to 3 days. Accuracies can be improved if relative positioning is performed in the 'translocation mode'. This involves simultaneously observing at two or more stations and leads to a reduction of ephemeris and atmospheric errors. A further improvement can be achieved if the translocation data is processed in the 'orbit relaxation model'. This allows the satellite orbits to move slightly following a mathematical model. Field tests have shown that using the broadcast ephemeris in orbit relaxation mode, relative positioning accuracies of the order of 30 centimetres for baselines up to 200 kilometres can be achieved with observations taken over 2 to 3 days *[Ashkenazi et al, 1977]*.

However, Transit has been superseded by VLBI, SLR and in particular GPS, with the last Transit satellite being launched in 1988. The US Navy plans to terminate operation of the Transit system by 1996 *[FRP, 1992]*.

2.4 Global Navigation Satellite System (GLONASS)

A corresponding system to GPS, GLONASS (Global Navigation Satellite System) is under development in the USSR, and planned to be fully operational by 1995. Its space segment will consist of 21 satellites plus 3 spares, at an altitude of 19100 km, arranged in three circular orbits 120 degrees apart and with an inclination of 64.8 degrees. Each satellite transmits two carrier waves (nominally 1.6 and 1.25 GHz) and are used for time (pseudo-range), carrier phase and Doppler shift measurements. Receivers are currently being developed to integrate GPS and GLONASS and utilize 21 + 21 satellite constellation *[Blanchard, 1993]*.

2.5 Very Long Baseline Interferometry

In Very Long Baseline Interferometry (VLBI), networks of radio telescopes located thousands of kilometres apart simultaneously track radio signals from extragalactic sources. These signals are assumed to arrive at the Earth as plane wavefronts due to the extreme distance of the source. Each station records the microwave signals received in digital form on magnetic tape, along with precise time using a Hydrogen MASER frequency standard. Usually, 10 to 20 sources are observed for 10 minutes, several times per day. The data tapes are then sent to a correlator centre where they are replayed and each source signal processed to determine the differences in arrival times (delay) and the changes in delays with time (delay rates) between each pair of stations. These delays and delay rates are then input into a least squares adjustment to estimate the baseline vector components between the two stations and various other pertinent geophysical parameters. These parameters include the coordinates of radio sources (which define the celestial reference frame), polar motion (motion of the Earth's rotation axis with respect to a fixed axis), Universal Time (which describes variations in the orientation of the Earth), offsets between the station clocks and finally atmospheric refraction corrections.

To avoid singularity in the adjustment, the right ascension of one radio source and the X and Y coordinates of the pole at a single epoch are fixed to define the orientation, the coordinates of one station are fixed to define the origin and the scale is defined by the speed of light. This implicitly defines the reference frame and for consistency, VLBI processing centres have adopted the same values. However, these values are quoted at the epoch 1988.0 so to realise this reference frame all VLBI adjustments involve data at this epoch. The advantage of VLBI is that, unlike satellite based systems, it does not require the use of an inevitably imperfect gravity field, however, this means that it is insensitive to the origin (geocentre).

VLBI is currently one of the principal contributors to the International Earth Rotation Service (see section 3.4.2.4) providing Earth rotation parameters, radio source coordinates and inter-continental baseline vectors. The latest VLBI instruments can produce inter-continental baselines with a precision of 1-2 ppb in scale and 0.05 arc-seconds in orientation.

2.6 Satellite Laser Ranging

In Satellite Laser Ranging (SLR) very short pulses of light, less than one billionth of a second, generated by a laser are transmitted to retroreflectors on the surface of an artificial satellite. A high resolution interval timer is used to measure the round trip travel times, which are converted into distances from station to satellite using the speed of light. A series of such observations from a network of stations eventually makes it possible to estimate orbital parameters and perturbations of the satellite well enough to derive a precise ephemeris, which is then used as a reference frame for determining polar motion and universal time. SLR observations to the Laser Geodynamics Satellite (LAGEOS) are currently the most common and as well as being used to determine the parameters listed above can be used to estimate geocentric coordinates of tracking stations, which defines the geocentre of the Earth.

As with VLBI and any other three-dimensional network, the normal equations are singular with seven degrees of freedom and hence seven constraints must be applied. In a SLR adjustment, or any other satellite system, the origin is defined by the model of the earth's gravity field and can be fixed to the geocentre. The orientation is defined in either of two ways. Firstly, by fixing one longitude and adopting one value of polar motion or secondly, by fixing the latitude of two stations and the longitude of one station. Finally, the speed of light and the geocentric gravitational constant provide a constraint on the scale of the adjustment.

As with VLBI, SLR is also a principal contributor to the International Earth Rotation Service, providing earth rotation parameters and geocentric coordinates, defining the origin and scale for the annual realisations of the International Terrestrial Reference System (see section 3.4.2.5). The latest SLR instruments can provide geocentric coordinates to an accuracy of better than 10cm.

2.7 Lunar Laser Ranging

Lunar Laser Ranging (LLR) is identical in concept with SLR, except that the targets are arrays of retroreflectors placed on the Moon by United States Apollo astronauts and by unmanned probes from the Soviet Union. The extreme distance to the Moon (400,000 kilometres) makes it more difficult to reliably point the laser at the retroreflectors and even when they do hit their target, the return signal is very weak compared to SLR. The technical difficulties in producing stronger lasers have limited the development of LLR and today there are only 3 permanently operating stations. These are used to investigate the long term variations in the rate of rotation of the earth and the effects of the tidal coupling between the earth and the moon.

2.8 Global GPS Networks

Recent advances in GPS technology and processing techniques have led to the computation of two global GPS networks. The first, GPS IERS and Geodynamics Experiment 1991 (GIG 91), was set up by the Jet Propulsion Laboratory to assess the ability of GPS to provide global coordinates with centimetre-level accuracy, for the establishment of a global GPS reference frame and the monitoring of large scale geophysical movements such as global sea-level rise and post-glacial crustal rebound. It was observed in early 1991 using twenty-one globally distributed Rogue receivers and processed by fixing only one station, Westford (USA). Once significant biases had been removed the RMS differences in latitude, longitude and height between GIG 91 and ITRF 91 were at the 2 cm level *[Boucher and Altamimi, 1992]*. This showed that on a global scale GPS could achieve accuracies comparable with VLBI and SLR.

The success of GIG 91 led to the establishment of the International GPS Geodynamics Service Experiment 1992 (IGS 92) to provide a GPS service to support geodetic and geophysical research activities (Resolution No.5, XXth General assembly, IUGG, Vienna, Austria, August 1991). The service depends on a global network of permanent GPS tracking stations and provides products such as earth rotation parameters, precise orbits and station coordinates. The first campaign was observed over a three month period from June to September 1992 using approximately thirty GPS receivers distributed world-wide. Within this campaign there was a two week period of intense activity, known as Epoch 92, during which the network was densified with some two hundred receivers operating simultaneously to enable the precise orbits to be tested on a regional scale, and provide a readily available terrestrial reference frame for geodetic and geophysical activities.

The data from the IGS 92 campaign has been processed by seven centres and the results are given in *[Beutler and Brockmann, 1993]*. These show an agreement between the precise orbits from the processing centres of better than 1 metre, once significant biases had been removed due to the use of different reference frames, and a 1 cm RMS agreement between ITRF91 and a combined GPS solution from five processing centres. The results of the IGS 92 campaign further proved the capabilities of GPS to compete on a global scale with VLBI and SLR and validated the concept of providing a permanent GPS service.

This permanent service is due to start in January 1994, and to bridge the gap between the end of IGS 92 and this permanent service the IGS Pilot Service was established. This meant that from June 1992 there is an uninterrupted series of GPS orbits and Earth rotation parameters for geodesy and geophysics activities. The latest results form the Pilot service have shown an improvement in the agreement between the precise orbits to the 0.5 metre level.

2.9 The Future

The application of VLBI and SLR to geodesy and geophysics was initially restricted to measurements on a global scale, such as polar motion, earth rotation and inter-continental plate motion. The introduction of mobile VLBI and SLR enabled measurements on a continental scale, such as the monitoring of crustal dynamics or the densification of reference frames. However, their application was limited by their large costs, long observation periods (months) and restricted mobility.

Even though GPS has not yet been declared fully operational (September 1993), it is already competing with VLBI and SLR on a global scale , ie IGS. Its low cost and portability have also enabled GPS to be used for the measurement of dense continental and national networks in shorter observations periods (a few days), when combined with recent advances in the fiducial processing technique, described in chapter 5, it has produced results comparable with mobile VLBI and SLR systems.

On a more local scale, the use of GPS for mapping and engineering surveying has highlighted many problems with existing terrestrial geodetic networks. Firstly, the GPS observations are of a superior quality and secondly, the transformation parameters between the GPS datum and the terrestrial datum are usually only known to a poor level of accuracy. The existing terrestrial networks and these associated problems are described in Chapter 3 and the establishment of a new GPS based datum for Europe, to overcome these problems, is described in Chapter 4.

CHAPTER 3

Reference Systems and Coordinate Datums

In the past, little regard was given to the meaning of the terms used to describe a position. Latitude was latitude, longitude was longitude and that was the end of the matter. However, the positioning methods used and objectives of the positioning activity were so approximate that exact definitions were not necessary, and this caused no problems. Today the situation is rather different due to the advances in positioning systems and careful attention must be given to the terms that are used to describe a position. Coordinates must be expressed in a known reference system or coordinate datum and should be accompanied by an indication of their quality. The level of understanding of the reference system or coordinate datum is proportional to the accuracy required, but even when working to a few hundred metres, care must be taken.

There are many such reference systems or coordinate datums available. In the United Kingdom one might express coordinates on a National datum, eg OSGB36, OSGB70(SN) or OS(SN)80, on a Continental datum, eg for Europe ED50, ED79 or ED87, or in a Global system, eg WGS 84 (or one of its predecessors WGS 60, WGS 66 or WGS 72) or the 'relatively' new ITRS. This chapter reviews the National, Continental and Global reference systems or coordinate datums that can be used in the United Kingdom. It starts in section 3.1 by defining the difference between reference systems, reference frames and coordinate datums. Section 3.2 describes the National Datums used in Great Britain and the Continental Datums of Europe and North America are described in section 3.3. Global systems are covered in section 3.4 and the chapter is concluded in section 3.5.

3.1 Definitions

With the development of geodetic space techniques an associated new vocabulary has evolved, and has caused much confusion when combined with the old 'terms' used for classical triangulation networks. This section briefly defines two of these new 'terms,' reference system and reference frame, which the author has found have varied definitions, and their relationship to an old 'term', coordinate datum. These definitions are the authors interpretation and their definition applies only to this thesis.

A **Reference System** is a 'conceptual view' consisting of a three-dimensional frame, with origin and a vector defining scale and orientation. It includes models, algorithms and constants that contribute to the realisation of the system, and for high accuracy work it will also contain a kinematic model.

A **Reference Frame** is the 'physical realisation' of a Reference System at a specific epoch. This realisation can be achieved through the time-tagged coordinates of a number of ground stations or a fixed satellite ephemeris.

A **Datum** is a term associated with classical triangulation networks, it consists of an reference surface (normally an ellipsoid) with defined shape and origin as well as the coordinates of a number of ground stations. It is, therefore, a reference frame and also to a lesser extent a reference system.

3.2 National Datums used in Great Britain

In the past, the horizontal and vertical networks for Great Britain, as with most other countries, have always been treated separately. Horizontal positions were defined by projecting a real ground station onto the surface of a reference ellipsoid (or plane). The networks were computed using distance, horizontal angles or directions and azimuths, which required intervisibility between stations and therefore traditionally located on hill tops. Heights were defined above mean sea level, an approximation of the geoid, and measured using the technique of spirit levelling. The horizontal and vertical networks of Great Britain, which are both the responsibility of the Ordnance Survey of Great Britain, are described in sections 3.2.1 and 3.2.2 respectively.

3.2.1 Ordnance Survey Horizontal Networks

Since the establishment of the Ordnance Survey over two hundred years ago, two independent triangulations have been observed in Great Britain. The first was completed in the 19th Century and became known as the Principal Triangulation. Although a remarkable achievement for its time, the Principal Triangulation suffered from a number of serious defects particularly in its relationship with the secondary and tertiary triangulations. The Retriangulation was observed between 1936 and 1951, and the lower order triangulations adjusted onto it. The development of improved instrumentation has shown that due to the method of adjustment, the Retriangulation suffers from a variable scale. The desire to attain greater accuracy and precision has led to periodic readjustments of the Retriangulation, namely OSGB70(SN) and OS(SN)80. This section briefly describes the development of the Ordnance Survey horizontal network. The reader is referred to the following for more information *[OS, 1957; Ashkenazi et al, 1972; Ashkenazi et al, 1985; OS, 1991b]*.

3.2.1.1 The Principal Triangulation (Clarke 1850)

The first triangulation was started by Major-General William Roy in 1784 at the request of the Royal Society and was funded by the Board of Ordnance. The triangulation was justified by the need to obtain an accurate connection between the two observatories of Paris and London (Greenwich). By 1822 the triangulation had been extended to Scotland. The reason for this extension was to obtain a more precise knowledge of the shape and dimensions of the Earth, and as a by-product the framework obtained was used to control the production of a 'One Inch to One Mile' map. Stations were marked with two feet square stones with a one inch diameter hole in the centre. Initially the triangulation was adjusted on Bouger's ellipsoid and the method of computation depended on the route chosen along the interconnecting triangles. Thus, the coordinate values of the check base at Salisbury Plain differed from the computed by either a positive or negative amount depending on the triangles used to traverse from the main base at Hounslow Heath. The maximum discrepancy at Salisbury Plain was of the order of seven inches, which was a considerable achievement since the bases were measured using wooden and glass rods!

From 1805 to 1853, a large amount of triangulation had been accumulated in a somewhat piece-meal fashion to cover the whole of Great Britain. Through a process of selection and rejection from this huge mass of data, Clarke created what is known as the Principal Triangulation of Great Britain. He produced an interlocking network of well conditioned triangles. Unlike the previous triangulation the adjustment was based on statistical theory by the method of least squares in twenty-one separate, but not entirely independent figures (sixteen covering England, Scotland and Wales). The corrections obtained from the solution of one figure being substituted in the condition equations of adjoining figures and held fixed.

The magnitude of the average triangular misclosure was 2.8 seconds. The scale was fixed by the weighted mean of two bases at Salisbury Plain and Lough Foyle, and the origin and azimuth were derived from the Royal Greenwich Observatory, London, and used to position the Clarke 1850 ellipsoid. In 1960, the Lough Foyle base was re-measured using Electromagnetic Distance Measurement (EDM) to check the scale of the Principal Triangulation. The EDM value differed from the 1827-1828 original measurement by 1 part in 520 000.

The Principal Triangulation was carried out as a scientific project, and the lower order triangulations used to control mapping were never adjusted to it. Inconsistencies in the County Series Maps, therefore, became evident when crossing county boundaries. Furthermore, in 1935, since most of the Principal Triangulation stations had been lost it was decided to observe and compute a Retriangulation.

3.2.1.2 The Retriangulation of Great Britain (OSGB36)

The observation of the new triangulation began in 1936 and ended in 1953. This included nineteen hundred horizontal directions and seven Laplace azimuths. In order to avoid disturbing the latitudes and longitudes on published maps, eleven of the remaining Principal Triangulation stations were incorporated into the Retriangulation. The Retriangulation was then adjusted onto these eleven stations which provided orientation as well as a mean origin. Thus, sympathy with the Principal Triangulation was maintained, whilst preserving the internal consistency of the Retriangulation.

The Retriangulation, like the Principal Triangulation, was divided into computing figures. With the introduction of calculating machines more equations could be handled simultaneously, hence, seven computing figures covered England, Scotland and Wales (see Figure 3.1) instead of the sixteen employed by Clarke. Ireland was not included because of the introduction of Home Rule in 1922 when all surveying responsibilities had been handed over to Dublin and Belfast. By 1937, four of the figures had been computed using condition equations, the same method as had been used by Clarke one hundred years previously. The Retriangulation had been completed by 1951, with the three remaining figures being adjusted using variation of coordinates. The adjustment was performed on the Airy Spheroid and is known as Ordnance Survey of Great Britain 1936 (OSGB36).

As inconsistencies within the lower triangulation were one of the reasons for the Retriangulation, the second order networks were divided into blocks and adjusted, using least squares, within perimeter primary stations. Tertiary and lower order stations were established as required to control mapping programmes using semi-graphic techniques. In 1938 the Davidson Committee was set up to consider how the effectiveness of the Ordnance Survey could be improved to deal with the increased demand for accurate maps. It recommended that "a National Grid should be superimposed on all large scale plans, to provide one reference system for the whole of the country". This National Grid (Transverse Mercator) *[OS, 1991a]* is based on OSGB36, and although there have been later adjustments, OSGB36 is still the basis for Ordnance Survey mapping today (1994).

A comparison between the Principal Triangulation and the Retriangulation is shown in Figure 3.2. The Retriangulation was fitted by least squares for position, scale and azimuth to the Principal Triangulation at eleven common stations along the 'back bone' of England (adjustment figures 1 and 2 of Figure 3.1). Therefore, little discrepancy between the two triangulations can be found in this area. Elsewhere, the differences are of varying magnitude and display a marked regional correlation. The Orkney to Shetland chain, Western

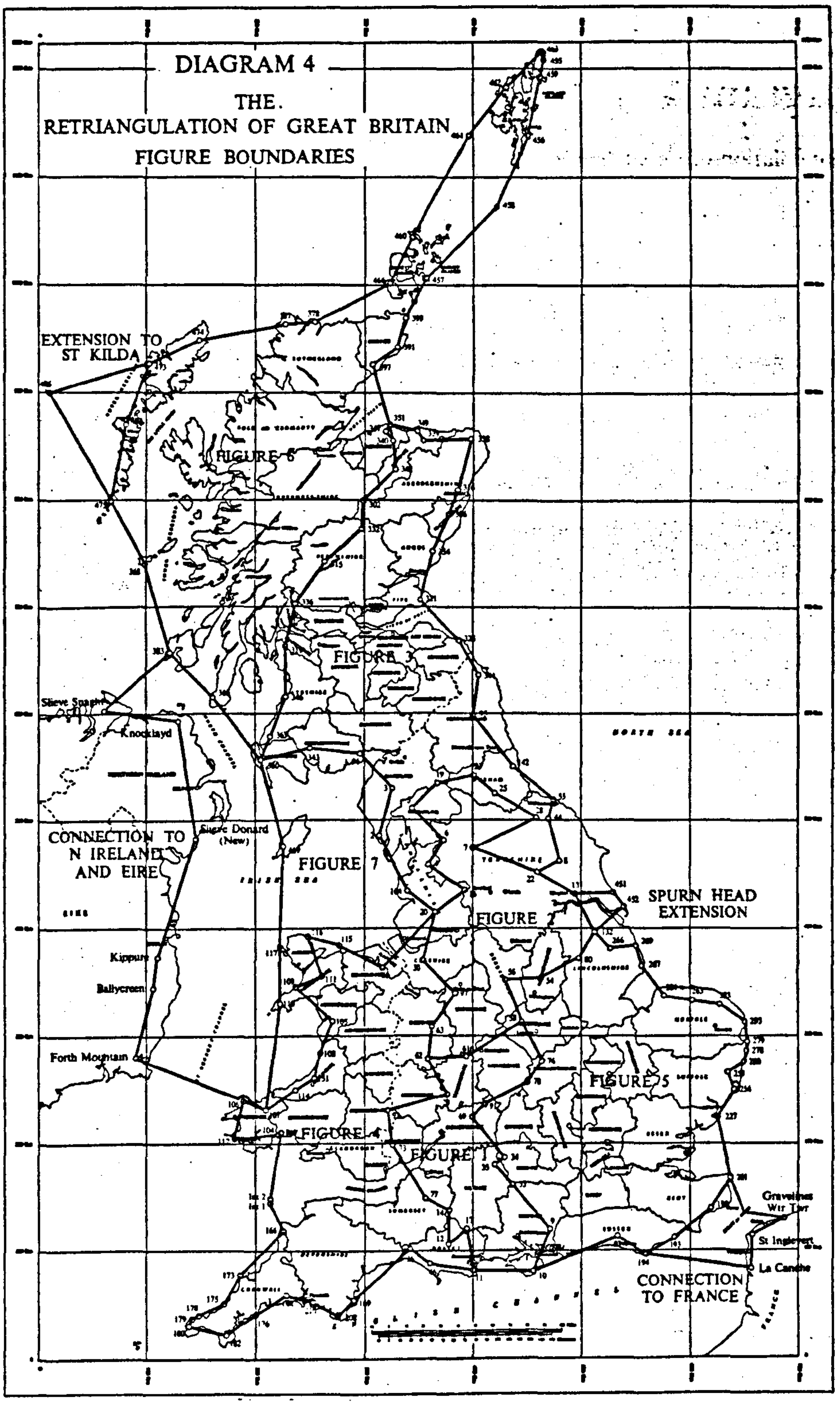


Figure 3.1 The Adjustment Figures of the Retriangulation of Great Britain *[OS, 1967].*

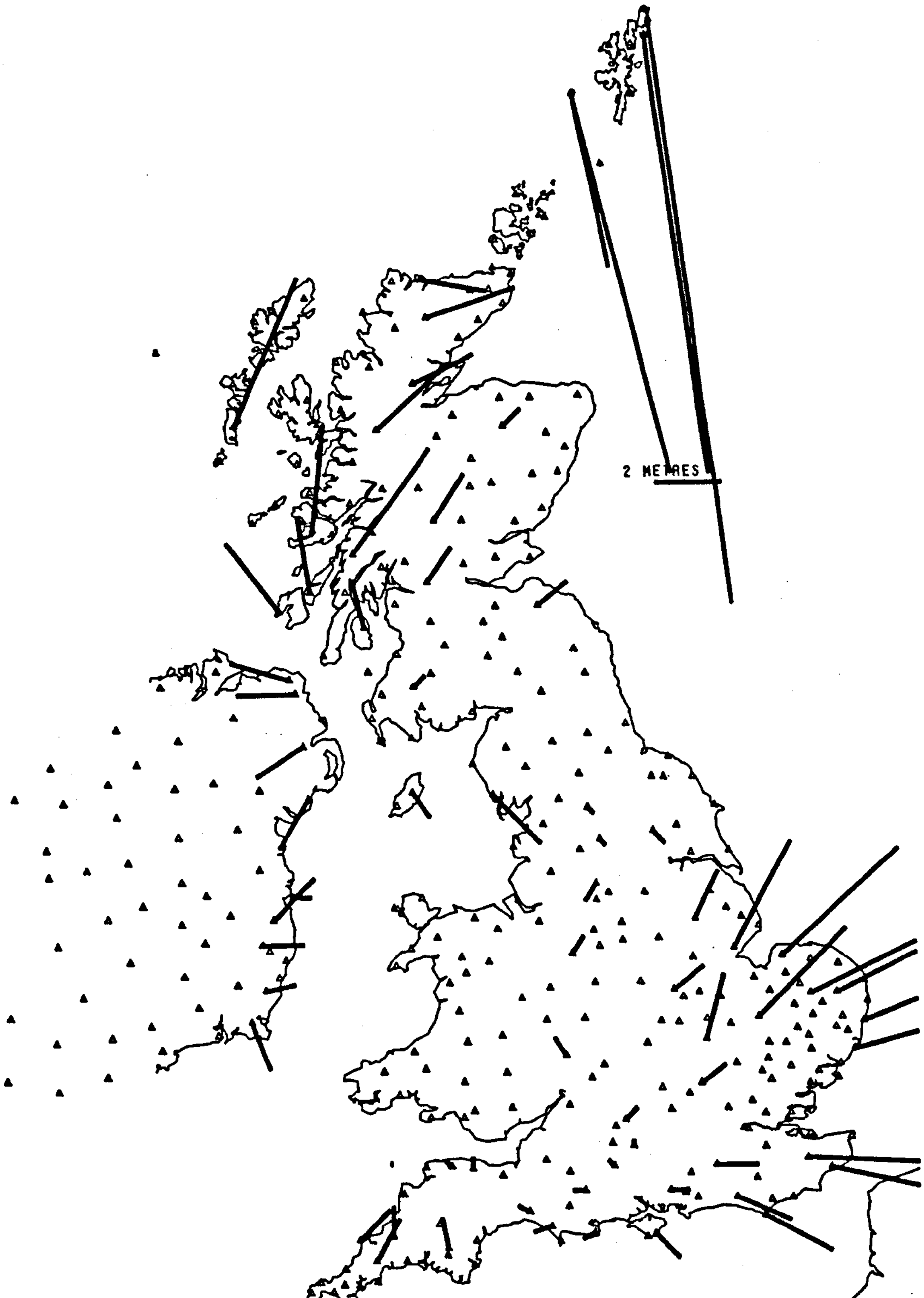


Figure 3.2 Differences at Selected Stations between the Principal Triangulation and the Retriangulation.

Isles of Scotland and Northern Ireland show large discrepancies of up to 18 metres, which were attributed to the poorly conditioned figures and large distances involved when crossing water. The discrepancies in East Anglia and Kent were not so easy to explain and blamed upon the varying scale of the Principal Triangulation *[OS, 1967]*. However, over most of the country the differences were small, 2 metres or less, which is quite remarkable considering the survey instruments used and the method of adjustment of these two triangulations.

During the 1950's, incompatibility between the OSGB36 datum and the European Datum (ED50) (see section 3.3.1.1) became serious enough for the Ordnance Survey to make the following statement.

> "...in the interest of geodetic knowledge a readjustment is being carried out to take account of the measured bases and azimuths, and to convert the triangulation to terms of the new European Datum reference." *[OS, 1955]*

Later measurements using EDM between primary stations confirmed the scale of OSGB36 was too large, and varied significantly with errors of 1 to 50 ppm. However, Laplace azimuths observed over six lines indicated that the orientation of OSGB36 agreed to within one arc-second.

3.2.1.3 The 1970 Readjustment (OSGB70(SN))

In 1968, the Ordnance Survey had their first computer installed at head office, Southampton, and were now in a position to simultaneously readjust the Retriangulation as one figure. During 1970, both the University of Nottingham and the Ordnance Survey performed independent adjustments of the Retriangulation. The difference between the two methods was in the weighting of the observations, although the final results agreed closely (10 - 20cm). The adjustment included two hundred and ninety-two stations connected not only by the nineteen hundred horizontal directions, but also by one hundred and eighty EDM distances and fifteen Laplace azimuths, to control the scale and orientation of the adjustment. This readjustment is known as Ordnance Survey Great Britain 1970 (Scientific Network), or OSGB70(SN), and covered England, Scotland

and Wales. The Republic of Ireland and Northern Ireland were adjusted to fit stations on the west of the OSGB70(SN) network. Herstmonceux was defind as the origin for the Airy ellipsoid, where the geoid-ellipsoid separation was set to zero and the geodetic latitude and longitude were defined. This was the first time since the foundation of the Ordnance Survey that Greenwich was not used as the origin, since the Royal Greenwich Observatory (RGO) had moved to Herstmonceux, Sussex, in 1953. To maintain consistency with OSGB36 the geodetic coordinates for the primary pillar at Herstmonceux were adopted from the OSGB36 adjustment. The results of the OSGB70(SN) adjustment confirmed earlier fears of a varying scale error (1 to 50 ppm) in OSGB36 (see Figure 3.3).

The OSGB70(SN) adjustment had been a considerable improvement over OSGB36 especially in terms of scale. However, comparison between the two types of EDM instrument used revealed that the scale of OSGB70(SN) was too small by 3 ppm. This discrepancy was attributed to the distances measured with the Tellurometer (microwave) instrument and confirmed by tests performed by the Ordnance Survey, which found that this instrument had a constant error of -2.6 ± 0.4 ppm *[Williams, 1979]*.

Since the Principal Triangulation all computations have been reduced and performed on the geoid rather than the spheroid. It was considered that Airy's spheroid was a good approximation to the geoid in the UK although no attempt had been made to determine the relationship between the two. Between 1969 and 1978 the Ordnance Survey observed deviations of the vertical at one hundred and ninety stations whose geodetic coordinates had been determined in OSGB70(SN). Using this data and geoidal sections, an astrogeoid was computed by Ordnance Survey and Oxford University *[Dean, 1980]*. This showed that, at no point, did the geoid/spheroid separation exceed 5 m. This allowed future adjustments to be reduced to the spheroid.

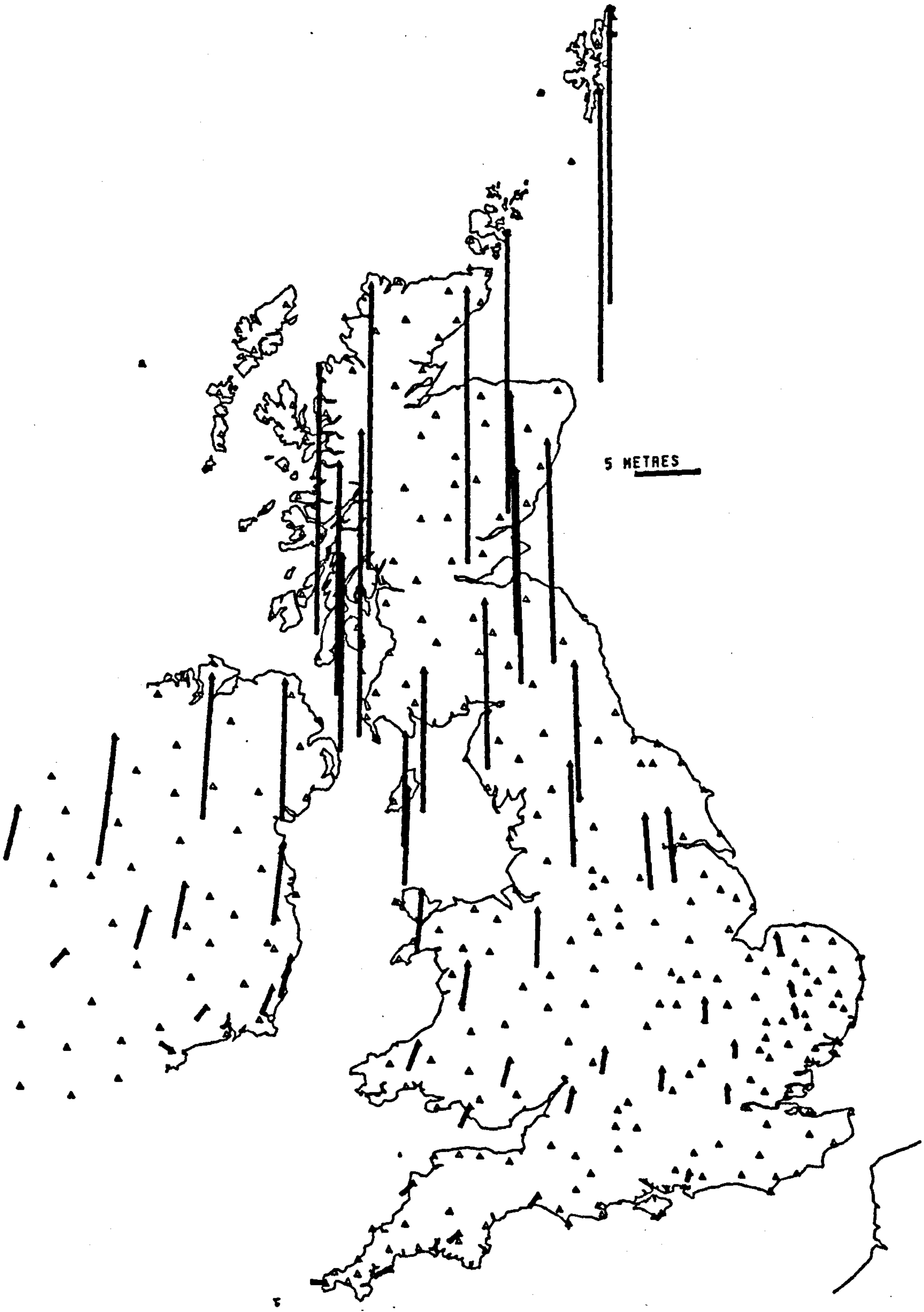


Figure 3.3 Differences at Selected Stations between the Retriangulation and the 1970 Readjustment.

3.2.1.4 The 1980 Readjustment (OS(SN)80)

The Transit Doppler System, described in section 2.3, was released for commercial use in the 1960's. Through observing the Doppler shift of signals transmitted by these satellites, and using the precise ephemeris it was possible to determine the absolute coordinates of a point to an accuracy of 1 metre. By 1977 the Ordnance Survey had eleven triangulation stations for which precise ephemeris Transit positions had been computed.

The existence of systematic errors in OSGB70(SN), and the availability of the Ordnance Survey 1977 Geoid Map, stimulated the need for a new adjustment. A readjustment of the Retriangulation was carried out in 1977 to compare the results obtained from Transit with the terrestrial network. The results confirmed the systematic errors in the OSGB70(SN) adjustment. In order to remove these systematic biases another adjustment was performed including the Transit coordinates. However, these coordinates altered the scale of OSGB70(SN) very little and this adjustment was never published.

The results of the 1977 adjustments and the accumulation of thirty-one extra EDM distance observations and nine Laplace azimuths, from the Edinburgh-Malvern-Dover precise traverse, led to the 1980 readjustment referred to as the Ordnance Survey (Scientific Network) 1980 (OS(SN)80). This adjustment was again the result of close co-operation between the Ordnance Survey and the University of Nottingham. OS(SN)80 included not only England, Scotland and Wales as did OSGB70(SN), but also Northern Ireland and the Republic of Ireland. This was the same area as covered by the Principal Triangulation of 1850.

The method of least squares with variation of coordinates was used to adjust OS(SN)80 but the observation equations contained extra unknowns, when compared with OSGB70(SN). These are known as bias parameters and their function is to allow for systematic errors in the two different types of EDM measurements (Tellurometer - microwave and Geodimeter - lightwave) and the Laplace azimuths to be modelled. To avoid rank deficiency due to the bias parameters, the scale and orientation of the network were controlled by the eleven

Transit positions and should, therefore, be free of any such errors. However, comparison of Transit and VLBI systems showed both scale and orientation errors in Transit. It was, therefore, decided to apply corrections to the Transit positions to account for these small errors (-0.4 ppm in scale and +0.8 arc-seconds in longitude), as well as transform the Transit coordinates from the mass centred reference system (NSWC 9Z2) to the geometrical centre of Airy's spheroid, a shift in origin of approximately 570 metres. This transformation was calculated using only the OSGB36 and Transit coordinates for Herstmonceux.

To retain consistency with previous triangulations and readjustments, Herstmonceux was adopted as the origin, with the geodetic coordinates remaining the same as those from OSGB36 (and OSGB70(SN)) and the Airy ellipsoid used. The Transit position at Herstmonceux was excluded and a free adjustment performed. The resulting coordinates of Herstmonceux differed from OSGB36, so a translation (dx, dy, dz) was calculated at Herstmonceux and applied to the adjusted coordinates, enabling Herstmonceux to retain its OSGB36 values. The OS(SN)80 readjustment has shown a-posteriori standard errors of azimuth and distance of the order of 0.4 arc seconds and 2 ppm respectively. The bias parameters showed the Tellurometer microwave measurements to be 3 ppm too short and the biases for the Geodimeter and Laplace azimuths were insignificant since they were smaller than the standard error of their determination.

The results of this adjustment confirmed the 3 ppm scale bias in the Tellurometer EDM distances, and showed that there were no significant biases in the Geodimeter distances or the Laplace azimuths. This scale bias can clearly be seen in Figure 3.4, which shows the difference between the 1970 and 1980 readjustments, as a radial pattern increasing in magnitude from the origin at Herstmonceux. However, there remains the possibility that the Transit coordinates are themselves contaminated with further systematic errors. If this is the case, then clearly the terrestrial bias parameters will be in error by a similar amount. The possible errors in OS(SN)80 are explored in section 6.1.

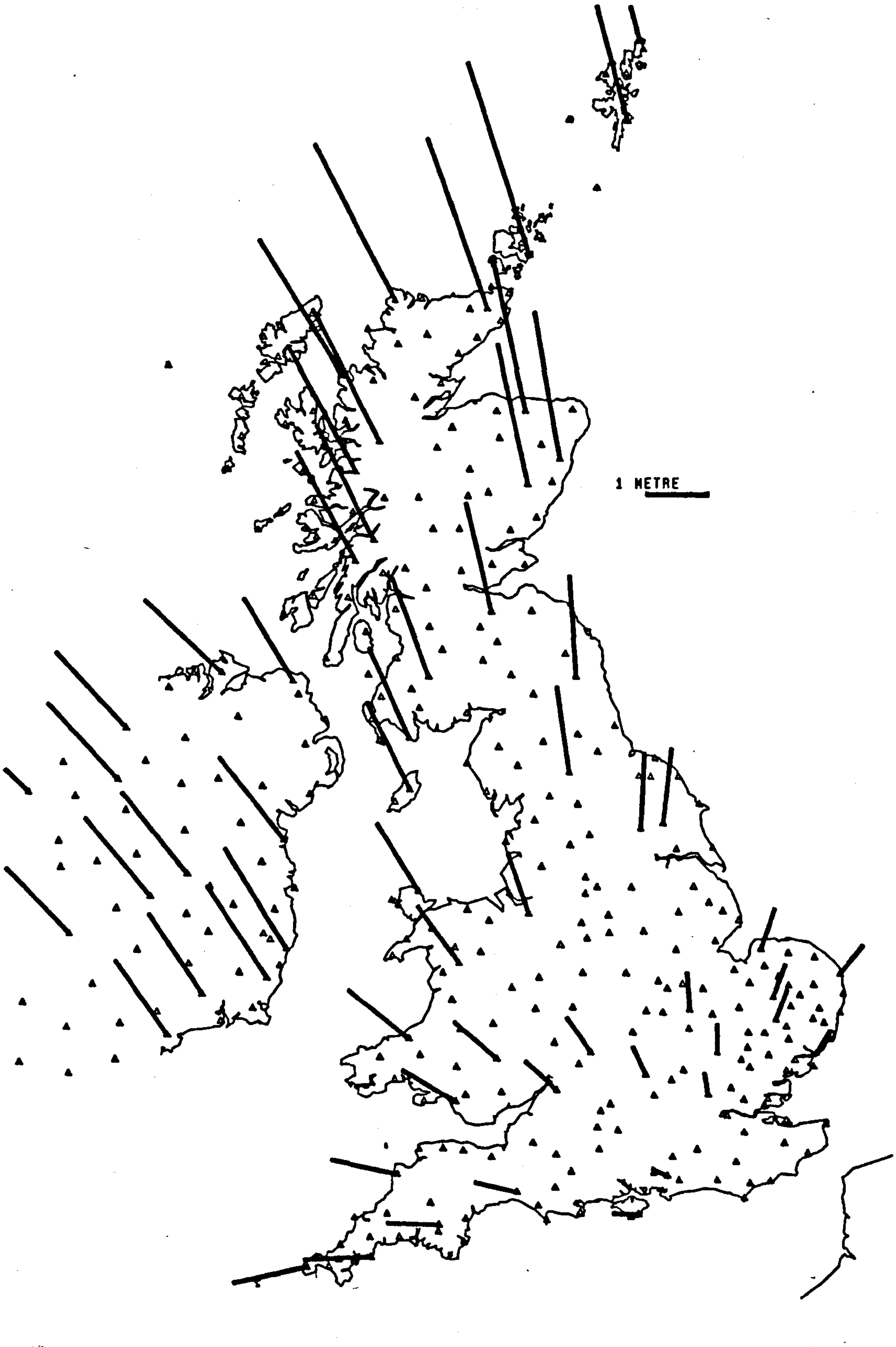


Figure 3.4 Differences at Selected Stations between the 1970 and 1980 Readjustments.

3.2.1.5 The Future of Ordnance Survey Horizontal Networks

As mentioned earlier, OSGB36 is still (1994) the datum used for the national mapping of Great Britain, despite the more recent adjustments namely, OSGB70(SN) and OS(SN)80. To date OSGB36 has served the needs of topographers and surveyors well, however, with the rapid development of geodetic space techniques, such as GPS, the problems of OSGB36 can no longer be overlooked. Great Britain is in need of a datum compatible with GPS for the 1990's and beyond. The aims and developments of this new mapping datum are described in Chapters 4 and 6.

3.2.2 Ordnance Survey Vertical Networks

To date (1994) there have been three geodetic levellings of Great Britain. The first was observed between 1840 and 1860 and used as its datum the mean tide pole observations at Liverpool from 7th to 16th May 1844. Unfortunately, this was never adjusted as one homogeneous network and no orthometric corrections were applied. In 1911 due to the loss of a large number of original benchmarks and problems with re-levelling it was decided to carry out the Second Geodetic levelling of Great Britain. This was observed across England and Wales during 1912 to 1921. This network used the mean hourly sea level values between 1st May 1915 and 30th April 1921 at Newlyn as its datum (Ordnance Datum Newlyn) and was adjusted, with orthometric corrections, as one free figure. Scotland was added between 1936 and 1952 and adjusted as four independent figures, holding 'junction stations' fixed from the adjustment covering England and Wales.

In 1950, a review of the Second Geodetic Levelling of Great Britain found that mean sea level was 0.8 ft higher at Dunbar than at Newlyn, and recommended the network was reobserved again. The Third and Final Geodetic levelling was divided into three sections, England, Wales and Scotland, and observed between 1951 and 1958. The network has been adjusted three times using different procedures. Firstly, an adjustment was performed holding the fundamental benchmarks fixed to Second Geodetic Levelling values in order to minimise the difference between the two levellings. Secondly, in the late 1960's, the first scientific adjustment was performed. This involved adjusting England and Wales as a free figure and then holding three of the resulting heights fixed for the adjustment of Scotland. The third adjustment was performed in 1970 and involved the whole network being adjusted as a free figure, using condition equations and the method of least squares.

A comparison of the results of the Second and Third Geodetic Levellings revealed several anomalies. Both showed that the mean sea level at Dunbar was significantly higher than at Newlyn by 0.81 ft (0.25 m) and 1.20 ft (0.37 m), in the Second and Third Geodetic Levellings respectively, suggesting an apparent rise of sea level of 0.39ft (0.12m) at Dunbar in 40 years. Further examination of the Second and Third Geodetic Levellings showed that the land at Dunbar had risen by 0.18 m relative to Newlyn over the same period. This uplift of about 5 mm/year is inconsistent with the mean sea level observations at Scottish tide gauges or geological evidence which showed that Scotland was rising by approximately 2mm/year *[Woodworth, 1987]*.

Mean Sea Level was calculated from the tide gauge records for 1960 to 1975 at twenty-nine tide gauges around the coast of Great Britain. This was calculated with respect to Ordnance Datum Newlyn, as defined by the Second Scientific Adjustment of the Third Geodetic Levelling of Great Britain, and is shown relative to the outline of Great Britain in Figure 3.5. The value of 96 mm at Newlyn indicates that mean sea level at Newlyn rose by this amount between the establishment of the Ordnance Datum (1915-21) and the new mean sea level observations (1960-75), ie 1-2 mm/year. The values for the other tide gauges were obtained by combining the results of the Third Geodetic Levelling and the local tide gauge records. The values in Figure 3.5 translate into an apparent northward rise in mean sea level of 5.3 ± 0.4 cm per degree of latitude with no difference across the country.

This apparent northward slope is considered difficult to explain oceanographically, and conflicts with the results from three independent oceanographic levelling techniques which agreed within 6 cm and have determined the net mean sea level slope to be zero between Southern England and Scotland *[Thompson, 1980; Davies,*

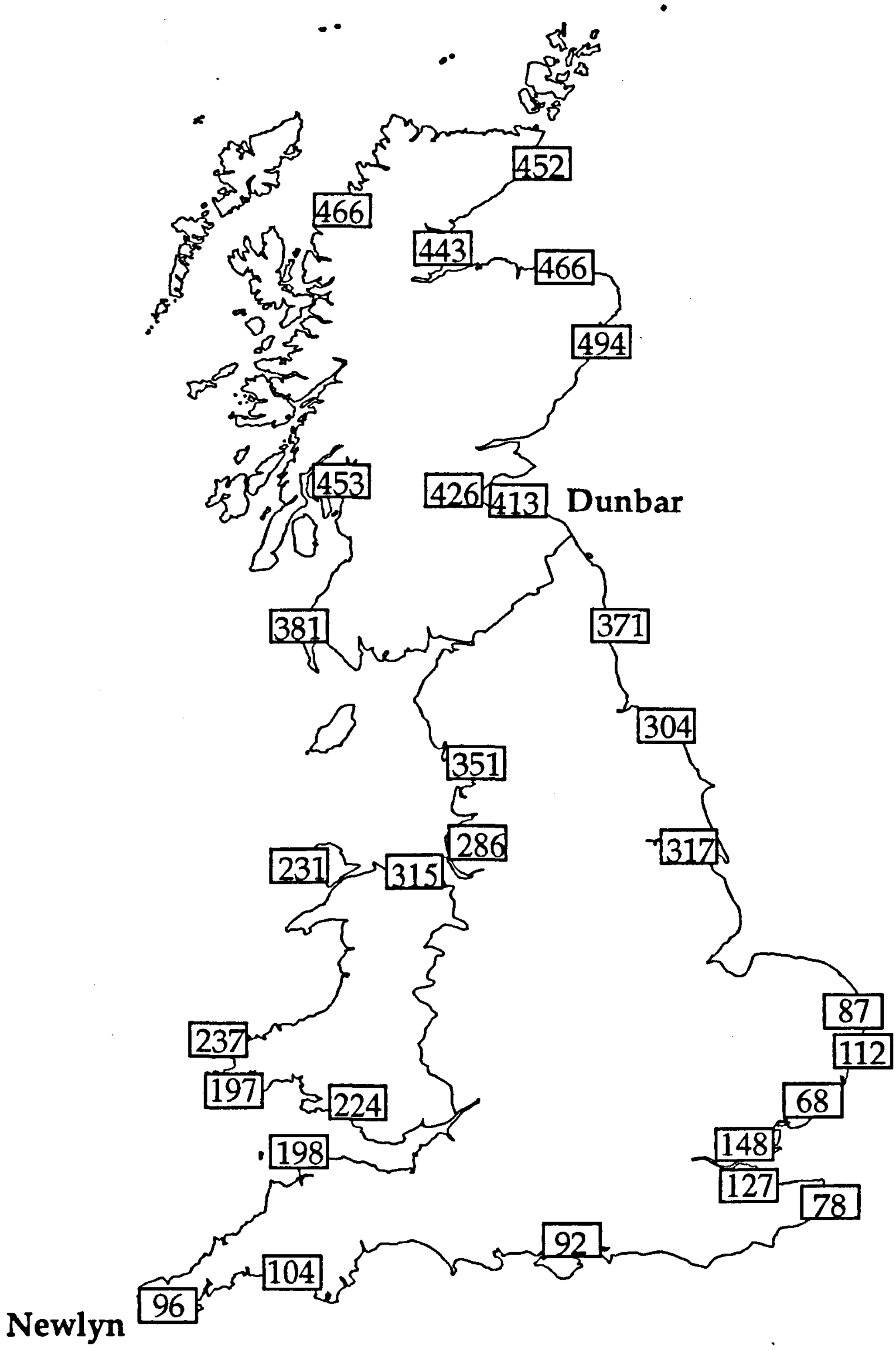


Figure 3.5 Mean Sea Level 1960-75 (mm) above Ordnance Datum Newlyn (3rd Geodetic Levelling, 2nd Scientific Adjustment) *[Thompson, 1980].*

1983; Amin, 1988]. It has, therefore, been suggested that it is due to a systematic error in the geodetic levelling, but to date (1994) this has not been proven.

Spirit levelling has always been a time consuming, elaborate and expensive operation. It is for this reason and despite the suspected systematic errors in the Third Geodetic Levelling, that the Ordnance Survey do not intend to re-observe this network. Instead, it is envisaged that GPS in conjunction with a precise relative geoid will be used for levelling. It will, therefore, be necessary to know the ellipsoid-to-geoid separations to the same high precision as the GPS ellipsoidal heights. The problems of using GPS for heighting are discussed further in section 6.2.

3.3 Continental Datums

This section describes the development of the European and North American datums. Although the North American datum can not be used for positioning in the UK, it has been included for comparison purposes. It has evolved differently and overcome many of the problems of the European and British Datums associated with using GPS as a positioning tool.

3.3.1 The European Datum

The first European Datum (ED50) was computed in the 1950's by the US Army to control military mapping, and declassified in the 1960's to define international boundaries for exploration of the North Sea. Inconsistencies in ED50 led to the establishment of the IAG Subcommission, RETrig, which performed several readjustments, namely ED77, ED79 and ED87. These involved using the latest surveying techniques and adjustment models. Unfortunately, the progress of RETrig was slow, since it depended on international co-operation, and its work had been overtaken by advances in geodetic space techniques. This section briefly describes the development of the European Datum from ED50 through to the conclusion of RETrig in 1988. The reader is referred to the following for more information *[Bordley and Christie, 1989; Poder and Hornik, 1989a].*

3.3.1.1 European Datum 1950 (ED50)

In the period following the Second World War, the US Army Map Service were presented with an opportunity to standardise the datum for the military mapping of Europe. The data used was acquired from German geodesists who had collected triangulation observations of all previously occupied countries. It consisted of horizontal directions, bases, and Laplace azimuths. The adjustment was performed using the "Modified Bowie Method", a computational process that is not considered rigorous. The International (Hayford) ellipsoid 1924 was used and fixed at the famous Helmert Tower in Potsdam. This defined the "Central European Net" (CEN) which extended from Pinsk in the West, to Bonn in the East, and from Kiel in the North, to Budapest in the South.

The adjustment of the CEN was carried out from June 1945 to June 1947 involving fifty geodesists and mathematicians together with twenty supporting staff. Even before this adjustment was completed there was considerable pressure to extend the network, and although the computational method was not rigorous, it was decided that the chance to standardise the datum for European mapping should not be missed. With the assistance of the International Association of Geodesy (IAG), the south-eastern block (Yugoslavia, Romania, Bulgaria and Greece) was added to the central block. Subsequently, the southwest block (France, Spain, Portugal, Switzerland, Italy, and North Coast of Africa) and northern block (Denmark, Norway, Sweden, Finland, and Estonia) were added (see Figure 3.6). This final adjustment was performed in 1950 and the complete network became known as the European Datum 1950 (ED50).

In the 1960's, exploration for oil and gas on the European Continental shelf highlighted the need to define internationally agreed median lines between nations bordering the North Sea. At the time, ED50 was the only datum suitable for this purpose and hence it was de-classified and became available for civilian usage. However, none of the stations in the British Isles were included in the original ED50 adjustment, but several links were observed across the Channel to connect Great Britain into ED50. These links were used to determine transformation parameters between OSGB36 and ED50. OSGB36 coordinates were then transformed to determine a consistent set of ED50 coordinates for all Ordnance Survey primary triangulation stations.

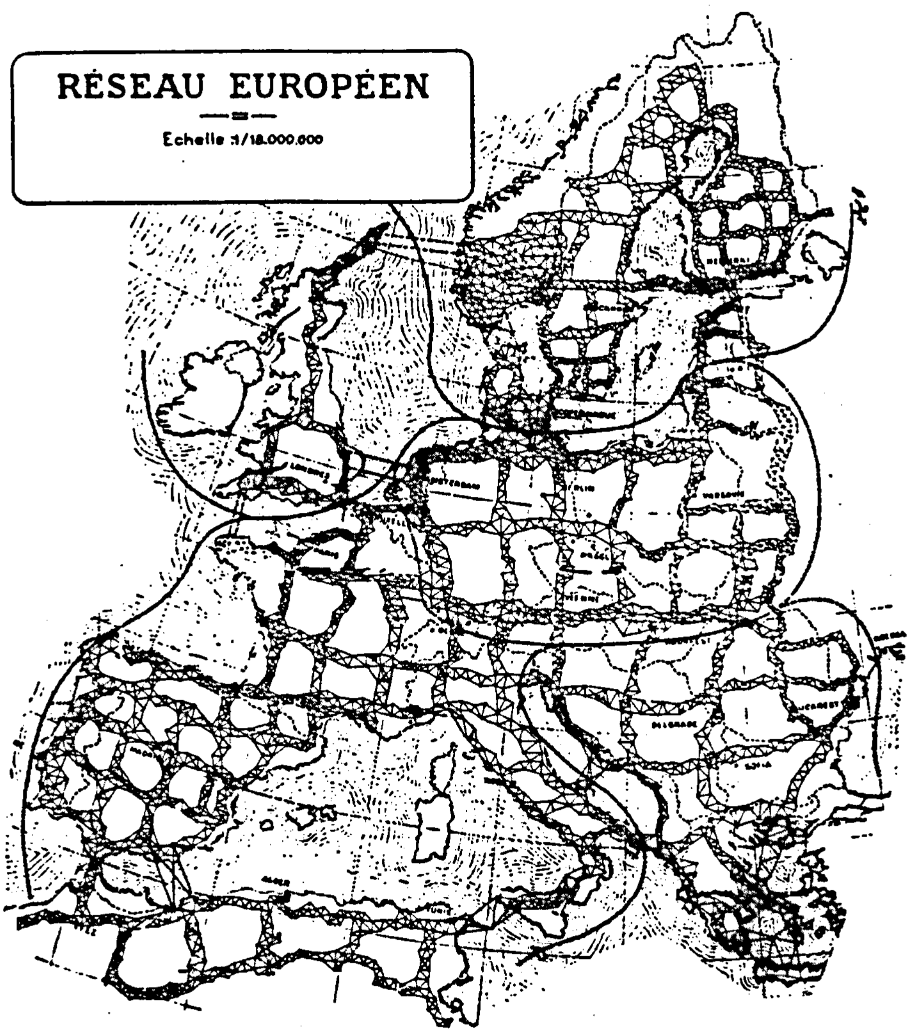


Figure 3.6 The European Datum 1950 *[Poder, 1989].*

ED50 coordinates were only developed to provide a datum for military mapping of continental Europe, a task for which they were perfectly adequate. However, their use for defining international boundaries in the North Sea soon showed their lack of internal consistency. This became apparently obvious when using ED50 coordinates with geodetic space techniques. These techniques such as Transit and GPS produce geocentric coordinates, which before they can be used must be transformed into ED50. However, these transformation parameters can differ by up to tens of metres when calculated using different subsets of stations. This has resulted in the production of a plethora of transformation parameters being published and the situation has been described as a 'minefield'.

The problems of ED50 were further compounded each time the Ordnance Survey readjusted their national network, since this would lead to a corresponding readjustment of the derived ED50 coordinates. As was shown in Figure 3.3, the difference between OSGB36 and OSGB70(SN) was up to twenty-three metres in Scotland and led to much confusion over which coordinates should be used. This caused particular problems in the Central and Northern North Sea because Scotland is the area from which ED50 positions are extrapolated.

The accuracy of ED50, not including the UK, ranged from a few metres at best to over ten metres at worst. This was due to its poor method of adjustment. In 1954, the IAG recognised the future needs of such a European Datum, and since ED50 had only used part of the available data, established a new subcommission to continue the work of ED50 and readjust the triangulation of Europe using all available data. This subcommission was known as Réseaux Européenes de Triangulation or Readjustment of the European Triangulation Network and commonly abbreviated to RETrig.

3.3.1.2 European Datum 1987 (ED87)

The ED50 coordinates had been of immense value to the geodetic community, so it was decided that the coordinates resulting from this new adjustment should be as close as possible to those of ED50. This was achieved by holding the coordinates of the station at Munich fixed to its ED50 value, and using the same reference ellipsoid. Potsdam the fundamental station for ED50 was not used since it was now in East-Germany, behind the Berlin Wall ! However, unlike the ED50 adjustment, the RETrig subcommission used all the available data from approximately five thousand five hundred stations, which for ease of handling was divided into six blocks. This included terrestrial data, horizontal directions, astronomical azimuths, electromagnetic distance measurements and invar bases, as well as space data, ie three dimensional coordinates from SLR, VLBI and Transit. The space data resulted from several campaigns known as MEDOC, MERITDOC and

RETDOC (see Figure3.7) *[Poder, 1988]*.

Figure 3.7 The RETrig Network ***[Poder 1988].***

Since ED50 suffered from its blockwise method of adjustment it was decided that this new adjustment would be based upon the Helmert blocking technique. Each country produced a normal matrix pertaining to stations within their country. This matrix was then reduced using Helmert blocking to only contain junction stations in common with neighbouring countries. A Central Computing Centre then merged these junction station matrices into one matrix called the 'Buffer Matrix'. This Buffer Matrix was then solved, and the coordinates of the junction stations were back substituted into the original National normal matrices to determine the remaining station coordinates. The advantages of using this procedure were that the coordinates of national stations were only disclosed to that country, and secondly that this procedure was less computationally demanding than performing one complete adjustment.

The computation of RETrig involved three phases:

Phase I - a test phase including only terrestrial observations (directions) to experiment with weighting schemes. This was finished in 1977.

Phase II - as Phase I plus azimuths and bases, but no satellite information. This resulted in the adopted coordinate set ED79 (an intermediate solution ED77 was also published)

Phase III - as Phase II, plus space data, and the adjustment included bias parameters (following the example of North American and British adjustments). Three intermediate solutions were produced between 1983 and 1987, known as the Copenhagen, Hague and Paris solutions after the city in which the symposium was held. They were used only for discussion and evaluation and not attempts to produce a 'publishable' solution. The final results of this phase, ED87 have been adopted as the final results of the RETrig subcommission.

The RETrig subcommission was terminated in May 1988 producing ED87, a rigorously adjusted datum for the whole of Europe, with scale and orientation provided by geodetic space techniques. The accuracy of ED87 was better than 2 metres *[Poder and Hornik, 1989a]*, a considerable improvement over ED50. The results from several doppler campaigns showed that the origin of the ellipsoid was offset from the geocentre by about 100 metres. However, even after 34 years, ED87 still contained many 'black spots' *[Poder, 1988]*. To attempt to correct all of these would have taken many years and resulted in a datum based on obsolete data and methods, and void of any practical value when finally finished. RETrig had been overtaken by the advances in geodetic space systems capable of producing high precision three-dimensional positions. Although RETrig had not completely finished its work, it gave invaluable experience in international cooperation and the infrastructure was ready for the new IAG subcommission, EUREF (European Reference Frame) to construct a unified three-dimensional European Datum based on geodetic space techniques. The work of EUREF is described in detail in Chapter 4.

3.3.1.3 Unified European Levelling Network

National Levelling Networks are usually measured with respect to mean sea level, observed at a single tide gauge for a particular period of time. However, due to the effect of such forces as wind and barometric pressure, the determination of mean sea level will differ form country to country, and hence, so will the reference equipotential surfaces. The Unified European Levelling Network (Réseau Européen Unifié de Nivellement, REUN) was formed in 1954 to solve this problem. The network consisted of selected first order levelling lines, of western European countries, connected to sixty tide gauges along the Mediterranean, the Atlantic, the North Sea and the Baltic. Several adjustments have been carried out using varying amounts of data, and defining different tide gauges as the datum. However, as with RETrig, the work of REUN has been overtaken with the advancement of geodetic space techniques. The Comité Européen des Responsables de la Cartographic Officielle (CERCO) are now considering the computation of a European wide geoid to enable the use of GPS as a levelling tool.

3.3.2 The North American Datum

The first North American Datum, NAD27, was comparable with OSGB36 and ED50. Advances in positioning and adjustment techniques led to the re-adjustment of this network resulting in NAD83. This differed from OS(SN)80 and ED87 because it was adjusted on a geocentric ellipsoid. This allows GPS positions to be plotted directly on to maps and charts based on this datum. This section briefly describes the development of the North American Datum from NAD27 to NAD83 and the decision to adopt NAD83 as the new mapping and charting datum for North America. The reader is referred to the following for more information *[Schwarz, 1989; Wade, 1986].*

3.3.2.1 The North American Datum 1927 (NAD27)

The North American Datum of 1927 (NAD 27) consisted of twenty-five thousand stations and forty-one triangulation loops ranging in circumference from several hundred to three thousand kilometres. NAD 27 was computed on the Clarke 1866 ellipsoid positioned to best fit the North American Continent, with the origin and orientation defined at Meades Ranch, Kansas. The adjustment was completed in 1931 and any new observations were forced to fit the NAD 27 network. This expansion of the NAD 27 network continued without difficulty until the 1950's when major advances in surveying technology meant that surveys were now of a higher accuracy than the NAD 27 control on to which they were mounted. This was particularly evident with the introduction of EDM's which showed scale distortions of up to one metre in 15 kilometres, ie 66 ppm.

The large discrepancies found in NAD 27 led to several partial readjustments using new survey data. This disclosed errors of up to ten metres in Northern Michigan and complaints were soon received about the frequency with which coordinate values were changing. This caused a loss of confidence in NAD 27 and prompted the National Geodetic Survey to write in defence of NAD27,

> "The discovery of the weakness did not indicate careless planning or execution of the adjustment but rather represented the significant improvement in surveying methods in the following decades." *[Schwarz, 1989]*

The problems of NAD 27 were described in the 1971 Report by the Committee on the North American Datum. Since 1927 some ninety-nine thousand new stations had been added, many of the original stations had been destroyed, and in some areas tectonic movements of approximately 5 cm per year had been recorded. The report concluded by calling for a new datum.

3.3.2.2 The North American Datum 1983 (NAD83)

The 1983 adjustment involved the horizontal geodetic networks of the United States, Canada, Mexico, Central America, the Caribbean and Greenland. NAD 83 was computed on GRS80, a geocentric ellipsoid designed to best fit the ellipsoid for the whole Earth, whereas NAD 27 was computed on the Clarke 1866 Ellipsoid, which was implicitly positioned to best fit the North American Continent. The advantage of using this geocentric ellipsoid was that Transit, SLR and VLBI coordinates could be used to strengthen the network without the need to use poorly determined transformation parameters, and in the future, GPS positions could be plotted directly onto the maps and charts based on this datum. The adjustments also differed in origin and orientation, NAD 27 was defined using one datum point and one orientation, whereas NAD 83 used satellite determined positions to define its origin and orientation.

The adjustment of NAD 83 took from 1976 to 1985. It included horizontal directions from NAD 27, astronomical azimuths and electronic and taped distance measurements, more than one thousand Transit stations (200 km average spacing), to provide control and translation to the geocentre, and a number of VLBI baselines to give the system scale and orientation. The network contained 1.8 million observations, more than two hundred and sixty thousand stations and was, therefore, adjusted using the Helmert Blocking technique, used for the adjustment of ED87. The adjustment used the height-constrained, three-dimensional method *[Vincenty, 1979]*, in which three-dimensional equations are used but holding the height component fixed. This avoided reducing observations to the ellipsoid. This adjustment also included solving for bias parameters following their successful use in the OS(SN)80 adjustment.

The final results showed shifts between NAD 27 and NAD 83 of

-24 to +40 metres in latitude,
-40 to +110 metres in longitude
-40 to -4 metres in height.

Comparisons of NAD83 with GPS results have shown RMS discrepancies of ± 1-2 to ± 20 centimetres for distances from 10 to 300 kilometres. In 1992 the Defence Mapping Agency announced that they would be readjusting the mapping datum from NAD27 to NAD83, a datum consistant with WGS84 and hence GPS positions *[Ashkenazi, 1992]*.

3.4 Global Systems

Global Systems are used by all satellite positioning techniques. Generally they are defined with their origins as close as possible to the centre of mass of the Earth (Geocentre), and their axes orientated to the corresponding axes of the Conventional Terrestrial Reference System. An associated ellipsoid is then chosen to provide the best fit to the geoid throughout the world. Global systems are realised by assigning cartesian coordinates to a number of stations around the world, which implicitly define the origin and orientation of the axes. However, unless these coordinates are based on very high precision observations, the resulting reference frame will be distorted by measurement errors. As measuring techniques improve such distortions are inevitably discovered and systems have to be re-realised.

In addition to coordinates, global systems also have other physical parameters associated with them, such as a description of the Earth's gravity field, an equipotential reference ellipsoid or a global geoid. Hence, when coordinates are quoted in a global system it implies that these physical parameters have been used in the determination of this position. For example if coordinates are determined in a global system using satellite data then the associated system's gravity field must be used. As measurement techniques become more accurate, global systems are having to account for phenomena never accurately measured before. One example of this is plate tectonics, or continental drift. Stations, even in the most stable parts of the World, can move by a few centimetres per year due to the movements of the continental plates. This can cause serious distortions of a high precision network after several years if this network is not 're-realised' or the movements modelled.

Two global systems are in common use, firstly the World Geodetic System for mapping and navigation and the International Terrestrial Reference System used for precise surveying, geodesy and geophysics.

The following sections describe these systems and methods that can be used to access them.

3.4.1 World Geodetic Systems

The US Department of Defence (DOD) has been developing, for just over thirty years, a World Geodetic System (WGS) for cartographic applications where mapping and charting with respect to the centre of mass of the Earth are required. Although primarily for military use, their use in navigation systems such as Transit and GPS has made them accessible to the civilian community. Several WGS's have been developed to date (WGS60, WGS66, WGS72 and WGS84) each using contemporary state-of-the-art technology, and therefore being progressively more accurate. The latest, WGS84, has been well documented for the civilian community *[DMA, 1987]*. This section briefly reviews the WGS60, WGS66 and WGS72 and then describes the development and components of WGS84.

3.4.1.1 World Geodetic Systems 1960, 1966 and 1972

During the late 1950's the US DoD developed its first geocentric reference system known as the World Geodetic System 1960 (WGS60). This was calculated by combining surface gravity data and astrogeodetic data. In 1966 the WGS Committee was set the task of developing an improved WGS to satisfy the improved techniques used in mapping and charting. Using newly available Transit and optical satellite data, in addition to the existing surface gravity and astrogeodetic data, and employing new adjustment techniques, the US DoD produced WGS66. This included a reference ellipsoid, gravitational model and geoid. WGS66 could be accessed through published Molodensky transformation parameters (origin and ellipsoid change only) from the North American Datum 1927 (NAD27), European Datum 1950 (ED50), Toyko Datums (TD) or Australian Geodetic Datum (AGD) *[Seppelin, 1974]*.

In 1970, the WGS committee started work on a replacement for WGS66. Large quantities of additional data had become available from both Transit and optical satellites, surface gravity surveys, triangulation and trilateration surveys, high precision traverses and astronomic surveys. In addition improved computer hardware and software, and adjustment techniques incorporating error analysis, meant it would be possible to combine larger data sets. After three years, the DoD produced WGS72, using a least squares adjustment of selected data. WGS72 also included an ellipsoid (GRS 67), a gravity model and a geoid.

WGS72 could be accessed by using Transit, via transformation parameters, or GPS until they were switched to WGS84. Today it can only be accessed using published Molodensky transformations to either the 'major' datums (NAD27, ED50, TD or AGD) or sixteen 'minor' local datums, including OSGB36. Seppelin *[1974]* states the accuracies of the transformations to the 'major' datums to be ± 5 metres for NAD27, ± 10 metres for ED50 and to ± 15 metres for TD and AGD. Comparison of the transformation parameters from WGS66 and WGS72 to ED50 showed differences of up to 20 metres in the position of the origin.

3.4.1.2 World Geodetic System 1984 (WGS84)

The World Geodetic System 1984 was made available for use in 1985, and through its use as the reference system for Transit and GPS (since 1st January 1989) it is becoming an internationally adopted standard for navigation and positioning.

WGS84 consists of the following components.

An Ellipsoidal Model: The choice of the WGS 84 Ellipsoid was greatly influenced by the IUGG recommendations to adopt the Geodetic Reference System (GRS80).

An Earth gravity model: This is complete through degree and order twelve and based on three independent types of data; satellite doppler and laser ranging data, satellite altimetry data from the oceans and surface mean anomaly data.

A Worldwide Geoid: This was calculated using the WGS84 Earth Gravity Model. The worldwide RMS geoid ellipsoid separation is ± 30.5 metres and the error ranges from ± 2 to 6 metres (1σ). The accuracy of the geoid is approximately ± 4 metres over 93 % of the globe.

A Reference Frame: The WGS reference frame was realised using the Naval Surface Weapons Centre (NSWC) 9Z2 system based on fifteen hundred and ninety-one worldwide Transit precise ephemeris positions. NSWC 9Z2 was the coordinate system used by the Transit tracking network for the computation of the precise ephemeris and evolved from the development of WGS72. By collocating the Transit receivers with Very Long Baseline Interferometry and Satellite Laser Ranging stations, any systematic errors in the NSWC 9Z2 coordinate system could be determined. The NSWC 9Z2 system was then scaled, rotated and translated to produce the WGS84 reference frame. These transformations are described below.

(i) Origin

With the development of Satellite Laser Ranging (SLR), it became apparent that the origin of the NSWC 9Z2 system was offset from that of the SLR system. The magnitude of this discrepancy was determined by the National Geodetic Survey (NGS) who found that the NSWC 9Z2 equatorial plane was north of that of the SLR coordinate system by amounts varying from 3.6 to 4.08 metres. A similar study by the Bureau International de l'Heure (BIH) found that using two different data sets, the Z-axis bias was either 4.36 or 5.61 metres. Following these results the Defense Mapping Agency (DMA) and NGS adopted a Z axis shift of 4.5 m with a one sigma uncertainty of ± 0.5 m for the correction of NSWC 9Z2 to WGS84. Similarly, tests showed that the X and Y axis offsets are approximately zero with an uncertainty of ± 0.5m, and hence no corrections were applied.

(ii) Orientation

The WGS 84 system was developed so that its zero meridian would be coincident with the Conventional Zero Meridian (CZM) defined by the BIH (see section 3.4.2). However, the zero meridian of the NSWC 9Z2 was offset to the East of the CZM. VLBI observations suggested that the NSWC 9Z2 X axis was East of the VLBI zero meridian by 0.77 to 0.88 seconds. The BIH, using three seven parameter transformations between NSWC 9Z2 and the VLBI coordinate systems, showed that Z axis rotation was between 0.8079 and 0.8243 seconds and also that the rotation between the VLBI zero meridian and the CZM was between 0.0057 and 0.0087 seconds. The BIH defined this Z axis rotation to be 0.8137 seconds and the DMA adopted a rounded version of this number, namely 0.814 seconds. The DMA found no rotation necessary about the X and Y axis to bring the NSWC 9Z2 coincident with the BIH Terrestrial System.

(iii) Scale

Due to neglected ionospheric terms, the computed heights of Transit stations were high in periods of high solar activity. Therefore, a scale modification was made to NSWC 9Z2 to produce WGS84. The scale of NSWC 9Z2 is based upon the Earth's gravitational constant (GM) used in the orbit computations and the speed of light (c) used in conversion of Transit Doppler data to range difference data. To validate the scale of NSWC 9Z2 Transit chord distances were compared with VLBI and SLR. The results showed a scale difference of between -0.53 to -0.69×10^{-6} ($\pm 0.1 \times 10^{-6}$). The DMA adopted a value of -0.6×10^{-6}, corresponding to a height correction of -3.8 m, in developing WGS84 from NSWC 9Z2.

These three corrections to the NSWC 9Z2 system define the WGS 84 Reference Frame.

$\Delta Z = 4.5 \pm 0.5$ metres (origin at Earth's centre of mass)

$\Delta Zr = 0.814 \pm 0.2$ seconds (axes coincident with BTS at epoch 1984.0)

$\Delta\lambda = -0.6 \pm 0.1$ ppm (compatible with VLBI and SLR)

These values differ, particularly in scale, from those values used to correct the Transit positions included in the OS(SN)80 adjustment. This is discussed further in section 6.1.

3.4.1.3 WGS84 Datum Transformations

One of the objectives of defining a world geodetic system is to produce a geocentric geodetic system onto which local geodetic systems can be transformed. To achieve this, it is necessary to coordinate as many sites as possible in both the geocentric and local systems. WGS84 included a total of fifteen hundred and ninety-one Transit stations in eigthy-three local geodetic datums, over six continents. This ranged from a high of four hundred and five stations in common with NAD27, to twenty-nine datums with only one common station. Furthermore, local geodetic datums are generally defined only horizontally, and heights are defined separately as part of a vertical datum relative to 'mean' sea level. It was therefore necessary to use the WGS84 geoid to determine N (geoid-ellipsoid separation) to enable semi-rigorous transformations to be determined.

There are four basic methods, specified by *[DMA, 1987]*, of obtaining WGS84 coordinates which reflect the data and equipment available. However, each method will produce a slightly different set of WGS84 coordinates and this must be remembered when using coordinates from different sources. The four methods are: satellite point positioning directly in WGS84 (see Appendix A), NSWC 9Z2 to WGS84 coordinate conversion, WGS72 to WGS84 coordinate conversion and Local Geodetic System Transformations. The value and accuracy of WGS84 coordinates of a station are significantly influenced by the technique used to obtain them. WGS84 was realised using Transit and therefore is consistent at the 1-2 metre level. Therefore, if a higher accuracy is required, this global system must be realised in a more refined way. This could be done using VLBI, SLR and GPS, to realise WGS84 to an accuracy of a few centimetres.

However, during the XXth General Assembly of the International Union of Geodesy and Geophysics (IUGG) and the International Association of Geodesy (IAG), Vienna, August 1991, personel from the National Geodetic Survey revealed that there were working on 'WGS 90', to be realised using VLBI, SLR and GPS. To date (September 1993) the author has heard nothing further.

3.4.2 International Terrestrial Reference System

The first global terrestrial reference frame was defined in 1968 by the Bureau International de l'Heure (BIH). It consisted of the adopted astronomical coordinates of a network of sixty-eight optical instruments contributing to the monitoring of the Earth's rotation. This initial realisation was concerned only with the tie to the pole, the directions of the axes, and the definition of the origin of longitudes. The pole used was the Conventional International Origin (CIO) defined by the IAU in 1967 by adopting values of astronomical latitude for five observatories. The BIH adopted a new zero meridian which became known as the BIH-zero longitude or the Conventional Zero Meridian (CZM).

Since then the techniques of VLBI, SLR and recently GPS have provided several orders of magnitude improvement in positioning and the measurement of the Earth's rotation parameters, when compared to astronomical techniques. This section describes the development leading to the establishment of the International Terrestrial Reference System realised annually using these geodetic space techniques.

3.4.2.1 Monitor Earth Rotation and Intercompare the Techniques (MERIT)

The full description of the rotation of the Earth in space is given by the motion of the axis of rotation with respect to an axis fixed in the Earth (Polar Motion), the motion of this rotation axis with respect to the celestial sphere (precision and nutation) and by the angular movement around the rotation axis (Universal Time, UT1). Precession and nutation can be modelled 'fairly' accurately and only require the occasional improvements to the models. However, Polar Motion and Universal Time are unpredictable and can only be determined by continuous monitoring.

An accurate knowledge of universal time and polar motion is not only required for geodetic surveying and precise navigation, but is of great scientific value since it provides information about the interior of the Earth and geophysical phenomena acting on the Earth. It was for these reasons that the participants of the 1978 IAU Symposium on "Time and the Earth's Rotation" set up a working group to investigate the development and availability of new techniques, such as VLBI, SLR and LLR, which could provide an order of magnitude improvement in the precision of this data. The aim of the working group was to perform a "comparative evaluation of the techniques for the determination of the Rotation of the Earth, and to make recommendations for a new international program of observation and analysis, in order to provide high quality data for practical applications and fundamental geophysical studies" *[Mc Carthy and Pilkington, 1979]*.

The working group met in October 1978 and agreed on a campaign of international collaboration to Monitor Earth Rotation and Intercompare the Techniques of observation and analysis, which became known by the acronym MERIT. The first campaign was the three month MERIT short campaign which started in August 1980. The aim of which was to test and develop the organisational arrangements that would be required during a main camapign, and show where improvements in operational and data analysis procedures were needed. Details and results from the short campaign can be found in *[Wilkins and Feissel, 1982]*.

There was an interval of three years before the MERIT main campaign which started in September 1983, and lasted for fourteen months. This campaign achieved its objectives in demonstrating that the new techniques could be used to provide high precision data on Earth rotation, and as a result stimulated the faster development of these techniques. A valuable by-product from this campaign is the set of MERIT standards *[Melbourne, 1983]* which were originally prepared for the MERIT processing.

3.4.2.2 Conventional Terrestrial Reference System (COTES)

In 1980, with the rapid development of geodetic space techniques, it became clear that the current realisation of the terrestrial reference system was inadequate. Hence, the IAU Colloquim No. 56 on "Reference Coordinate Systems for Earth Dynamics" *[Gaposchkin and Kolaczek, 1991]* set up a working group to establish and maintain a new Conventional Terrestrial Reference System that would be based on new geodetic space techniques. This working group became known by the acronym COTES.

The MERIT and COTES working groups held discussions about a possible co-operation in May 1981. It was decided that there were two ways in which differences between reference frames determined by different techniques could be investigated. Firstly, collocation, by determining the coordinates of stations simultaneously using two or more techniques, or secondly, by determining the differences between Earth rotation parameters obtained by each of these techniques. The working groups held a special three month intensive campaign during the MERIT main campaign when all stations observed as frequently as possible, and a special effort was made to collocate mobile systems with permanent systems for different techniques.

The results of the MERIT/COTES campaign clearly showed that VLBI and SLR could provide more precise estimates of polar motion, universal time and the length of the day, than could optical astronomy and Transit which at the time were the principal measurement techniques *[MERIT, 1985]*. Based on these results the two working groups recommended the establishment of an International Earth Rotation Service (IERS) to be based on both VLBI, SLR and LLR. This new service would not only be concerned with, as its name suggests, the determination of all aspects of the rotation of the Earth, but also the establishment and maintenance of a conventional terrestrial reference system. This will consist of the permanent stations used for monitoring Earth rotation and densified partly by mobile systems using the same techniques, but mainly by the use of other developing geodetic space techniques, such as GPS.

3.4.2.3 The Bureau International de l'Heure (BIH) Terrestrial Reference Frame

In 1985, the BIH initiated the realization of the Conventional Terrestrial Reference System. This was achieved, in the frame of the MERIT/COTES project, by the least squares combination of sets of station coordinates and earth rotation parameters, from VLBI, SLR and LLR solutions. This led to a series of realisations of the BIH Terrestrial system: BTS84, 85, 86 and 87. The BTS was realised such that the origin of the pole and zero longitude were consistent with previous realisations of the BIH terrestrial reference system, ie constrained to the CIO and CZM. This maintained continuity with published pole coordinates and universal time.

3.4.2.4 The International Earth Rotation Service (IERS)

Following the recommendations of the MERIT and COTES working groups the International Earth Rotation Service (IERS) was established by the IAU and IUGG and started operation on the 1st January 1988. It continued BIH activities concerning the definition and maintenance of the terrestrial reference frame.

The IERS Reference System is composed of two parts, namely the IERS standards and the IERS reference frames. The IERS standards *[McCarthy, 1992]* are a set of state-of-the-art constants and models used by IERS analysis centres for VLBI, SLR, LLR and GPS data processing, and by the Central Bureau of the IERS for the combination of analysis centre results. The values often differ from the IAG conventional ones where deficiencies have been found. The IERS reference frames consist of the IERS Terrestrial Reference Frame (ITRF) and the IERS Celestial Reference Frame (ICRF) which are realised annually through lists of coordinates of terrestrial sites or compact extragalactic radio sources respectively. In addition the IERS determines the Earth Rotation Parameters (ERP) which connect the ITRF and the ICRF. The realisations of the ITRF and ICRF and observed ERP's are published in the IERS Annual Reports, the latest being *[IERS, 1992]* and further information can be found in the accompanying technical notes.

3.4.2.5 The Realisation of International Terrestrial Reference System

The ITRS is realised yearly by combining sets of station coordinates from VLBI, SLR, LLR and recently GPS analysis. The origin, orientation and scale of ITRF are implicity defined by the adopted coordinates of the terrestrial sites. The origin is located at the geocentre, using SLR, the orientation is defined to be consistent with the CZM and the length unit is the SI metre.

Each set of station coordinates derived from the different techniques, and even coordinate sets from different observation campaigns of the same technique, exhibit small but significant systematic differences in reference system definition. These manifest themselves as non-geocentricity in the origin, non-parallelism of the axes and differences in scale. In order to allow a direct combination of these coordinate sets it is necessary to solve for seven parameter transformations between them. This is achieved as part of a least squares adjustment including all coordinate sets and local survey ties connecting different coordinated points at a single site. Further details of the the compuation of ITRF including model used weighting of coordinate sets can be found in *[Boucher et al, 1992]*.

The successive realisations of the ITRS are

(i) **ITRF0**, the initial realisation which linked the ITRS to the final realisation of the BTS. This was achieved by adopting the origin, orientation and scale of BTS87 *[Boucher and Altamimi, 1989]*.

(ii) **ITRF88**, the first realisation of the ITRS, the origin, scale and orientation adopted are those of ITRF0 *[IERS, 1989]*.

(iii) **ITRF89**, the origin and scale are defined by the 1989 SLR solution of the Centre for Space Research, Texas, the orientation was defined such that no global rotation existed with respect to ITRF88 *[IERS, 1990]*.

(iv) **ITRF90**, the origin and scale are defined by the 1990 SLR solution of the Centre for Space Research, Texas, the orientation was defined such that no global rotation existed with respect to ITRF89 *[IERS, 1991]*

For all the above realisations, no velocity field has been adjusted and so the AM0-2 model is recommended.

(v) **ITRF91**, the origin and scale are defined by the 1991 SLR solution of the Centre for Space Research, Texas, the orientation was defined such that no global rotation existed with respect to ITRF90. In addition a global velocity field was derived from combination of site velocities estimated by SLR and VLBI analysis centres and the Nuvell-1 NNR plate motion model *[IERS, 1992]*

Table T-2 of *[IERS, 1992]* shows the transformation parameters output from the ITRF91 adjustment. These show that the SLR solutions, which define geocentric frames, only show offsets of up to 4 cm in the geocentre (with one exception), the VLBI analysis centres have constrained their solutions close to the ITRF origin (ie within 5 cm) although this is arbitrary and the LLR origin is about 10 cm from SLR, due to the weakness of the LLR network. The SLR scales differ at the 1×10^{-8} level, this is due to the different values of GM used by the analysis centres, and the VLBI scale differs by less than 1×10^{-8}. The orientation is arbitrary, although with the exception of one solution, all SLR and VLBI solutions agree within a few milli-arc-seconds. The weighted 3-d RMS means for the ITRF91 adjustment (table 4 *[Boucher et al, 1992]*) are 1 cm for VLBI and 3 cm for SLR and LLR. This is clearly the most precise coordinare reference system available today.

Access to ITRS can be achieved in two ways. Firstly, connecting the point to one or more points which already exist in the ITRS using either terrestrial survey of geodetic space techniques, or secondly, using published transformation parameters between realisations of the ITRS and WGS84 or BTS87. Transformation to national and continental datums can be achieved through WGS 84.

During the XXth General Assembly of the International Association of Geodesy (IAG), Vienna, August 1991, the IAG Special Study Group (SSG5.123) specified the requirements for the definition and realisation of the Conventional Terrestrial Reference System (CTRS). They considered the implementation of such a system under the name ITRS and recommended (IAG Resolution No. 2) that "groups making highly accurate geodetic, geodynamic or oceanographic analysis should either use the ITRS directly or carefully tie their own system to it" *[Boucher and Altamimi, 1992]*.

3.5 Summary

(1) From the coordinate reference systems described in this chapter it is quite clear that in the UK, any one point can have many different sets of geodetic and cartesian coordinates. In fact, one for each coordinate system, differing by up to hundreds of metres. Therefore, coordinates are not unique and meaningless unless quoted in a reference system or coordinate datum.

(2) The datums for the United Kingdom and North America differ widely. OSGB36 and NAD 27 were both recognised to be in error due to the method of adjustment, however, only NAD 27 was re-defined using a new ellipsoid, GRS80, whereas OSGB70(SN) and OS(SN)80 were again adjusted on the Airy ellipsoid. Since GRS80 is almost identical to the WGS84 reference ellipsoid it is possible for coordinates obtained from satellite systems in WGS 84 to be plotted directly onto the maps and charts of North America. Whereas in the UK, OSGB36 is still the datum for national mapping and multiple regression

equations must be used in order to transform from WGS84 to OSGB36, to allow for the scale variations in the latter *[DMA, 1987]*.

(3) In the UK, as with Europe, transformation parameters between different coordinate reference systems are either unknown or at best only known to a poor degree of accuracy. Europe is in need of a new reference frame which has sufficient stations colocated with each coordinate system to accurately determine these transformation parameters.

(4) Even today, over 30 years later, ED50 is still the internationally accepted datum for positioning in the North Sea and used by NATO for mapping control in Europe. Unfortunately, by the time ED87 was available (34 years after ED50) there was no need for such a terrestrial network. This was due to the advances in satellite positioning which had proven themselves to be accurate and reliable. Europe is, therefore, in urgent need of a satellite based reference system.

(5) With the development of geodetic space techniques such as GPS, which can produce millimetre level relative positioning accuracies, careful consideration must be given to the choice of datum to eliminate, as much as is possible, any errors.

(6) WGS84 (and WGS72) have played an important role in the use of GPS and Transit, and WGS84 is becoming an 'international adopted standard'. Therefore, the UK and Europe are in need of a new mapping and charting datum that can be considered 'identical' to WGS84.

(7) Through the use of GPS many positioning operations are covering areas of more than one country, which have different national datums upon which their maps are based. The digitisation of cartographic data by many countries has provided the ideal opportunity to unify national mapping onto a single European datum.

(8) The adjustments of the Ordnance Survey triangulations have always resulted from improvements in instrumentation and

techniques detecting flaws in the old network. The advances in positioning with GPS will soon detect the errors in OS(SN)80.

(9) The scale correction applied to the Transit Doppler coordinates included in the OS(SN)80 adjustment (-0.4 ppm) differed from the value used in the realisation of WGS 84 (-0.6 ppm) by 0.2 ppm. Clearly one or both of these coordinate sets has a scale error. This is explored further in Chapter 6.

(10) If the whole world used the same reference system for all applications (navigation, cartography, surveying and geodesy) then the problems of datum transformation would disappear. However, this is a very long way ahead and until then the determination of high accuracy transformations is crucial to the success of high accuracy positioning.

CHAPTER 4

The New European Reference Frame

The conclusion of RETrig in 1988, with the publication of the European Datum 1987 (ED87), paved the way for the establishment of the EUREF subcommission to construct a 3-dimensional European Reference Frame (EUREF), to be based on geodetic space techniques. The aim of EUREF was to be achieved through an extensive GPS network, which would be controlled by VLBI and SLR. EUREF will enable transformations between European or National coordinate systems and WGS 84, will provide a geocentric datum for national mapping, and control for further National GPS measurements.

This chapter discusses the processing of the EUREF 89 GPS data set, as carried out by the author at the IESSG in Nottingham and by the 'Bernese Group', and the adoption of the final coordinate set. It starts in sections 4.1 and 4.2 by describing the establishment and work of the new EUREF subcommission. Section 4.3 describes the EUREF 89 GPS campaign and data set, details the stations involved in the UK and the processing centres. Sections 4.4 and 4.5 cover the processing strategy, the types of solutions performed, and the methods used to assess the quality of the results by the IESSG and Bernese Groups respectively. Section 4.6 compares the results from the two independent processing centres and section 4.7 discusses the resolutions passed at the March 1992 EUREF Subcommission Symposium. Finally, section 4.8 describes some further GPS campaigns extending EUREF in to Eastern Europe and North America and the chapter is concluded in section 4.9.

4.1 The European Reference Frame (EUREF)

Europe was in need of a unified geocentric reference frame that is accurate and dense enough to satisfy all the requirements highlighted in chapter 3. The impetus for this came from the rapid development of geodetic space techniques and in particular the adoption of GPS for surveying and navigation. To avoid the confusion that would be caused by national survey agencies and private companies each establishing their own satellite based reference frames, EUREF needed to be established by the scientific community so that it was unique and freely available to all.

In 1987 the IAG urged the RETrig Subcommission to finish its work, namely the readjustment of the first order triangulation network of Europe and the definition of a set of homogeneous coordinates. This resulted in the publication of ED87, a two-dimensional terrestrial network computed using horizontal angles, distances and azimuths, and controlled by geodetic space techniques such as Transit Doppler, VLBI and SLR. As described in chapter 3, ED87 had taken 34 years to compute and still contained many 'black spots'.

During the penultimate Symposium of the RETrig Subcommission in Paris, May 1987 *[Poder and Hornik, 1988]*, the following resolution was passed. The IAG RETrig Subcommission *recognised* "that there is a need for a continuing evaluation and maintenance of three-dimensional reference frames", *noted* "the rapid development of geodetic space techniques" and *recommended* "the establishment of a new subcommission to replace the RETrig subcommission". The RETrig subcommission was disbanded in May 1988.

During the International Union of Geodesy and Geophysics (IUGG) XIXth General Meeting in Vancouver, Canada, August 1987, the International Association of Geodesy (IAG), recognising the conclusion of RETrig and the above resolution, set up a new subcommission called EUREF (EUropean REference Frame). The task of this new subcommission was to establish a three-dimensional reference frame for Europe, accurate to a few centimetres, and maintain it for all types of future needs. Furthermore, the subcommission should define standards for the application of new measuring techniques such as GPS for surveying. This reference frame would be achieved principally through an extensive GPS network and controlled by VLBI and SLR. Since the necessary infrastructure already existed from the RETrig Subcommission, work on EUREF started immediately.

The name 'European Reference Frame' consists of three words which were chosen to have a specific meaning.

European a unified effort involving as many European countries as possible.

Reference connected to a high precision global datum.

Frame the realisation of a geocentric 3D system with stations at a density that will meet all its requirements.

The need for a modern, all-European, precise Terrestrial Reference Frame within the accuracy requirements of navigation and cartography (1-2 m), and that can be considered identical with WGS84, was also recognised by CERCO. In September 1987, during the CERCO Plenary meeting in Athens, it was decided to establish a new Working Group VIII on GPS. This Working Group (WG) was to study the practical consequences which resulted from the growing capabilities of GPS as a high precision positioning system for surveying and mapping. The two commissions, IAG and CERCO, were immediately aware that their work overlapped and that only one future reference system for Europe needed to be established. Therefore, the EUREF Subcommission and CERCO WG VIII agreed to work together on the scientific and practical task of defining and realising, as quickly as possible a new European Reference Frame.

The EUREF Subcommission started work during the final RETrig Symposium, Lisbon, May 1988, and continued with a joint meeting with CERCO in Munich in October of the same year. The outcome of these meetings was firstly, to organise a GPS campaign for May 1989 to occupy about one-hundred first order stations in Europe for several days each (this campaign is described in section 4.3 and the results presented in sections 4.4, 4.5 and 4.6), and secondly that the results must be available within a few years. This was crucial if EUREF was to be a success and not turn out to be an 'academic exercise' like RETrig. The next symposium was held in Florence in May 1990 which dealt with the reference system and the reference frame.

4.2 The European Terrestrial Reference System (ETRS)

The need for a unified European Reference Frame arose from the need to determine transformation parameters between National Coordinate Systems and to establish a satellite based coordinate system which is consistent with WGS84, and hence GPS. The only critical disadvantage of WGS84, as far as precise geodesy is concerned, is that it was realised using Transit Doppler measurements (see section 3.4.1.2), which on a global scale can only provide an accuracy of 1-2 m in all three dimensions. Therefore, WGS84 itself cannot be of a higher accuracy. Since future requirements of a continental reference frame will be significantly higher than 1 m, WGS84 cannot be adopted as the new European Reference System. Therefore, a reference frame accurate to a few centimetres and coincident with WGS 84, at the 1-2 m level, is required. These requirements can only be met by a reference frame based on the techniques of VLBI and SLR which can produce precisions of 1-3 cm over distances of up to 5000 km. Global networks such as the International Terrestrial Reference Frame (ITRF) are realised yearly by combining VLBI and SLR positioning results (see section 3.4.2).

Resolution Number 1 adopted at the EUREF symposium in Florence, 28 - 31 May 1990 *[Gubler et al, 1992a], recognised* "the availability of the International Terrestrial Reference Frame which has been established by the International Earth Rotation Service, is accepted world-wide, and uses VLBI, SLR, and LLR observations" *noted* "that in such a system, station positions in Europe have a common rotation of the order of one centimetre per year" *recommended* "that the system to be adopted by EUREF will be coincident with ITRF at the epoch 1989.0 and fixed to the stable part of the Eurasian plate, and will be known as the European Terrestrial Reference System 1989 (ETRS 89)" and *accepted* "that this geocentric system will coincide with WGS 84 at the one metre level and, for most applications, the coordinates will have no time variation".

The following is a summary of the reference systems and frames involved in the realisation of EUREF. The reader is refered to section 3.1 for an explanation of the conceptual difference between a reference system and a reference frame.

ITRS (89) the implementation of the Conventional Terrestrial Reference System by the IERS (see section 3.4.2).

ITRF 89 realisation of ITRS by the IERS, through combining VLBI, SLR and LLR station coordinates up to 1989 at the epoch of 1988.0 (see section 3.4.2).

ETRS 89 a coordinate system coincident with ITRF 89 at the epoch

1989.0 and fixed to the stable part of the Eurasian plate.

ETRF 89 realisation of ETRS 89, achieved by adopting the ITRF 89 European VLBI and SLR station coordinates, mapped from epoch 1988.0 to 1989.0 using the AMO-2 plate motion model.

The determination of transformation parameters between National Coordinate Systems and ETRS 89 requires at least three but ideally more points common to both systems. The number of points in ETRF is not sufficient to fulfil this requirement, therefore, ETRF was densified using the EUREF 89 GPS campaign.

EUREF 89 realisation of ETRS89, consisting of the coordinates of the European stations resulting from the densification of ETRF 89 by the EUREF 89 GPS campaign. ETRF 89 will allow orbit improvement and define the reference frame for the GPS adjustment. In addition ETRF 89 will be realised by the publication of the following transformation parameters,

- ETRF 89 to ED 50 and reverse
- ETRF 89 to ED 87 and reverse
- ETRF 89 to National Coordinate Systems and reverse
- ETRF 89 to ITRF and reverse (re-computed every year)
- ETRF 89 to WGS 84 and reverse (once differences become significant)

There were two additional resolutions passed during the EUREF symposium, Florence 1990 *[Gubler et al, 1992a]*, which are worth mentioning. Resolution No. 4 *recommended* "that data collected during GPS campaigns (ie EUREF 89 and National densifications) should be retained in a form suitable for future use", ie so that it can be re-processed if a new European Reference System was defined or WGS 84 superseded. Resolution No. 5 *recognised* "the need for a European directory of EUREF stations, including site descriptions, common European coordinates and station eccentricities". This is particularly important since high accuracy GPS controlled by VLBI and SLR is highly dependent upon the accurate measuring and recording of local site offsets between the VLBI/SLR and the GPS reference marks.

4.3 The EUREF 89 GPS Campaign

Each participating country was asked to propose GPS stations that were part of their first order triangulation network at distances of about 300 to 500 km apart. Based on these proposals the EUREF Subcommission selected 93 stations. In central Europe with many small countries, a certain concentration of stations could not be avoided (see Figure 4.1).

The EUREF 89 GPS campaign took place in May 1989, as a joint effort between the participating National survey agencies. Due to the limited number of GPS receivers available at the time, the network of 93 stations was observed as two 6 day phases. The duration was choosen to ensure, even with receiver failure on one or two days, that sufficient observations would be collected at each station to allow for accurate positioning. Phase A included 62 stations, and was observed on 16 - 21 May 1989. Phase B included 55 stations, and was observed on 23 - 28 May 1989. There were 23 stations common to both phases, which included 15 VLBI/SLR reference stations, and 8 other additional stations to help in the combination of the two phases.

A total of sixty-nine GPS receivers were used during the two week campaign, including; twenty-one TI-4100 (dual frequency with P code L2), fifteen WM102 (dual frequency with P code L2), four Minimac 2816 (dual frequency with P code L2), and twenty-nine Trimble 4000 SLD (dual frequency with L2 squaring). Seven spare receivers were distributed across Europe and all were needed. As mixed receiver types were used, an antenna calibration exercise was performed in Wettzell prior to the campaign. This was used to compare the antenna phase centre offsets of the different receiver types, and to test newly developed software for transforming the raw receiver data into a Receiver Independent Exchange Format (RINEX). RINEX was developed by the Astronomisches Institut Universität Berne, and adopted by the GPS community during a symposium in Las Cruces, New Mexico, March 1989 *[Gurtner et al, 1989a]*. The results of the antenna calibration exercise *[Gurtner et al, 1989b]* agreed well with results from a similar calibration exercise observed in Greece during the same year, and computed at the IESSG *[Ashkenazi et al, 1991c]*. This suggested that different receiver types could successfully be mixed at the 1 cm level.

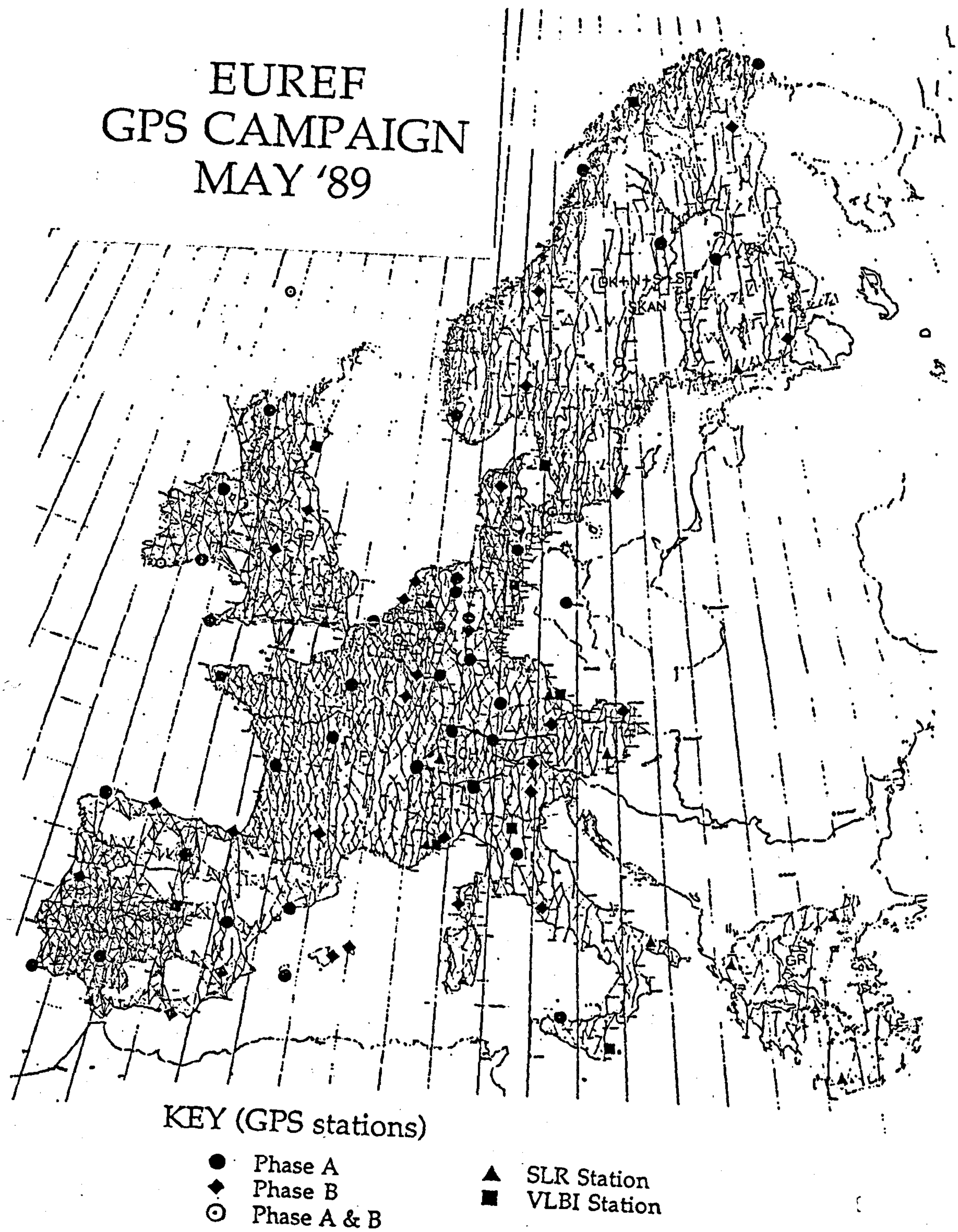


Figure 4.1 EUREF 89 GPS Campaign Stations *[Poder et al, 1986b]*.

The limited satellite constellation over Europe in May 1989 meant that only seven Block I satellites were available. These included SV08 which was using its quartz clock and therefore its observations are less reliable. The observation window used in the EUREF 89 GPS campaign was 1100 - 1600 UT for phase A and 1000 - 1500 UT for phase B. Figures 4.2 and 4.3 show the limited constellation and poor geometry of the satellites, compared to what is possible with the current constellation (see Figure 5.3 for the 1991 constellation and the improvement over only two years).

Twelve preprocessing centres were established to screen the data, convert the receiver binary files into RINEX and load onto magnetic tape. These tapes were then sent to the Astronomisches Institut Universität Berne for distribution to the processing centres on request.

4.3.1 The 1989 Mobile VLBI (MVLBI) Campaign

ITRF 89 and consequently ETRF 89 contained many VLBI and SLR stations in Southern and Central Europe, due to previous campaigns such as the WEGENER-MEDLAS program (Workshop of Européan Geoscientists for the Establishment of Networks for Earthquake Research - MEDiterranean LASer project) which uses mobile SLR systems to occupy 11 sites every two years. However, there was an obvious lack of stations in Northern and Western Europe. Since these stations were necessary to define the reference frame and allow orbit improvement in the GPS adjustments it was considered necessary for the reference station network to be densified.

A mobile VLBI unit was hired from the United States National Geodetic Survey and visited 6 sites, for 5 days each, from June to September 1989. These sites were, Hohenbunstorf (Germany), Metsahovi (Finland), Tromsoe (Norway), Buddon (Scotland), Brest (France) and Grasse (France). The results of the mobile VLBI campaign produced coordinates with precisions of 1 cm in plan and 3 cm in height *[Abell and Morrison, 1990]*. These stations were included in the ITRF 89 adjustment and all EUREF 89 reference stations are shown in Figure 4.4.

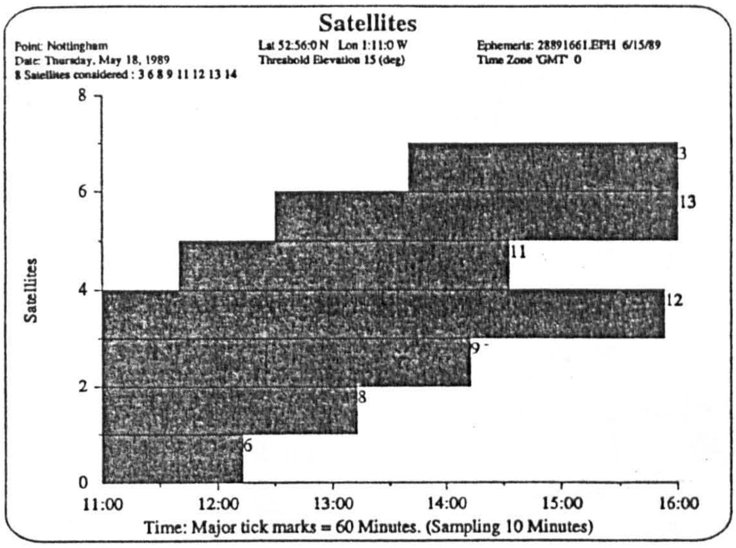


Figure 4.2 EUREF 89 GPS Campaign Satellite Availability Plot.

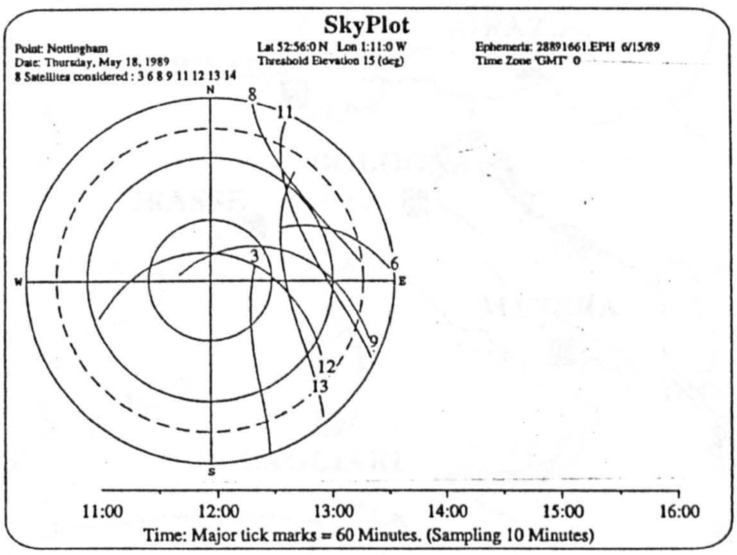


Figure 4.3 EUREF 89 GPS Campaign Sky Plot.

Figure 4.4 EUREF 89 Reference Stations.

4.3.2 The UK GPS Data Set

The EUREF 89 GPS Campaign included 9 UK stations; Bartinney, Herstmonceux, and Collier Law in England, Moel Fammau in Wales, Buddon and An Cuaidh in Scotland, and Carrigfadda, Carrigaderragh, and Crockinagoe in Eire (see Figure 4.5). All UK Stations were occupied with Trimble 4000 SLD receivers. Hardware problems with this particular model of Trimble receiver unfortunately meant that a large amount of the second (L2) frequency phase data was lost. The percentage of L2 data (compared with the total of L1 data - assumed perfect) is summarised in table 4.1, on a station-by-station basis. This hardware problem also affected about 80% of the Trimble receivers used across mainland Europe.

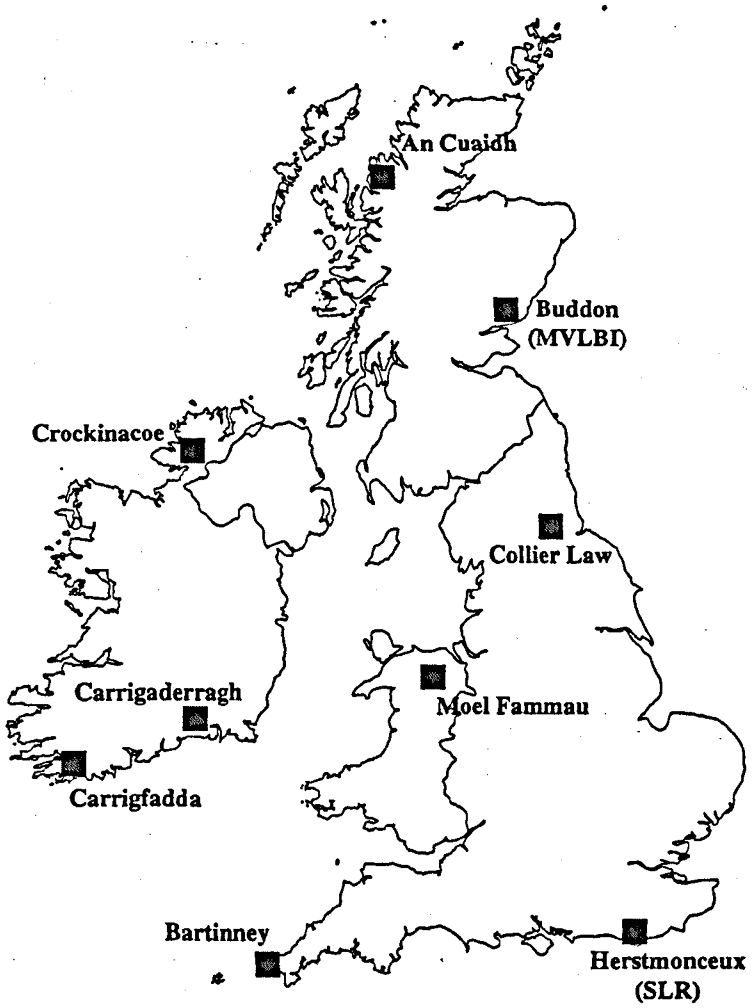


Figure 4.5 EUREF 89 UK Stations.

	Phase A	Phase B
Bartinney	65	-
Herstmonceux	55	58
Collier Law	-	15
Moel Fammau	-	56
Buddon	87	90
An Cuaidh	42	-
Carrigfadda	20	20
Carrigaderragh	63	-
Crockinagoe	59	-
Average	56	48

Table 4.1 Percentage of L2 data compared with L1 data for EUREF 89 UK stations.

The clear conclusion from table 4.1 is that only about half of the intended data set for the UK was available for dual frequency processing. Moreover, station Carrigfadda (one of the 4 non-VLBI/SLR EUREF sites selected for inclusion in both phases of the campaign) and station Collier Law had so little dual frequency data available that the resulting coordinates for these stations would be poorly determined. It later transpired that the receiver at Collier Law suffered interference from local telemetry links blocking the GPS signals. This UK EUREF station has since been relocated to Danby Beacon in subsequent campaigns.

4.3.3 Processing Centres

The EUREF 89 GPS Campaign was processed at a number of geodetic centres across Europe, including:

Astronomisches Institut Universität Berne (AIUB),
Bayerische Kommission für die Intern. Erdmessung, Munchen (BEK),
Institut für Angewandte Geodäsie, Frankfurt (IfAG),
Institut Géographique National, Paris (IGN),
Faculty of Geodesy, University of Delft, The Netherlands,
Faculty of Aerospace, University of Delft, The Netherlands,
Institute of Engineering Surveying and Space Geodesy, University of

Nottingham (IESSG).

Due to the size of the EUREF 89 GPS campaign the first four of these institutions (AIUB, BEK, IfAG and IGN) combined resources to produce a joint solution for the whole network. They all used the Bernese GPS software and, therefore, were collectively known as the 'Bernese Group'. The two Delft Groups concentrated on processing the EUREF data from the Netherlands along with data from the reference stations. Data processing at IESSG concentrated on obtaining a solution for the UK stations, through the use of the reference network. Five of the seven processing centres used Bernese Version 3.2 GPS software and Delft Aerospace used the Jet Propulsion Laboratory GIPSY software. The IESSG used in-house developed GPS Software, as detailed in section 2.2.8, and are the only check on the Bernese Group solution for the UK.

4.4. The IESSG Processing of the EUREF 89 GPS Campaign

The raw data was provided in RINEX format, at a 30 second epoch separation with meteorological data from selected stations. The data pre-processing strategy employed at IESSG was as follows:

(i) Convert the RINEX format data into NOTTM1 format.
(ii) Define the optimum baseline configurations for the UK station sub-networks.
(iii) Clean the UK station sub-network data by performing cycle slip editing on the defined UK station sub-network baselines.
(iv) Define the optimum baseline configurations for the 'inter-reference station baselines'.
(v) Clean the reference station data by performing cycle slip editing on the defined inter-reference station baselines.
(vi) Select the data down to a 'common' one minute time-tag.

Within the UK network, the normal process of selecting baselines to minimise their length could not be followed because of the gaps in the L2 data, ie the shortest baselines did not share sufficient common data. Instead, baselines had to be selected to give the most common data, with the result that all baselines had a common station, Buddon. The length of the resulting baselines meant that ambiguity resolution within the UK could not been performed. Therefore, all solutions described here are 'ambiguity free' solutions, using the following options

- ionospherically free observable (L1/L2)
- Magnet Tropospheric Model *[Curley, 1988]*, with
- Tropospheric scale factor per station (per session)

The empirical Magnet tropospheric model was selected in preference to the use of the surface meteorological data provided due to the problems experienced on previous campaigns with the calibration of meteorological instruments. Results obtained *[Ashkenazi et al, 1988]* indicated that this model will very often yield solutions which are better than those obtained with surface meteorological values.

4.4.1 Types of Solution

Once the data had been cleaned of cycle slips, four types of solution were performed, namely;

- NGS precise ephemeris 'free network' solutions,
- IESSG precise ephemeris 'free network' solutions,
- IESSG precise ephemeris 'constrained network' solutions,
- Rigorous fiducial network solutions.

All solutions were carried out with the sole purpose of determining the coordinates of the UK stations, although each solution required the inclusion of the reference network data in order to allow the estimation of the satellite orbits and/or to define the required reference frame.

In Phase B apart from Buddon and Herstmonceux (whose ETRF 89 coordinates will not be improved by the EUREF 89 GPS campaign), there was only one station (Moel Fammau) which had sufficient data to produce a solution. For this reason, the processing at IESSG has not addressed Phase B.

The amount of data available at the UK stations in Phase A meant that the optimal orbital span for each solution was three days. Shorter orbital spans gave very poor repeatability from one solution to the next, whilst longer periods gave no extra improvement, and further compromised the ability to perform repeatability tests on the results. The data from Phase A was, therefore, treated as three 3-day arcs, each overlapping the next by two days (see Table 4.2). The results from each of these solutions are, therefore, not entirely independent, but repeatability tests could still be used to give some measure of their precision.

Arc 1	Day 136	Day 137	Day 138	-	-
Arc 2	-	Day 137	Day 138	Day 139	-
Arc 3	-	-	Day 138	Day 139	Day 140

Table 4.2 Data Arcs used in the processing of Phase A.

For those solutions requiring the calculation of the satellite ephemerides the same 3-day arcs were used for each ephemeris determination.

- **NGS Precise Ephemeris 'Free Network' Solutions**

In an NGS (National Geodetic Survey) precise ephemeris 'free network' solution, the UK station data and reference station data were included. All station coordinates were adjusted in the solution (ie free), and the satellite orbits were held fixed to the NGS precise ephemeris (post computed, $\sigma < 10$ m). However, the reference frame of the NGS orbits could not be guaranteed to conform to the ETRF. Moreover, so-called 'free networks', in which no receiver coordinates are held fixed, usually display systematic reference frame biases (3 translations of the origin, 3 rotations of the axes and scale), even when the orbits can be guaranteed to conform with the reference frame. For these reasons, seven parameter transformations had to be computed in order to transform the derived reference station coordinates back into the ETRF 89 coordinate reference frame. This transformation was then applied to the UK station coordinates. This procedure was calculated to give the best definition of the ETRF, so that the derived UK coordinates would conform with similar solutions submitted by other centres.

- **IESSG Precise Ephemeris 'Free Network' Solutions**

For an IESSG precise ephemeris 'free network' solution, an ephemeris was first generated using the reference station data alone, by holding the reference station coordinates fixed, adjusting an initial integrated orbit and solving for corrections to the corresponding satellite state vectors (position and velocity). This procedure was iterated until no further corrections were required to the state vectors. They were then integrated and converted into precise ephemeris format to produce the IESSG Precise Ephemeris. The same free network approach as for the NGS precise ephemeris solution was then employed to define the coordinate reference frame, even though the ephemeris reference frame was consistent with ETRF.

- **IESSG Precise Ephemeris 'Constrained Network' Solutions**

For an IESSG precise ephemeris 'constrained network' solution, the ephemeris calculated for IESSG precise ephemeris 'free network' solution was used and selected stations were fixed in order to remove the need for the post adjustment transformations. In general, if the satellite positions are held fixed and in addition some of the stations are fixed the result is a clash between the fixed ranges (fixed satellite to fixed station) and the measured value. This difference is then forced into the free ranges (fixed satellite to free station) and consequently into the free station coordinates. These free stations will move in a direction either towards or away from the centroid of the satellite constellation resulting in a scale type error, affecting predominantly the height component. In theory this orbit is in harmony with the reference frame so fixing the reference stations should not cause clashes in the network. The advantage of this technique is that a single precise ephemeris could be computed for EUREF 89 and then distributed to processing centres for computation of sub-sections of the whole network.

- **Rigorous Fiducial Network Solutions**

In the rigourous fiducial network solutions, UK station data and reference station data were included, all of the reference stations were held fixed, and the integrated satellite orbits were adjusted as part of the solution. This procedure corresponds to the rigorous fiducial approach (see section 2.2) which theoretically produces the most accurate receiver coordinates.

4.4.2 Quality Assessment Criteria

The results from the different solutions described above are assessed according to the following criteria.

i) Reference station coordinates

The accuracy of the recovery of the reference station coordinates, assuming the VLBI/SLR coordinates are error free, is presented as an RMS difference from the known coordinates. This criterion gives an indication of the likely accuracy of the UK coordinates.

ii) UK station coordinates

The repeatability of the UK coordinates between the three 3-day arcs gives an indication of their precision. This criterion can be applied to all categories of solution.

iii) Satellite Ephemerides

For the IESSG Precise Ephemeris approach, the recovered orbits can be used as an indication of the stability of the solutions. The agreement between overlapping portions of the three IESSG 3-day arcs can be used to indicate precision, and agreement between the NGS orbits and the IESSG orbits can be used to assess accuracy.

iv) Buddon - Herstmonceux

From the point of view of the UK data set, a comparison between the known coordinates of the two ITRF stations (Buddon and Herstmonceux) and their recovered coordinates can be made. This could not be done for the rigorous fiducial network solution, since these two stations were held fixed in the solution.

v) OS(SN)80 coordinates

Finally, as the only practical external check on the accuracy of the results, the UK coordinates can be compared with their national geodetic (OS(SN)80) values. This test will probably say more about the quality of the OS(SN)80 coordinates, rather than the EUREF values themselves since they are of a lower accuracy (see section 6.1).

4.4.3 Results

- **NGS Precise Ephemeris 'Free Network' Solutions**

These solutions were the most straightforward to produce, requiring none of the complex orbit integration software.

(i) Reference station coordinates

The accuracies with which the three 3-day arcs recovered the reference frame coordinates are shown below. These represent the RMS differences in all components for all stations.

Arc 1	ΔX:	15 cm	Δφ:	13 cm
	ΔY:	10 cm	Δλ:	10 cm
	ΔZ:	7 cm	Δht:	10 cm
	ΔLength:	17 cm		
Arc 2	ΔX:	18 cm	Δφ:	16 cm
	ΔY:	12 cm	Δλ:	12 cm
	ΔZ:	7 cm	Δht:	11 cm
	ΔLength:	20 cm		
Arc 3	ΔX:	7 cm	Δφ:	4 cm
	ΔY:	5 cm	Δλ:	4 cm
	ΔZ:	6 cm	Δht:	8 cm
	ΔLength:	6 cm		

(ii) UK station coordinates

The repeatability of the UK coordinates is expressed as RMS differences between all combinations of the three 3-day arcs.

Arc 1 - Arc 2	ΔX:	14 cm	Δφ:	9 cm
	ΔY:	12 cm	Δλ:	11 cm
	ΔZ:	8 cm	Δht:	14 cm
	ΔLength:	12 cm		

Arc 1 - Arc 3	ΔX:	19 cm	$\Delta\phi$:	17 cm
	ΔY:	14 cm	$\Delta\lambda$:	13 cm
	ΔZ:	13 cm	Δht:	16 cm
	ΔLength:	18 cm		
Arc 2 - Arc 3	ΔX:	21 cm	$\Delta\phi$:	20 cm
	ΔY:	13 cm	$\Delta\lambda$:	13 cm
	ΔZ:	8 cm	Δht:	10 cm
	ΔLength:	18 cm		

(iii) Buddon - Herstmonceux

The baseline between Buddon and Herstmonceux was one of the most poorly determined of all the baselines, reflecting the small quantity of data at Herstmonceux. The comparisons with the known coordinates of these stations ranged from around 30 cm for Arc 1 to 10 cm for Arc 3, approximately twice the RMS for the complete reference network. This is important as an indication of the effect of the missing L2 data, since the other UK stations were similarly affected.

- **IESSG Precise Ephemeris 'Free Network' Solutions**

(i) Satellite Ephemerides

The ephemeris accuracy is first assessed by comparing the satellite coordinates given by the NGS precise ephemeris with those in the IESSG precise ephemeris. The comparisons are restricted to the portions of the orbit for which tracking data were available, since the orbit determination will not be reliable in untracked areas.

For most of the satellites, an average daily RMS agreement between the orbits of better than 3 metres was achieved for periods of tracking (see Figure 4.6), although there was a clear dependence on the amount of data available for each satellite, eg SV06, for which very little dual frequency data was available, agreed to little better than 8 m. When combined with Figure 4.7, these results seem to suggest that a minimum of two to three hours of data per day are needed for orbit improvement.

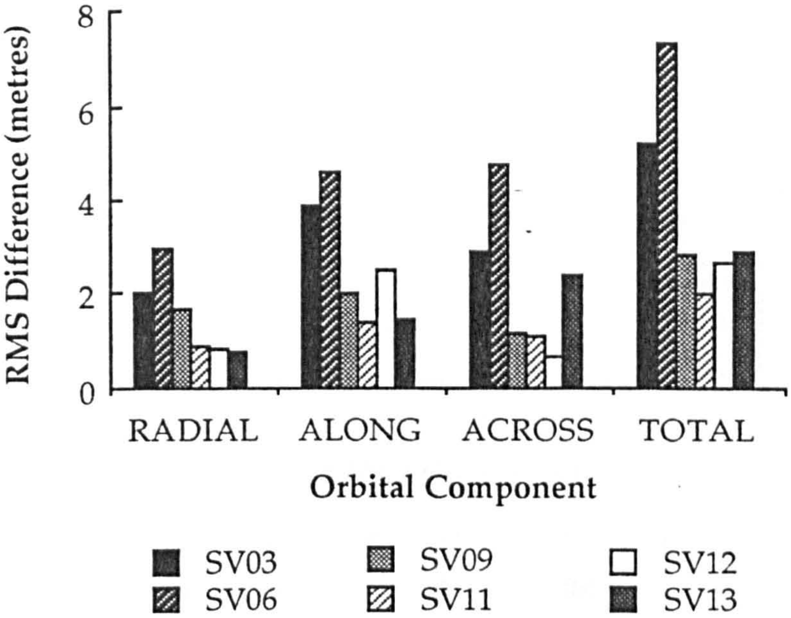


Figure 4.6 Average Daily RMS Differences between IESSG and NGS Precise Ephemerides for periods of tracking only.

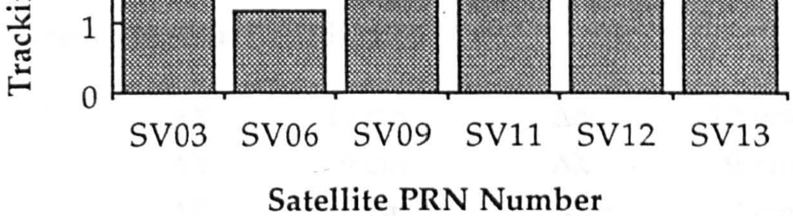


Figure 4.7 Average Daily Tracking time in hours for each satellite

Overlapping sections of the IESSG precise ephemerides also agreed to within 3 metres, with the exception of SV03 and SV06 (Figure 4.8). In conjunction with the NGS comparisons, this suggests an accuracy for the IESSG precise ephemerides of the order of 2 to 3 parts in 10^7.

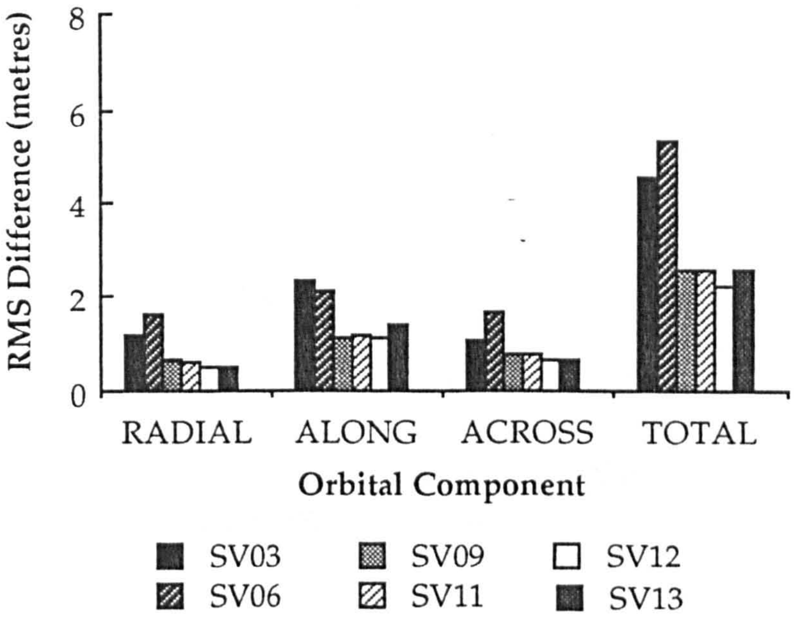


Figure 4.8 RMS Differences between IESSG ARC1 and ARC3 on Julian Day 138 for periods of tracking only

The two-step procedure employed in these solutions produced precise ephemerides based only on the reference station data, and then resulted in solutions which are conceptually the same as the above NGS precise ephemeris 'free network' solutions. The same comparisons, ie reference stations coordinates, UK station coordinates and Buddon-Herstmonceux, can therefore be applied, and give the following results.

(ii) Reference Station Coordinates

Arc 1	ΔX:	13 cm	$\Delta\phi$:	12 cm
	ΔY:	9 cm	$\Delta\lambda$:	9 cm
	ΔZ:	6 cm	Δht:	7 cm
	ΔLength:	14 cm		
Arc 2	ΔX:	16 cm	$\Delta\phi$:	15 cm
	ΔY:	10 cm	$\Delta\lambda$:	11 cm
	ΔZ:	6 cm	Δht:	10 cm
	ΔLength:	18 cm		

Arc 3	ΔX:	4 cm	Δφ:	3 cm
	ΔY:	4 cm	Δλ:	4 cm
	ΔZ:	5 cm	Δht:	6 cm
	ΔLength:	5 cm		

(iii) UK Station Coordinates

Arc 1 - Arc 2	ΔX:	13 cm	Δφ:	8 cm
	ΔY:	12 cm	Δλ:	11 cm
	ΔZ:	10 cm	Δht:	13 cm
	ΔLength:	13 cm		
Arc 1 - Arc 3	ΔX:	18 cm	Δφ:	16 cm
	ΔY:	14 cm	Δλ:	13 cm
	ΔZ:	15 cm	Δht:	17 cm
	ΔLength:	19 cm		
Arc 2 - Arc 3	ΔX:	18 cm	Δφ:	18 cm
	ΔY:	12 cm	Δλ:	13 cm
	ΔZ:	8 cm	Δht:	9 cm
	ΔLength:	17 cm		

As a preliminary conclusion, it can be seen that the reference station recovery is slightly improved over the NGS precise ephemeris 'free network' solutions, although the UK station precision seems unaffected.

(iv) Buddon - Herstmonceux

This baseline was again very poorly determined, with virtually the same results as the NGS Precise Ephemeris 'free network' solutions.

- **IESSG Precise Ephemeris 'Constrained Network' Solutions**

Tests were performed using the IESSG Precise Ephemeris and fixing one (Herstmonceux), two (Herstmonceux, Buddon), three (Herstmonceux, Onsala, Wettzell) and all the reference stations. The arc to arc UK station repeatabilities were degraded (22 to 35 cm) as the number of fixed reference stations increased. This confirmed that as the number of fixed stations is increased, more 'clashes' between the fixed ephemeris and fixed stations are introduced into the network. These distortions are then absorbed by the solution unknowns, ie the UK station coordinates. Because the results from this solution are of a lower quality than those from the IESSG Precise Ephemeris 'Free Network' they have not been included.

- **Rigorous Fiducial Network Solutions**

The fiducial solutions have a principal advantage over fixed ephemeris approaches, no matter how accurate the fixed ephemeris is claimed to be. The inclusion of additional unknown ephemeris parameters in the least squares solution, whilst allowing the orbits to adjust to slightly different positions, provides a means of absorbing small unmodelled error sources into parameters other than the station coordinates. Tests with the IAG standard data set have shown that this method of solution can provide a very precise solution for receiver coordinates *[Dong and Bock, 1989]*. However, it must be remembered that any error in the fixed fiducial stations will propagate directly into the solution.

(i) UK station coordinates

Since the solution requires that the reference station coordinates are constrained to their known values, there can be no test of the ability of the solution to recover the reference station coordinates. However, the UK coordinates are determined in each arc, and their repeatability from one solution to the next is summarised here.

UK coordinate repeatability (rms difference between pairs of arcs)

Arc 1 - Arc 2	ΔX:	12 cm	$\Delta\phi$:	8 cm
	ΔY:	9 cm	$\Delta\lambda$:	7 cm
	ΔZ:	5 cm	Δht:	12 cm
	ΔLength:	12 cm		
Arc 1 - Arc 3	ΔX:	11 cm	$\Delta\phi$:	7 cm
	ΔY:	8 cm	$\Delta\lambda$:	7 cm
	ΔZ:	6 cm	Δht:	12 cm
	ΔLength:	10 cm		

Arc 2 - Arc 3	ΔX:	1 cm	$\Delta\phi$:	2 cm
	ΔY:	1 cm	$\Delta\lambda$:	1 cm
	ΔZ:	2 cm	Δht:	1 cm
	$\Delta Length$:	3 cm		

The improvement over both of the fixed ephemeris approaches is clear, with baseline precision now around the 10 cm level.

(ii) OS(SN)80 coordinates

Most of the UK EUREF sites have coordinates in the Ordnance Survey Scientific Network OS(SN)80, and a comparison has been made between the 2 dimensional coordinates from the OS(SN)80 adjustment *[Christie, 1992]* and the results of the fiducial EUREF processing. The two solutions are based on different reference frames, and it was, therefore, necessary to attempt to remove the systematic biases between the solutions, before the relative coordinate agreement could be assessed. The only biases that were significant (ie 2-3 times larger than their standard error) were the three translations of the origin. Once these three biases had been removed the comparison between the solutions demonstrated that the plan coordinates differed by approximately 25 cm. However, since the standard errors for the biases were as large as one metre this comparison cannot give an indication of the quality of either set of coordinates.

(iii) Recovery of Reference Stations

In the original fiducial solution all nineteen reference stations were held fixed to their ETRF 89 values. To define a reference frame a minimum of only three fixed stations are needed, but the reference frame will be more accurately defined as the number of fixed reference stations increases. Tests were performed, releasing one reference station at a time in the fiducial adjustment and comparing their known to recovered coordinates. The recoveries differed, from arc to arc, by a few centimetres and the values for arc 2 are shown in Table 4.3.

Free Station	$\Delta\phi$ (cm)	$\Delta\lambda$ (cm)	Δht (cm)	Technique
Herstmonceux	3	-5	7	SLR
Buddon	3	4	-11	MVLBI
Brest	-3	6	-9	MVLBI
Wettzell	2	-3	4	VLBI/SLR
Onsala	2	-4	-4	VLBI

Table 4.3 Recoveries of free reference stations for Arc 2 (cm) (GPS - SLR/VLBI)

These show that the permanent VLBI and SLR stations are recovered to 4 cm in plan and 4 cm height, whereas, the MVLBI stations are recovered to 6 cm in plan and 11 cm in height. The recoveries indicate that there 'may' be a problem with the height of Buddon and to a lesser extent Brest. These discrepancies were thought to be either due to incorrectly measured antenna heights (GPS or Mobile VLBI) or high ionospheric activity during the Mobile VLBI campaign. This anomaly is explained in Chapter 5.

4.4.4 Summary

The IESSG precise ephemeris 'free network' solution slightly improved the recovery of the reference station coordinates when compared with the NGS precise ephemeris 'free network' solution, but did not improve the UK station coordinate determination (20 cm), probably due to the lack of L2 data in the UK. The rigorous fiducial network approach has, however, considerably improved the precision of the coordinates of the UK stations, with baseline repeatabilities of 10 cm being achieved. It is this solution which was therefore submitted to the EUREF subcommission. The fiducial network solutions from the three arcs, although sharing some common data, have simply been averaged to produce a single Phase A solution. The IESSG UK coordinates are published in *[Ashkenazi et al, 1992a]*.

The precision of the UK coordinates, whilst somewhat lower than anticipated, is still sufficient for the primary purpose of transforming between WGS84 and the National or European coordinate system and as a new geocentric datum for National mapping. However, the resulting coordinate precisions are not comparable with those of the VLBI/SLR stations, and are not suitable for the control of national GPS measurements.

There are a number of contributory factors to the lower than expected accuracy for the UK EUREF stations. The main reason is the quantity of missing L2 data, which shortened the span of data on each baseline. The effects of the shortened spans were minimised by selecting baselines with the maximum of common data, but this in turn meant that ambiguity resolution was impractical over the resulting long baselines. This problem was further compounded by the fact that the L2 data was not full-wavelength P code data, but 12 cm 'squared' data, meaning that the wide lane ambiguities were only 43 cm instead of 86 cm.

4.5 Bernese Group Processing of the EUREF 89 GPS Campaign

This section is a brief summary of the strategy used and results produced by the Bernese Group for the EUREF 89 GPS campaign, full details can be found in *[Gurtner et al, 1992]*.

In order to facilitate the data handling and the simultaneous processing of the whole campaign, by the four institutions, it was split into 6 sub-networks as shown in Table 4.4.

Subnetwork	Consisting of	Responsible
EUREF-TR	All VLBI and SLR sites (Tracking net)	IGN + AIUB
EUREF-CW	UK, Ireland, France, Belgium	BEK
EUREF-SW	Spain, Portugal	BEK
EUREF-CE	Netherlands, Germany, Austria, Switzerland	IfAG
EUREF-SE	Italy, Greece	IfAG
EUREF-NO	Denmark, Norway, Sweden, Finland	AIUB

Table 4.4 Bernese Group Subnetworks

4.5.1 Type of Solution

The Bernese Group adopted a strategy similar to that of the IESSG Precise Ephemeris 'Constrained Network' Solution. Firstly, the EUREF-TR sub-network, containing all the VLBI and SLR sites, was split into four periods of three days and by holding the stations fixed to ETRF 89 coordinates, each period was used to produce a precise ephemeris. These precise ephemerides were then used for the processing of the five individual sub-networks. Selected reference stations were held fixed in the processing of the individual sub-networks, and to reduce the effect of splitting the network some reference stations in surrounding sub-networks were included. Ambiguity resolution was not attempted due to the length of baselines and poor quality of the data.

4.5.2 Quality Assessment Criteria and Results

The Bernese Group used the following criteria to assess the quality of their solution.

i) Formal Errors

The variances output from the least squares adjustments give an internal 'relative' indication of the strength of weakness of any individual stations within the solution. Two stations in the UK showed large formal errors, namely, Colier Law and Carrigfadda. This was undoubtedly due to the missing L2 data.

ii) Repeatability

The average day to day repeatability for the whole network was approximately 2 - 3 cm and 15 cm for the UK stations. Again this was undoubtedly due to the missing L2 data.

iii) Recovery of VLBI/SLR coordinates

The RMS differences between ten VLBI/SLR coordinates and the GPS determined values was 4.3 cm and 6.4 cm in the horizontal and vertical components respectively. Assuming the ETRF 89 coordinates not be in error this suggests that the accuracy of the new GPS station solutions is about 4 cm and 6 cm in the horizontal and vertical components respectively. The recovery of three of the stations either in the UK or in close proximity are shown in Table 4.5. These stations were not fixed in the final Berne Group solution and their VLBI/SLR coordinates substituted into the final adopted coordinate set.

Free Station	$\Delta\phi$ (cm)	$\Delta\lambda$ (cm)	Δht (cm)	Technique
Buddon	-1	-4	-5	MVLBI
Brest	6	1	-11	MVLBI
Hohen'storf	2	-5	-3	MVLBI

Table 4.5 Recoveries of free reference stations (cm) (GPS - SLR/VLBI)

These suggests that there is a problem with the latitude and height of Brest which is explored further in chapter 5.

4.6 Comparison of Results

A direct comparison between the IESSG and Bernese Group final solutions is shown below in Table 4.6.

Station	$\Delta\phi$ (mm)	$\Delta\lambda$ (mm)	Δht (mm)
An Cuaidh	-5	-58	-127
Bartinney	109	155	34
Crockinnagoe	35	73	-79
Carrigaderagh	-20	-50	-78

Table 4.6 A Direct Comparison between IESSG and Bernese Group Final Coordinates (mm).

The reason for these differences may be manifold. The data set was of a poor quality, particularly in the UK, with large gaps and missing L2 observations. Combined with the high ionospheric activity (see Figure 4.8) it was extremely difficult to detect and accurately repair cycle slips, which was very much based on personal subjective judgement. Furthermore, the differences in software (single or double difference observables), processing strategy (rigorous or fixed ephemeris approach) and processing options (minimum elevation angle, epoch separation, tropospheric models, etc) will also cause differences. The influence of such differences being exaggerated with such a poor quality data set.

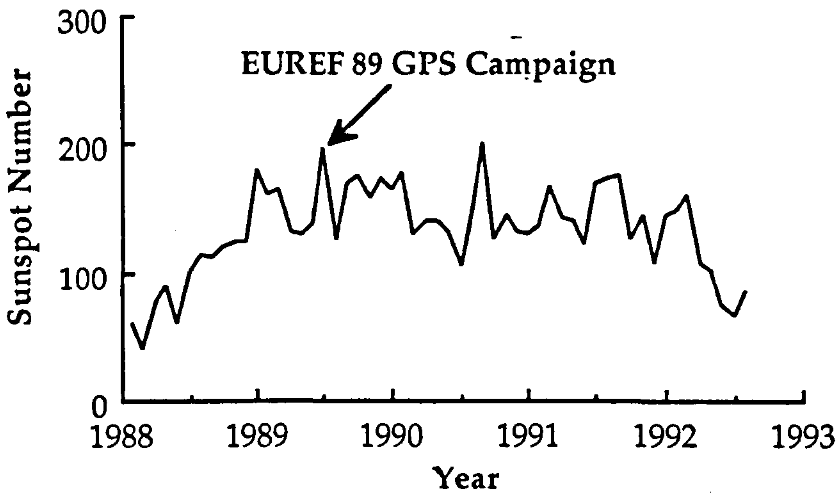


Figure 4.9 Ionospheric (sunspot) activity from 1988 to 1992.6
[Clarke, 1993].

One further comparison was performed between the two (IESSG and Bernese) Precise Ephemerides. The average daily RMS difference between the two orbits, for Phase A periods of tracking only, was 2 m with the exception of SV03 and SV06 which suffered due to the limited amount of data (see Figure 4.10). This agreement is quite remarkable considering the different softwares and poor quality data and suggest an ephemeris accuracy of 1 part in 10^7, leading to an expected agreement between coordinates of 3 to 5 cms over baselines of 300 to 500 km. The agreement between these two orbits was closer than between the IESSG ephemeris and the NGS precise ephemeris.

Despite these differences between the Nottingham and Bernese final coordinates, it was decided that further re-analysis of this poor quality data set would probably only result in small improvements. Furthermore, since many of the poorly determined stations had already been re-observed, re-analysis would not be beneficial. Nevertheless, these results were a significant step towards the realisation of a three-dimensional European Reference Frame and were obtained in less than three years.

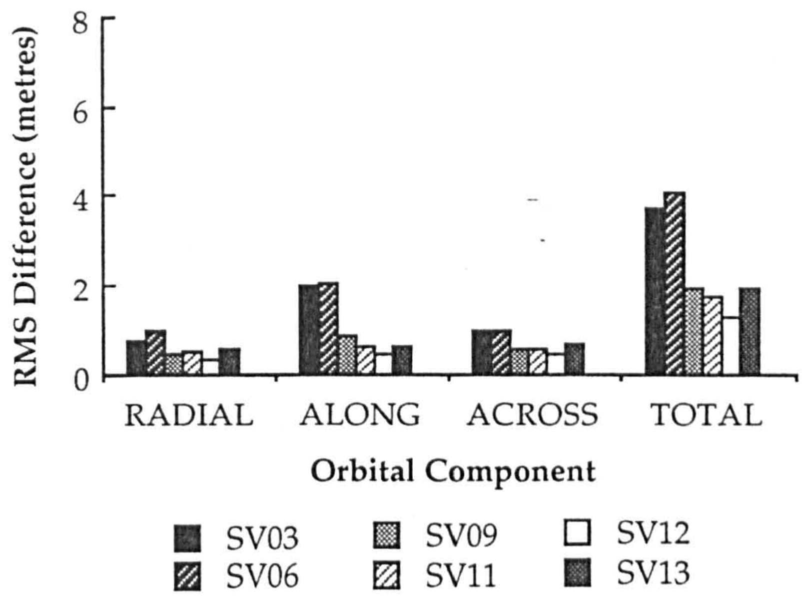


Figure 4.10 Average RMS difference between IESSG and Bernese Precise Ephemerides for Phase A periods of tracking data only.

4.7 Adoption of EUREF 89

In March 1992 the EUREF Subcommission held a symposium in Berne *[Gubler et al, 1992c]*. The purpose of which was to present and discuss the results of the EUREF 89 GPS Campaign and produce a final solution which could be adopted as a set of coordinates for the first realisation of ETRS 89. Results were presented by all processing centres and a comparison performed. The Subcommission then passed the following two resolutions.

Resolution No. 1 *recognised* "the results of the EUREF 89 GPS Campaign obtained by the processing centres" and *recommended* "that the final results to be adopted are those obtained by the Bernese Group and that comprehensive documentation should include the comparisons with the other solutions" and further *recommended* "that this solution be accepted as the current realisation of the ETRS 89 under the name EUREF 89".

Resolution No. 2 *recognised* "that the coordinates of stations determined by EUREF 89 will be subject to improvement, that the existing network will be extended, and that these improvements and extensions could affect the homogeneity of EUREF 89" and *accepted* "that new campaigns must include observations at sufficient primary stations and other neighbouring EUREF stations, to fulfil EUREF standards" and *recommended* "that a Technical Working Group is created which will define standards, monitor results from subsequent campaigns and evaluate whether the IGS-campaign should result in improvements of the EUREF 89 coordinates". The IGS Campaign is discussed in section 2.8.

The requirement of most users of EUREF 89 will involve expressing their positions in geodetic ellipsoidal coordinates (latitude, longitude and ellipsoidal height). The difference between the WGS 84 and GRS 80 ellipsoids are at the millimetre level. The EUREF Subcommission has adopted the use of the GRS 80 ellipsoid in conjunction with ETRS 89.

The final coordinate values for the ninety-three EUREF stations and comparisons with other solutions are published in *[Gurtner et al, 1992]*.

4.8 Further GPS Campaigns

The initial EUREF 89 network has since been supplemented by several further GPS campaigns, extending the network into Northern and Eastern Europe.

The EUREF NORTH GPS campaign was observed in July 1990, extending the initial EUREF network in to Iceland, Spitzbergen and Greenland, and connecting the network to the North American continent. EUREF NORTH included approximately 40 stations and was observed over a 10 day period using 6 different types of dual frequency receivers. Included in the 40 stations, further observations were performed at the UK stations of Herstmonceux, Buddon and An Cuaidh and the new EUREF station of Danby Beacon. However, due to the very long baselines and high ionospheric activity at these high northern latitudes the results of this campaign are not expected to improve the coordinates of the UK stations.

In August/September 1990 the TURK EUREF GPS campaign observed 16 stations to connect the Turkish first order triangulation to the

EUREF system.

Due to political changes in Eastern Europe, several countries have joined the CERCO community and asked for connection to the EUREF system. The EUREF EAST-91 GPS campaign was performed in October / November 1991, extending the initial EUREF network into Hungary and Czechoslovakia. This was followed by further GPS campaigns to connect Poland in July 1992, Bulgaria in September 1992, the new states of the Federal Republic of Germany and the new Baltic States in October 1992, and the Republic of Cyprus in February 1993. In the near future additional observational campaigns will be carried out to link the remaining Eastern and South-East Europe countries to EUREF with the aim of producing a common geodetic reference frame for the whole of Europe.

To date (September 1993) the author is unaware if any of these campaigns have or are being processed.

Reoccupation of UK EUREF Stations

The UK Gauge project has addressed the problems of the EUREF89 GPS campaigns and reoccupied the six UK EUREF sites. The campaign was designed so that ambiguity resolution would be possible, by including a good mix of baseline lengths, and by observing almost exclusively with dual frequency P code receivers of a single type, ie Trimble 4000 SST. Observations were made on five consecutive days, using an 8-hour window. The UK GAUGE Project and results are described in Chapter 5.

4.9 Conclusions

1. The EUREF Subcommission has successfully observed and computed a three-dimensional reference frame for Europe in under three years. A remarkable achievement considering the work involved and that RETrig took 34 years to produce a partially complete solution.

2. The IESSG processing has clearly shown that, even with this poor quality data set, the best results are obtained using the rigorous fiducial network technique rather than using the fixed ephemeris approach. Although this technique is

computationally demanding and a single 'European Wide' adjustment would be impractical, this approach could have been used on a country by country basis.

3 The IESSG and Bernese solutions agreed at the 10 cm level.

4 EUREF 89 is not as accurate as was originally hoped, due mainly to receiver problems, but can now be used to compute transformation parameters between national coordinate systems and WGS84.

CHAPTER 5

High Accuracy Fiducial GPS

The UK Gauge Project initially involves the observation of three fiducial GPS campaigns. The first was observed in September 1991 and initial indications, later confirmed by results presented in this chapter, showed that this data set was of a very high quality, comparing well with the 'famous' IAG Standard Data Set *[Dong and Bock, 1989]*. This has made it possible for the author to carry out a number of tests, the results of which are not only applicable to the coordinates and heights of the UK Gauge Project, but also conceptually relevant to other fiducial GPS networks.

This chapter describes the UK Gauge Project and presents the results of the processing of the first fiducial GPS campaign. It begins in section 5.1 with details of the aims of the UK Gauge Project, the design of the fiducial GPS network and the observation of the first fiducial campaign. Section 5.2 presents the results obtained from the conventional fiducial GPS processing of this data set and section 5.3 discusses the error sources in a conventional fiducial GPS adjustment. These errors sources are explored in sections 5.4 to 5.10 and the results have been used to produce a high accuracy fiducial processing technique. The data set has then been re-processed using this high accuracy fiducial GPS technique in section 5.11 and the resulting coordinates compared to the GPS coordinates from the EUREF 89 campaign, in section 5.12. The chapter is concluded in section 5.13.

5.1 The UK Gauge Project

The prime objective of the UK Gauge Project is, as the name suggests, to connect selected Class A tide gauges, around the coast of the United Kingdom, to a global reference framework. By performing three fiducial GPS campaigns, each one year apart, the aim is to prove the suitability of the fiducial GPS technique for deformation monitoring and provide a set of zero-order coordinates for long term monitoring. Additionally, observations of a national scale network presented the opportunity to obtain improved coordinates for the UK EUREF stations, which were poorly determined in the EUREF 89 GPS campaign.

5.1.1 Tide Gauge Monitoring

The global monitoring of mean sea level has become very topical due to its possible connection with the 'greenhouse effect'. The general consensus of opinion is that the greenhouse effect is causing a gradual 'global warming', which is resulting in the melting of mountain glaciers and the thermal expansion of the oceans resulting in a global rise in sea level. Currently, significant world-wide effort is being made to determine if sea level is rising, and if so, to measure the magnitude of this rise.

Mean sea level is measured by tide gauges, which lead to the values of sea level 'relative' to the level of the land at the tide gauge. On a local scale this may be all that is required. However, what is unknown is whether the change in mean sea level indicated by the tide gauge records is due to an actual rise in sea level, or due to ground movement, or (more likely) a combination of both. Figure 5.0 shows plots of average annual tide gauge readings from some of the longest tide gauge records available in Europe. In Southern Europe the tide gauge readings suggest that mean sea level is rising by 20 cm per century (ie 2 mm per year), which is typical of that found in other parts of the world. However, in Northern Europe the readings suggest that mean sea level is falling by up to 40 cm per century (ie 4 mm per year). On a global scale, predictions of mean sea level rise are approximately 1.5 mm per year, which suggests that for Northern Europe, the effect of post glacial rebound may be causing the land to rise at a faster rate than mean sea level.

In order to monitor 'absolute' changes in sea level, it is therefore necessary to monitor the land uplift or subsidence which occurs at tide gauge sites. This can only be achieved through the use of a global geodetic reference frame, in which the ground movements can be measured. In 1988, an international meeting of oceanographers and geodesists took place at the Woods Hole Oceanographic Institute in Massachussetts, to discuss methods for determining absolute sea level.

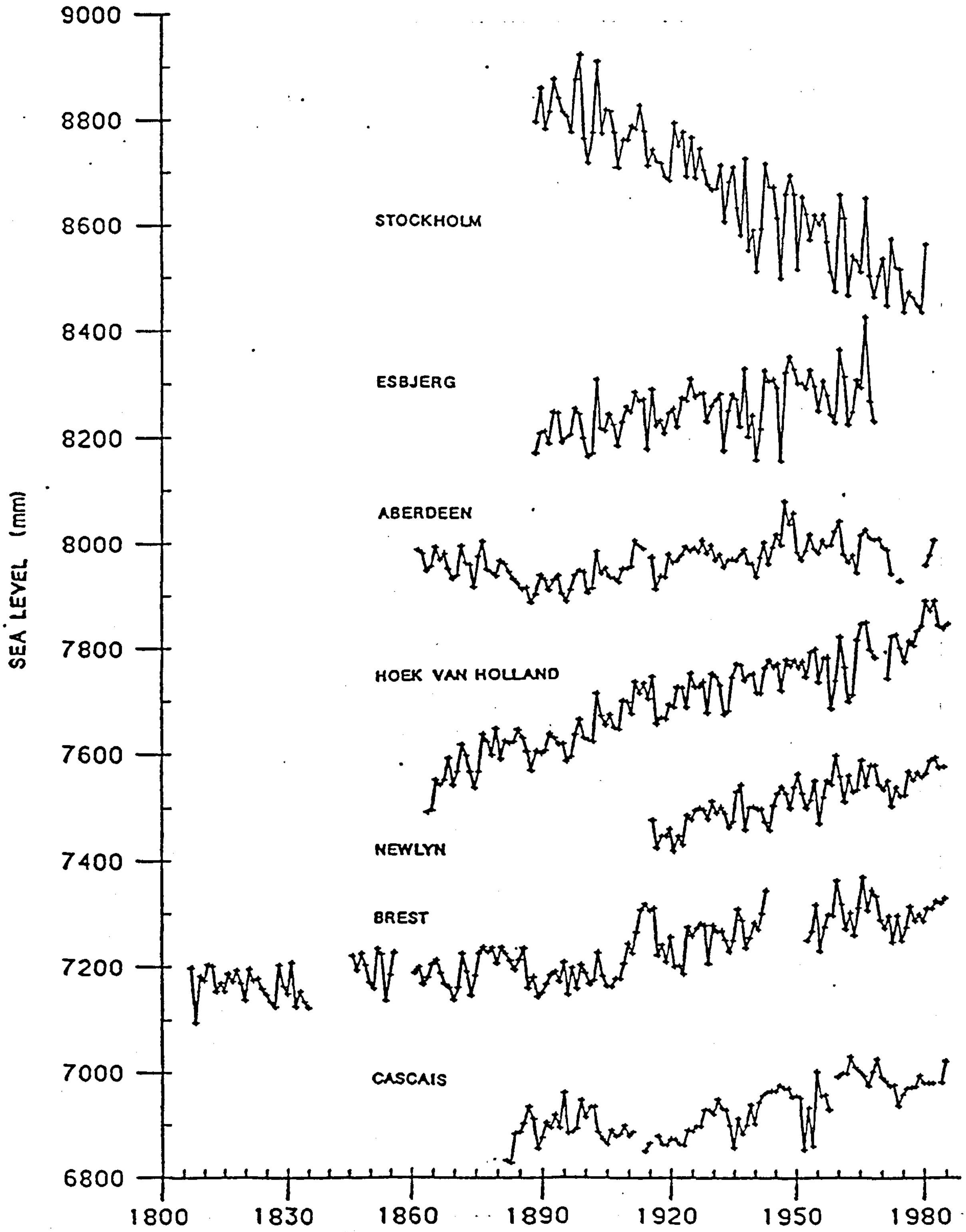


Figure 5..0 Time Series of European Annual Mean Sea Levels
(Note: Each record has been given an arbitrary vertical offset for presentation purposes) *[Baker, 1993]*.

This meeting led to the publication of a report *[Carter et al, 1989]*, which recommended various techniques for the geodetic fixing of tide gauge heights, and suggested the use of GPS to connect tide gauges to a global geodetic reference frame.

The effect of a rise in sea level over several decades could be catastrophic. For example, in the UK 57% of Grade 1 agricultural land lies below the 5 metre contour *[Whittle, 1989]* . Therefore, the realistic long term planning of coastal and river flood defences in the UK is essential. This planning necessitates the monitoring of mean sea level and hence the monitoring of vertical land movement at tide gauge sites. To this end, the Ministry of Agriculture Fisheries and Food (MAFF) and the Natural Environment Research Council (NERC), through the Proudman Oceanographic Laboratory (POL), contracted the IESSG to develop a strategy for monitoring vertical land movement at nine selected tide gauge sites in the UK, using the fiducial GPS technique.

5.1.2 UK EUREF Stations

The UK EUREF stations were initially observed during the EUREF 89 GPS campaign. However, as described in Chapter 4, receiver hardware problems at the UK stations combined with very high ionospheric activity, and baselines between 300-500 km meant that a very poor data set had been compiled. The resulting coordinates for the UK were not of a comparable accuracy with VLBI and SLR and therefore unsuitable as control for further national GPS measurements. The UK Gauge Project addressed the problems of the EUREF 89 GPS campaign, with the Ordnance Survey of Great Britain supplying personnel and receivers to re-occupy the six UK EUREF stations.

The objectives of the UK Gauge project were complementary to the Ordnance Survey requirements, with the UK EUREF stations being used to strengthen the UK Gauge network and reduce baseline lengths, while the tide gauge stations were being used to reduce the 300-500 km baseline lengths between the UK EUREF stations. Furthermore, the observation of three fiducial GPS campaigns, in quick succession between 1991 and 1993, will enable three independent computations of the coordinates for the UK EUREF stations, as opposed to a single computation from a one-off fiducial GPS campaign, such as EUREF 89.

5.1.3 The UK Gauge Fiducial GPS Network

The UK Gauge fiducial GPS network includes seven 'candidate' fiducial stations in Europe and seventeen regional stations in the United Kingdom. The seven candidate fiducial stations are Tromsoe (Norway), Onsala (Sweden), Wettzell (Germany), Herstmonceux (England) and Madrid (Spain), and the local mobile VLBI sites of Buddon (Scotland) and Brest (France) (see Figure 5.1). The seventeen regional stations include nine tide gauge stations, the six UK EUREF stations (including Herstmonceux and Buddon), and intermediate stations at Nottingham and Hermitage (see Figure 5.2). This meant that the maximum baseline length in the regional network was 300 km, and that a wide selection of baselines was available, with a particular cluster of short baselines in the South East of England, for the 'boot-strapping' integer fixing process (see section 2.2.5).

Due to limitations of the PANIC-1 software and for the purpose of obtaining new coordinates for the six UK EUREF stations, the author processed a sub-set of twenty stations from the twenty-two available. All seven candidate fiducial stations were used and since they all are primary ITRF stations, they have ETRF 89 coordinates to recover the original EUREF 89 reference frame. Fifteen regional stations were used (including Herstmonceux and Buddon), with the tide gauge station of Lerwick and the intermediate station of Hermitage being dropped.

5.1.4 The 1991 GPS Campaign - UK Gauge 1991

The first UK Gauge fiducial GPS campaign was observed in September 1991. Observations were taken during an 8 hour window, for five consecutive days. The campaign involved a total of 22 dual frequency GPS receivers, namely 4 Rogue SNR-8's (with P code L2) at the fiducial stations of Tromsoe, Onsala, Wettzell and Madrid, 3 Trimble 4000 SLDs (with L2 squaring) at the regional stations of Bartinney, Moel Fammau and An Cuaidh and 15 Trimble Geodesist Ps (with P code L2) at the remaining stations. In addition, data from the Rogue SNR-8

Figure 5.1 The UK Gauge Project Candidate Fiducial Stations.

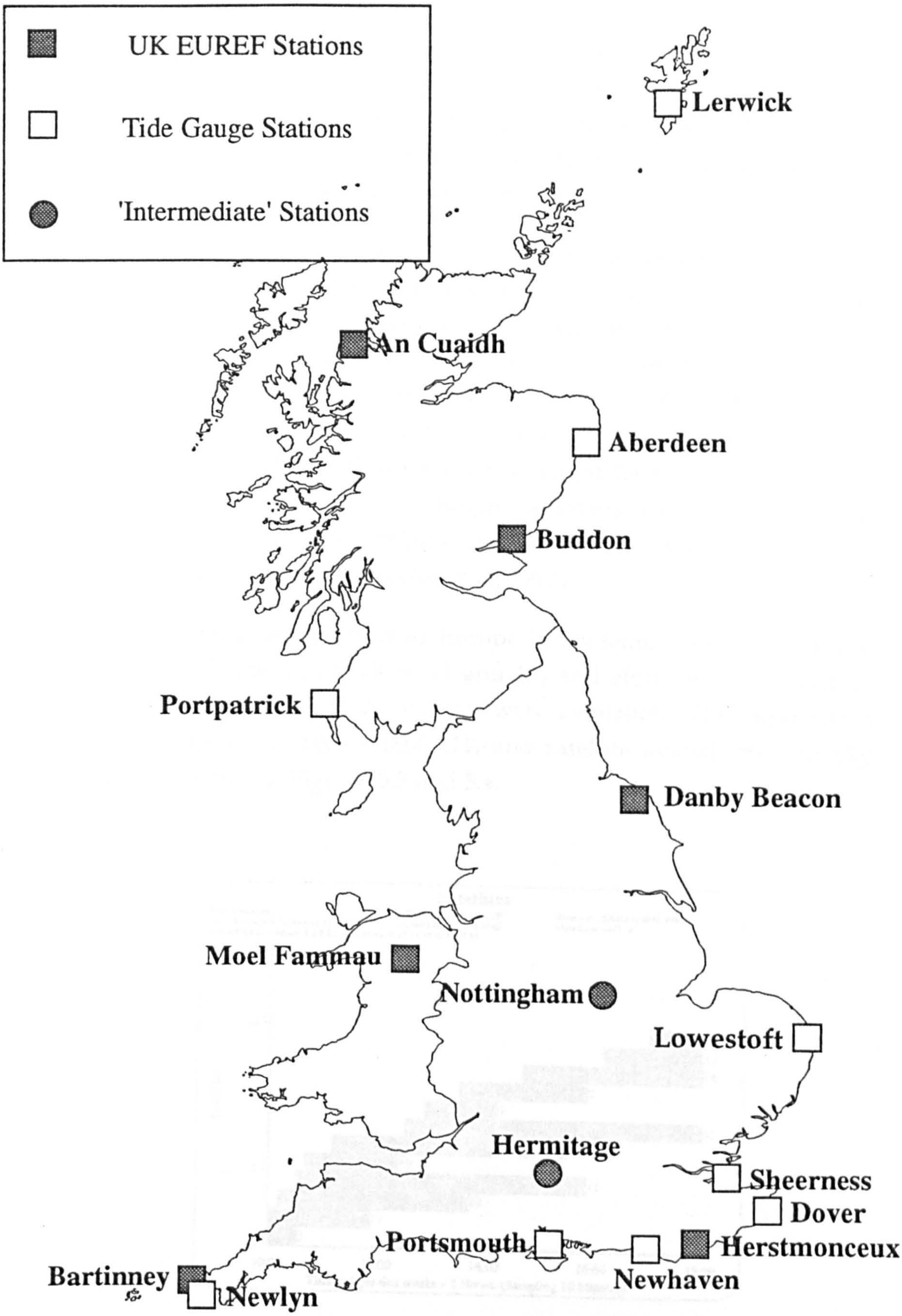


Figure 5.2 The UK Gauge Project Regional Stations.

receiver, which had just become operational at Herstmonceux, was also available. The data from the Rogue receivers was obtained from CIGNET (Cooperative International GPS Network), with the exception of Madrid which was obtained from the Jet Propulsion Laboratory as part of their Deep Space Network *[Starr, 1991]*.

An antenna phase centre calibration was performed prior to the campaign, using the Trimble receivers, on a series of pre-surveyed points located at the University of Nottingham. This indicated that the vertical phase centres of all of the Trimble antennas used in the campaign were within ± 2 millimetres of the manufacturer's claimed values and agreed well with the phase centre offsets determined during the EUREF 89 Calibration Campaign (see section 4.3). The phase centre values for the Rogue receivers were fixed to their published values *[CSTG, 1992]*, except for Madrid whose values were obtained from the IERS *[Boucher et al, 1992]*.

The satellite constellation over Europe in September 1991 meant that four Block I satellites (SV3, 6, 11 and 13) and eight Block II satellites (SV2, 14, 15, 17, 18, 19, 21 and 23) were available. The observation window used was 1000 - 1800 UT, and satellite availability and sky plots are shown in Figures 5.3 and 5.4.

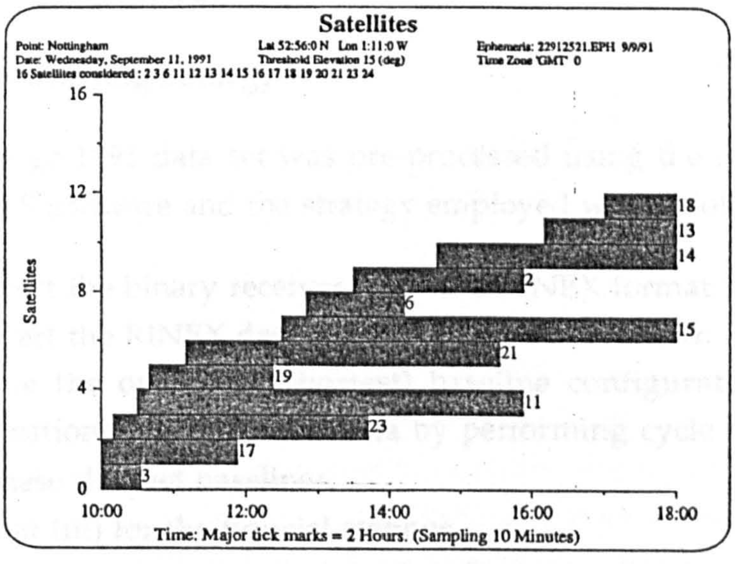


Figure 5.3 UK Gauge 1991 Satellite Availability Plot.

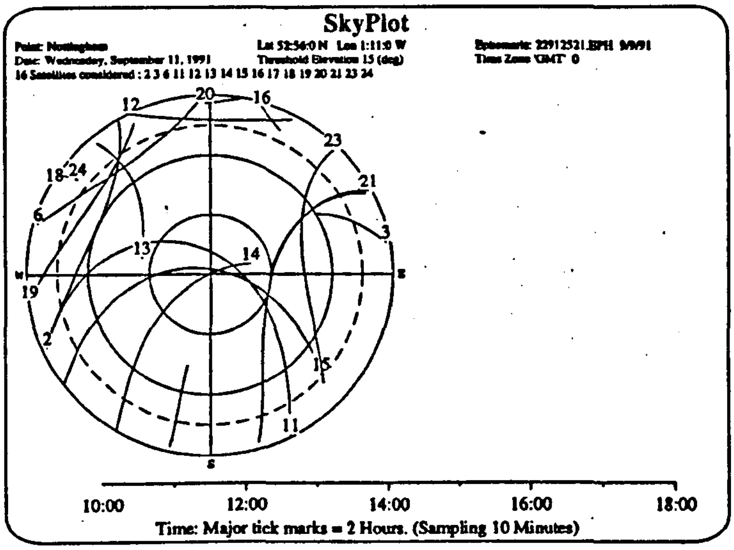


Figure 5.4 UK Gauge 1991 Satellites Sky Plot.

5.2 Conventional Fiducial Processing of UK Gauge 1991

This section describes the processing of the UK Gauge 91 data set using the conventional fiducial processing technique, as described in section 2.2 and used by the author for the processing of the EUREF 89 GPS campaign (see section 4.4).

5.2.1 Pre-processing Strategy

The UK Gauge 1991 data set was pre-processed using the Nottingham in-house GPS software and the strategy employed was as follows,

(i) Convert the binary receiver data into RINEX format.
(ii) Convert the RINEX data into NOTTM1 data format.
(iii) Define the optimum (shortest) baseline configuration for the UK stations and clean the data by performing cycle slip editing on these defined baselines.
(iv) Repeat (iii) for the fiducial stations.
(v) Perform preliminary network adjustments using the NGS Precise Ephemeris, and fixing one station (Herstmonceux), to check that the data is clean.

The presence of very few cycle slips and the results from the preliminary network adjustments indicated that a very high quality data set had been observed and should produce results far superior to those from the EUREF 89 data set. The author in indebted to Messrs Glen Beamson and Peter Clarke for carrying out the pre-processing, a very time consuming and laborious task.

5.2.2 Processing Strategy

The results from processing the EUREF 89 GPS campaign clearly confirmed that, even with a poor data set, the most precise solution was obtained using the rigorous fiducial approach. Therefore, this was the only type of solution performed. As with the EUREF processing, tests were carried out which involved fixing different subsets of fiducial stations and comparing the recovered coordinates of these free fiducial stations with their known values. These subsets of fixed fiducial stations are listed in Table 5.1 (TOWM = Tromsoe + Onsala + Wettzell + Madrid etc). To be consistent with the original EUREF 89 GPS campaign, the fiducial station coordinates were obtained at the epoch 1991.7 (UK Gauge 1991) by using the AM0-2 plate motion model and the ETRF 89 coordinates from the epoch 1989.0.

ID of Test	Fiducial Stations Held Fixed						
Ref Frames	Troms	Onsal	Wettz	Herst	Madri	Buddo	Brest
TOWM	•	•	•		•		
TOWHM	•	•	•	•	•		

Table 5.1 Reference Frameworks Tested
(• Fixed Station).

The UK Gauge 1991 data set was processed using the PANIC-1 software and the following options, L1/L2 observable, with ambiguities free, solving for 6 orbital parameters (position and velocity) per satellite, per session and a constant tropospheric scale factor per station, per session. The campaign was originally observed for 8 hours over five days. Due to limitations in the PANIC-1 software each day was processed as two independent 5-hour sessions, ie using two different base-satellites. This formed a total of 10 sessions. However, as it transpired, all four of the Rogue receivers (Tromsoe, Wettzell, Onsala and Madrid) were only tracking simultaneously during three of the five days. As a result, the author concentrated on these three days, ie six 5-hour sessions. The processing of each session led to a set of coordinate values for the unknown stations and a corresponding covariance matrix. These coordinates were combined together, with their covariance matrices, using the Nottingham 3-d network adjustment program (CARNET), into a set of weighted mean values.

5.2.3 Results and Analysis

The session to session differences in baseline components, from a weighted mean for the six sessions, are shown in Figures 5.5 and 5.6 for the fixed fiducial stations TOWM and TOWHM respectively. The horizontal line on each graph represents the 2σ confidence level, ie 95% of the points are contained in the area below this line.

These results indicate that a high precision (1-2 cm in plan and 3-5 cm in height) data set has been observed and correctly cleaned of cycle slips. A comparison between the networks TOWM and TOWHM shows improved repeatabilities in the height component when a vertical constraint is applied by fixing a regional station (Herstmonceux) within the area of interest. However, despite this improvement in the vertical component the precision is still two or three times worse than that of the horizontal components.

The recoveries of the 'free' fiducial stations are given in Table 5.2. This shows that even with this high precision data set, the resulting coordinate accuracies are only at the 10 cm level. When compared with the accuracies for the original EUREF 89 GPS campaign, this is of little improvement and not accurate enough for the control of further national GPS measurements, and certainly not accurate enough for the monitoring of vertical land movements. Clearly, the results suggest that conventional fiducial GPS processing cannot produce the 1-2 cm accuracies required. This has serious implications for many other fiducial campaigns which claim "millimetres over hundreds or even thousands of kilometres", based entirely on day to day repeatability results. Section 5.3 discusses some of the problems which may have contributed to these lower than expected accuracies.

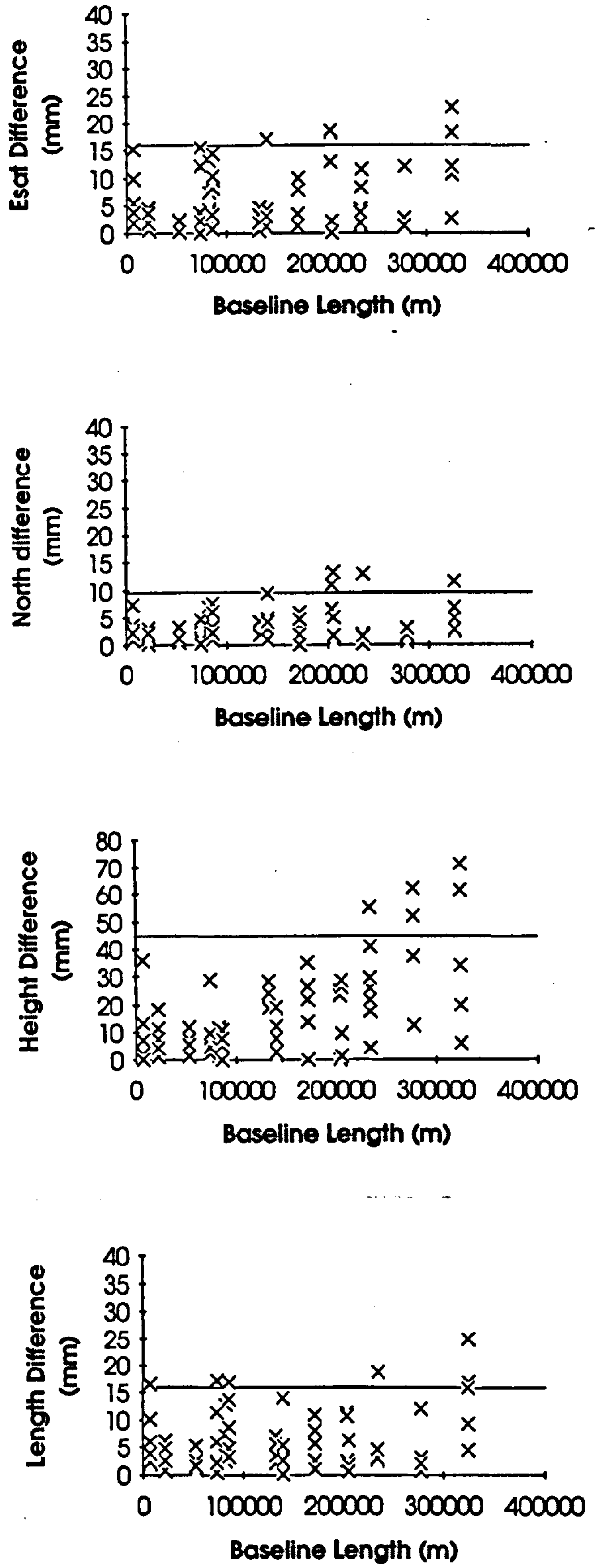


Figure 5.5 The Session to Session Differences (mm) in Baseline Components from a Weighted Mean for Fixed Fiducial Stations TOWM.

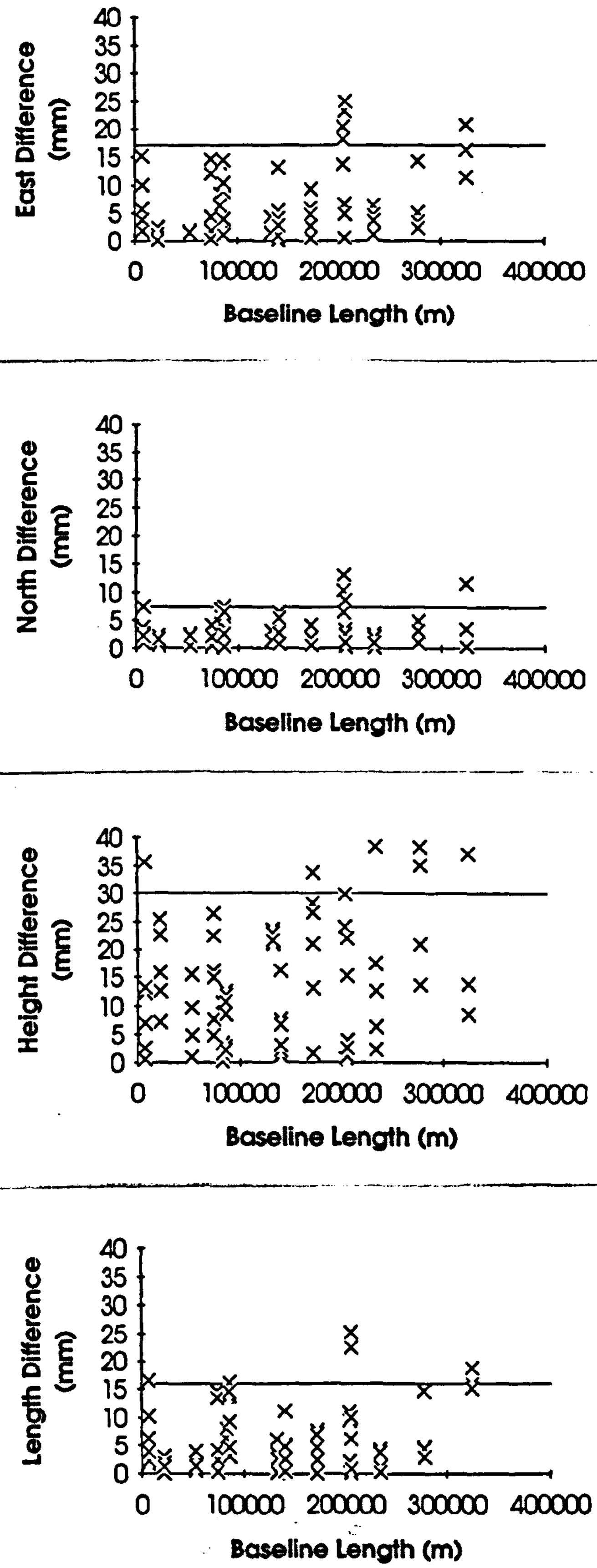


Figure 5.6 The Session to Session Differences (mm) in Baseline Component from a Weighted Mean for Fixed Fiducial Stations TOWHM.

Free Fiducial Stations	Fiducial Stations Held fixed					
	TOWM			TOWHM		
	dN	dE	dH	dN	dE	dH
Herstm	-4	66	83	•	•	•
Buddon	-13	50	-31	16	36	113
Brest	7	11	69	· 7	-55	-10

Table 5.2 Recovery of the Free Fiducial Stations (mm)
(• Fixed station).

5.3 Error Sources in a Conventional Fiducial GPS Adjustment

Coordinate values are of little practical use unless they are accompanied by an estimate of their quality. Coordinates derived from survey observations are affected by three types of error, namely, random, gross and systematic. The concepts of precision, reliability and accuracy are related to these types of errors and can be used, with caution, as estimates of quality.

5.3.1 Precision of a Fiducial GPS Adjustment

Precision is defined as a measure of the ability to repeat an observation and contains random errors, due to the noise of the observable. Repeatability is a further indicator of 'precision'. It can be defined as the root-mean-square (RMS) scatter about the weighted mean, but unlike variance-covariance, it also contains some of the effects from unmodelled or poorly modelled error sources. Assuming that several data sets can be treated independently, a weighted mean and hence repeatability, can be computed. However, the author has found that several institutions compute repeatability slightly differently *[Blewitt, 1989; Dong and Bock, 1989]*. Therefore, this measure of quality must be used with great care.

In fiducial GPS the data from a single campaign, performed over several consecutive days, can be used to compute 'short-term' repeatabilities. Whereas, data from several campaigns, spanning months or even years, can be used to compute 'long-term' repeatabilities. The latter will be a more realistic indicator of quality due to the presence of slowly varying systematic errors, such as the troposphere, systematic multipath (due to a permanent structure) or constellation dependencies which may appear constant in a single campaign.

The errors which affect the short term repeatabilities of a fiducial GPS adjustment are those which vary randomly from day to day. These include:

- Random unmodelled atmospheric variations (troposphere and ionosphere)
- Uncorrected cycle slips
- Missing or noisy data
- Random multipath (eg due to passing vehicles, not permanent obstructions)

Short-term repeatability is, therefore, a good indication of the quality of the data and that it is free of cycle slips

5.3.2 Reliability of a Fiducial GPS Adjustment

The reliability of an adjustment is its ability to detect a gross error or blunder. In a fiducial GPS adjustment there are normally thousands of observations for the determination of less than one hundred unknowns, therefore, gross errors due to cycle slips can usually be confidentally detected. In a fiducial GPS adjustment, examples of other blunders can be an incorrectly measured antenna height, or the GPS antenna being set up over the wrong point (it can happen!). For permanently located GPS receivers these blunders can be eliminated, but for temporary locations care must be taken in designing the observational procedure.

For the UK Gauge Project, each antenna was dismantled between observation windows, and the new antenna height measured in metres and inches at the start and end of every day. Therefore, any gross errors would show up as outliers in the repeatability analysis, when combining the results from several sessions. Similarly, gross errors due to a set up over the wrong point will be detected through the comparison of results from several observation campaigns.

5.3.3 Accuracy of a Fiducial GPS Adjustment

Accuracy can be defined as a measure of 'truth'. The only practical measure of accuracy is by comparison with an independent measurement, which must be of an equal, or preferably, higher order of accuracy. For a fiducial GPS adjustment, the available independent measurements are from VLBI, SLR or global GPS networks.

Error sources which affect the accuracy of a fiducial GPS adjustment are those which are either constant for a single campaign or have a repeat period coincident with the repeat period of the observation window (normally 24 hours). Such errors could be due to:

- Observational / Pre-processing Strategy
  - uncorrected cycle slips,
  - multipath,
  - gross errors in antenna set up.
- Reference Frame Definition,
  - errors in the VLBI/SLR coordinates,
  - errors in the plate motion model (velocity field),
  - errors in the local offset between the VLBI/SLR reference point and the GPS reference point.
  - geometrical weakness of the fixed fiducial station network
- Combination of different receiver types,
  - antenna phase centre variations,
  - receiver time-tag differences (satellite clock drift).
- Atmosphere (troposphere and ionosphere)
- Ocean Tide Loading Effects
- Earth Body Tides

The ways in which cycle slips are detected and repaired were discussed in section 2.2.8. The observation procedure employed to eliminate gross errors in antenna set up were given in section 5.3.2, and the reduction of systematic and random multipath errors through careful site selection are well known. The other error sources are explored further in the following sections.

5.4 Reference Frame Definition Tests

This section describes tests that were carried our in order to determine the quality of different reference frames based on different positioning techniques, ie VLBI, SLR or GPS, and hence determine the reference frame which produces the highest accuracies for the UK Gauge 91 data set.

5.4.1 Reference Stations in Europe

Europe appears to be fortunate (!) in having a plethora of candidate fiducial stations (see Figure 4.4). Some of these stations, particularly in South-East Europe, are not fixed to the stable part of the Eurasian plate and, since their tectonic movements are not easily modelled, are considered less suitable as fiducial stations. The most fundamental fiducial station in Europe is Wettzell in Germany, which has both permanent VLBI and SLR facilities, and a continuously tracking GPS receiver. Of the other fiducial stations, Onsala (Sweden) and Madrid (Spain) have permanent VLBI and GPS, Herstmonceux (England) and Kootwijk (The Netherlands) have permanent SLR and GPS, and Graz (Austria) and Zimmerwald (Switzerland) have permanent SLR only. A Mobile VLBI campaign, involving six sites in Europe was conducted in 1989 (see section 4.3.1). Of these, Tromsoe (Norway) and Metsahovi (Finland) have also been observed by mobile SLR and have a permanently tracking GPS receiver, and Grasse has a permanent SLR facility. The remaining three, Buddon (Scotland), Brest (France), and Höhenbunstorf (Germany), have not been re-occupied by mobile VLBI or mobile SLR and do not have continuously tracking GPS receivers.

5.4.2 Fiducial Station Problems

The selection of fiducial stations must be treated with great care, especially with regard to mixing VLBI and SLR derived coordinates and to the permanence of the VLBI/SLR facility. A permanent facility, such as Wettzell, obviously inspires more credibility than, say, a station which has only been observed at a single epoch by a mobile VLBI campaign, such as Brest or Buddon. In addition to considering which technique was used to derive the fiducial station coordinates, extreme care must also be taken in the application of the local offsets between a GPS receiver and the VLBI/SLR reference point. It is the authors experience that local offset values acquired from different sources can vary by several centimetres, over distances of tens of metres!

The outermost layer of the Earth, the lithosphere, is broken up into 11 major plates which drift around the Earth's surface. This movement is known as continental drift, and for some stations in tectonically active regions can be as large as 100 mm per year *[Cross and Sellers, 1991]*. Clearly, this has serious implications for high accuracy GPS since the stations defining the reference frame will be moving. Therefore, either the reference frame must be re-realised at the epoch of the GPS campaign, or some form of plate motion model (or velocity field) must be used to compute the fiducial station coordinates at the epoch of the GPS campaign. The plate motion models available at present, and used in conjunction with geodetic reference frameworks, are the Minster-Jordan AMO-2 *[Minster and Jordan, 1978]*, Nuvel 1 NNR *[Demetz et al, 1990]*, individual computing centre's VLBI / SLR velocity fields or the IERS 91 combined velocity field *[IERS, 1992]*.

Up to 1990, the IERS recommended the use of the AMO-2 model, which was used in many fiducial GPS campaigns in order to map the coordinates of the fiducial stations from the reference frame epoch (1988.0) to the epoch of the fiducial GPS campaign. Recently, the use of AMO-2 has been replaced by the Nuvel 1 NNR model. Arguably, the most representative form of plate motion model are the individual station velocities computed from many years of VLBI and SLR measurements, although the values derived by each individual computing centre can differ. A compromise, is the IERS 91 combined velocity field, which is based on the individual station velocities derived by the individual computing centres, and the Nuvel 1 NNR plate motion model. However, the main drawbacks of these models is that they do not account for vertical movement. Suffice to say that at the present time, the use of plate motion models must be carried out with great care if the high accuracy of the coordinates at the reference frame epoch are to be maintained.

5.4.3 Global Reference Frameworks

The fiducial station coordinates used in these tests were obtained from a number of different sources. They were based on either combined VLBI/SLR solutions (eg ITRF91), pure VLBI solutions (eg GSFC or NOAA), or a pure GPS solution (eg JPL IGS92). Where necessary, the coordinates were mapped, from their given epoch to the observation epoch of the UK Gauge 1991 campaign. The coordinate sets and the plate motion models used are described below, where they are assigned an acronym for identification in this and subsequent sections.

(a) **ITRF91N** ITRF 91 coordinates, mapped from epoch 1988.0 to 1991.7, using the Nuvel 1 NNR model *[Boucher et al, 1992]*.

(b) **ITRF91A** ITRF 91 coordinates, mapped from epoch 1988.0 to 1991.7 using the AMO-2 model *[Boucher et al, 1992]*.

(c) **GSFC** Goddard Space Flight Centre VLBI coordinates (GSFC 92 R 03), based on VLBI measurements from 1979 to 1991, given at epoch 1992.0, with no plate motion model used *[Altamimi, 1992]*. These coordinates were used by the IERS in the derivation of the basic ITRF91.

(d) **NOAA** National Oceanic and Atmospheric Administration VLBI coordinates, based on VLBI measurements and mapped from epoch 1988.0 to 1991.7 using the NOAA derived velocity field *[Abell, 1992]*. NB these coordinates were not used in the derivation of ITRF91.

(e) **JPL IGS92** Preliminary solution of the International GPS Geodynamics Service Experiment 1992 by Jet Propulsion Laboratory *[Blewitt et al, 1993]*, calculated from 70 daily (July to August 1992) solutions of the IGS global GPS network, using the 'no fiducial' technique, of not fixing any fiducial stations and a post adjustment transformation on to ITRF91.

5.4.4 Tests Performed

The tests carried out with the UK Gauge 1991 Data Set consisted of holding fixed different subsets of fiducial stations, assigning to them coordinate values chosen from several alternative global reference frameworks (see section 5.4.3) and determining the quality or 'goodness' of that solution. The full set of tests performed are listed in Table 5.3. The aim of these tests was to find the 'optimal' (ie internally the 'most consistent') global reference framework, which would henceforth lead to the 'best' values for the tide gauge stations and EUREF stations in the United Kingdom.

ID of Test Ref Frames	Fiducial Stations Held Fixed					Global Ref Frameworks
	Troms	Onsal	Wettz	Herst	Madri	
TOWM	•	•	•		•	ITRF91N, ITRF91A, GSFC, NOAA, JPL IGS92
OWM		•	•		•	ITRF91N, ITRF91A, GSFC, NOAA, JPL IGS92
TWM	•		•		•	ITRF91N, ITRF91A, GSFC, NOAA, JPL IGS92
TOW	•	•	•			ITRF91N,I TRF91A, GSFC, NOAA, JPL IGS92
TOM	•	•			•	ITRF91N, ITRF91A, GSFC, NOAA, JPL IGS92
TWH	•		•	•		ITRF91N, ITRF91A JPL IGS92
TOWHM	•	•	•	•	•	ITRF91N, ITRF91A JPL IGS92

Table 5.3 Reference Frameworks Tested
(• Fixed Station)

5.4.5 Results and Analysis

As described in section 5.3, repeatability cannot, on its own, point out any systematic biases, nor modelling errors, and hence gives no indication of accuracy. It is for this reason that the criteria chosen in these tests, to express the 'goodness' of the various global reference frameworks, was their ability to recover the known coordinate values of those fiducial stations which were not held fixed, but solved for during the adjustment. Each test consisted of three stages:

(a) Define the test reference frame by holding the coordinates of the fiducial stations fixed to values derived from a specific global reference framework.

(b) Carry out an adjustment of the UK Gauge 1991 data set onto this test reference frame.

(c) Compare the newly adjusted coordinate values of the remaining 'free' fiducial stations in that specific global reference framework to their 'known' values.

The full test results, for both the plan and the height coordinate components are tabulated in Appendix B and summarised in table 5.4 which contains the height components obtained from three of the five global reference frameworks.

The global reference frameworks listed in the first column of Table 5.4 are ITRF91N (combined VLBI/SLR), GSFC (pure VLBI) and JPL IGS92 (pure GPS). Each successive column represents a different test reference frame, and gives an indication of the fiducial stations held fixed (•), and the difference (in mm) between the 'adjusted' and 'known' coordinate values for the 'free' (fiducial) stations. The following is a summary of the conclusions we can draw from the results listed in Table 5.4.

(a) The GSFC (pure VLBI) solution leads to slightly smaller differences between 'adjusted' and 'known' heights than does the ITRF91N (combined VLBI/SLR) solution. The pure VLBI based reference framework is, therefore, more consistent internally and hence a better choice than a combined VLBI/SLR based reference framework.

(b) The improvement in internal consistency is much more significant when we compare either of these two reference frameworks, mentioned in (a), to JPL IGS92 (pure GPS) framework. It is clear that, except for Madrid and to a lesser extent Herstmonceux (Trimble), the differences between the 'adjusted' and the 'known' heights in the JPL IGS92 reference framework are the smallest, leading to the conclusion that a pure GPS based reference framework is very consistent internally, and therefore the optimal choice.

Global Ref Frameworks	Test Reference Frames						Stations
	TOWM	OWM	TWM	TOM	TWH	TOWHM	
ITRF91N (combined VLBI/SLR)	•	•	•	24	•	•	Wettz^{R}
	•	•	-1	•	-11	•	Onsa^{R}
	•	•	•	•	-169	•	Madri^{R}
	•	-37	•	•	•	•	Trom^{R}
	-10	-46	-7	-4	-85	-78	Budd^{T}
	40	13	41	62	-62	-15	Brest^{T}
	72	61	73	89	-	-	Herst^{T}
	54	39	54	70	•	•	Herst^{R}
GSFC (pure VLBI)	•	•	•	23	-	-	Wettz^{R}
	•	•	-11	•	-	-	Onsal^{R}
	•	•	•	•	-	-	Madri^{R}
	•	-17	•	•	-	-	Trom^{R}
	3	-23	1	-19	-	-	Budd^{T}
	55	33	54	43	-	-	Brest^{T}
	-	-	-	-	-	-	Herst^{T}
	-	-	-	-	-	-	Herst^{R}
JPL IGS92 (pure GPS)	•	•	•	20	•	•	Wettz^{R}
	•	•	-4	•	-8	•	Onsa^{R}
	•	•	•	•	-74	•	Madri^{R}
	•	-7	•	•	•	•	Trom^{R}
	-	-	-	-	-	-	Budd^{T}
	-	-	-	-	-	-	Brest^{T}
	35	41	39	36	-	-	Herst^{T}
	14	20	18	15	•	•	Herst^{R}

Table 5.4 Recovery of the Height Component of the Free Fiducial Stations (mm)

(• Fixed station, - not used, ^{T} Trimble receiver, ^{R} Rogue receiver)

The following are further conclusions that can be drawn from the full results in Appendix B.

(c) Table B.1 shows the recoveries for the global reference frameworks ITRF91N and ITRF91A. The differences between these two are due, entirely, to the different plate motion models used, ie Nuvel 1-NNR and AMO-2. Since neither set of recoveries are significantly better than the other, this raises the question of which model should be used and why ?

(d) Table B.2 shows the recoveries for the VLBI reference frame NOAA are similar to those for GSFC and supports the conclusion in (b) that JPL IGS92 is the optimal choice.

The problems with Herstmonceux and Madrid in the global reference frame JPL IGS92 can be explained as follows. At Herstmonceux when we substitute the Rogue receiver (the last row of Table 5.4) for the Trimble 4000 SST receiver, then clearly the recovery is as good as with any other station. This is not because one GPS receiver is better than another (although on some occasions this is the case !), but that combining GPS receivers of different types produces less consistent results than when using receivers of one type throughout. The poor recovery of Madrid is probably due to the fact that this station lies far outside the region defined by the fixed fiducial stations.

Another series of tests were carried out, to try and pinpoint the 'guilty' stations, ie those with patently poor coordinate values. The results of these tests are given in Appendix B and summarised (for heights) in Table 5.5. As with Table 5.4, the three alternative global reference frameworks considered are listed in the first column. In each successive column, we have the mean height differences (and corresponding standard deviations) between pairs of test reference frames. These pairs contain the same 'fixed' fiducial stations, bar one. For example, the column headed TOWM - OWM shows the mean value of the height differences, for the 15 regional stations (see Figure 5.2) obtained by using these two test reference frames. They have three of the same fiducial stations (Onsala, Wettzell and Madrid), but one of the two also has Tromsoe fixed, and therefore gives an indication of any potential difficulties with the coordinates of Tromsoe.

Global Reference Frameworks	Test Reference Frames				
	TOWM-OWM	TOWM-TWM	TOWM-TOW	TOWM-TOM	TOWM-TOWHM
ITRF91N	-22 ±10	2 ±1	-45 ±8	21 ±5	-19 ±3
GSFC	-17 ±7	-3 ±2	-27 ±5	-14 ±9	-
JPL IGS92	6 ±1	4 ±1	-56 ±10	-5 ±6	-29 ±4

Table 5.5 Mean Height Differences and Corresponding Standard Errors (mm) Derived by Fixing Different Subsets of Fiducial Stations
(Herstmonceux Trimble receiver used).

The following is a summary of the conclusions we can draw from the results listed in Table 5.5.

(a) The first column of Table 5.5 shows that holding Tromsoe fixed or free causes a substantial difference, in a combined VLBI/SLR (ITRF91N) or a pure VLBI (GSFC) solution. Clearly, either the heights of Tromsoe obtained by Mobile VLBI in 1989 and Mobile SLR in 1990 are not well determined, or the station could be contaminated by a local offset error.

(b) The results improve very significantly in the second column, namely TOWM - TWM. Clearly, the permanent VLBI coordinates and local offset values for Onsala are well defined, and holding Onsala fixed or free does not distort the network.

(c) The third column of results involves Madrid and confirms the conclusions drawn earlier, concerning the geographic position of Madrid vis-a-vis the rest of the network.

(d) The fourth column of Table 5.5 involving Wettzell, shows that the VLBI coordinates or local offsets are not well defined. The large differences in ITRF91N and GFSC coordinates could be due to contamination by the combination of VLBI and SLR, or errors in the Nuvel 1 NNR plate motion values for Wettzell used to obtain the ITRF91N coordinates.

(e) The large differences in the last column of Table 5.5 are due to the mixing different receiver types. This is confirmed in Table B.9 when the Rogue receiver is substituted for the Trimble 4000 SST and the difference for the JPL global reference frame is reduced from -29 ± 4 to -5 ± 1 mm.

(f) However, the most important conclusion which can be drawn from the summary Table 5.5 (as well as the complete set of results in Appendix B) is the **remarkable consistency of a pure GPS reference framework**. The reader is especially referred to the corresponding table in Appendix B, where the consistency, which is expressed in terms of 3-d coordinates, is even more striking. These differences are down to a few millimetres !

The tests described above were aimed at establishing the most consistent reference framework for the determination of high accuracy coordinates. They highlighted the potential problems in using networks based on pure VLBI or a combination of VLBI and SLR techniques. These usually require plate motion models and are dependent on a precisely determined local offset between the GPS receiver and the corresponding VLBI/SLR reference points.

The tests showed that the JPL IGS92 global GPS network provided the most consistent reference framework, particularly for heights. One of the advantages of using such a network for fiducial GPS is that potential errors in local offsets are eliminated. With the rapid densification of the CIGNET and IGS global GPS networks, continuous worldwide GPS data will become increasingly accessible, enabling the regular computation of global GPS solutions. These GPS data sets could be collected and solved for at the required epoch of observation, thus directly providing the information necessary for monitoring the movements of the fiducial stations themselves and eliminating the need for using plate motion models. This constitutes another clear advantage of using a pure GPS based reference framework.

5.5 Antenna Phase Centre Variations

The problem of antenna phase centre variations when mixing receiver types was first brought to the authors attention by Werner Gurtner, Astronomisches Institut Universität Berne, during the EUREF subcommision meeting in Paris, March 1992 *[EUREF 1992]*. He later described in a personal communication that he had found that:

> "the Trimble 4000 SST antenna shows an elevation angle dependence that can result in errors of up to 10 cm in the height component when combined with either a Rogue or Ashtech antenna. This effect is only seen when estimating tropospheric scale factors and is 3 times larger for the ionospherically free combination than for single frequencies *[Gurtner, 1993]*."

The UK Gauge 1991 data set, which involved mixing both Trimble and Rogue receivers, was processed using the ionospherically free observable (L1/L2) and estimating a tropospheric scale factor per station. This fulfilled the worst scenario as described above, hence its correction has proved crucial to the success of the UK Gauge Project.

To validate the problem described above, the author processed data on two
50 m baselines, combining Trimble and Rogue receivers with and without tropospheric scale factors (TSF). In theory, on such a short baseline any effect of the troposphere should be cancelled by double differencing, hence, the estimation of TSF's should produce a scale factor of one. However, in practice, the TSF will absorb any unmodelled errors, hence its value can be used as an indication of any problems. The baselines were all processed using PANIC-1 software with the NGS precise ephemeris and the MAGNET tropospheric model.

Baseline 1 A 50 m baseline at Herstmonceux, between the permanent Rogue receiver, operating as part of the CIGNET/IGS network, and the Trimble 4000 SST receiver located on the SOLAR pillar for the duration of UK Gauge 91 campaign.

Frequency	No TSF	TSF
L1	0	30 (-5.7)
L2	2	-12 (-2.2)
L1/L2	-4	57 (-11.1)

Table 5.6 The Difference in the Height Component (mm) Between the Solutions Assuming the L1 No TSF Solution to be 'Correct'.
(The values in brackets are the Tropospheric Scale Factors).

Table 5.6 shows that the difference in the height component between the L1, L2 and L1/L2 solutions was 6 mm when no TSF was estimated. However, when a TSF is estimated the difference between the solutions increases to 69 mm. This clearly verifies the effect reported by Werner Gurtner. Note: the horizontal components and baseline lengths were not significantly affected.

To confirm that this effect was due to the different receiver types and not a 'bug' in the software a second baseline was processed between two similar receiver types.

Baseline 2 A 50 m baseline at the University of Nottingham, between two Trimble 4000 SST receivers, observed as part of the calibration for UK Gauge 91. Note: this used the same constellation as for Baseline 1.

Frequency	No TSF	TSF
L1	0	3 (1.9)
L2	-3	3 (2.7)
L1/L2	4	3 (0.6)

Table 5.7 The Difference in the Height Component (mm) Between the Various Solutions Assuming the L1 No TSF Solution to be 'Correct'. (The values in brackets are the Tropospheric Scale Factors).

Table 5.7 shows that the height component is relatively unaffected by the estimation of TSF's when the same receiver types are used on such a short baseline. This clearly shows that for Baseline 1, the TSFs are absorbing some unmodelled error source which is not present for Baseline 2. These results indicate that an error of up to 7 cm in the height component can be caused, on baselines of 50 m, by solving for tropospheric scale factors when mixing different receiver types. This is believed to be caused by elevation angle dependent antenna phase centre variations, which cancel on such a short baseline when two similar antennas are used. Similar results were found when combining Ashtech and Trimble receivers and to a lesser extent when combining Ashtech and Rogue receivers.

In a recent study performed at Bendix , *[Schupler & Clark, 1991]* and *[Rocken, 1992]* used laboratory (chamber) tests to determine elevation and azimuth angle antenna phase centre variations for all commonly available antenna types. This data was very kindly supplied to the author by Rocken *[1993]*, and was used in the PANIC-1 software to apply corrections to the computed ranges as a function of elevation angle. Figures 5.7 and 5.8 show the L1, L2 and L1/L2 elevation angle dependent antenna phase centre variation model for the Trimble 4000 SST and Rogue Dorne-Magolin antennas respectively.

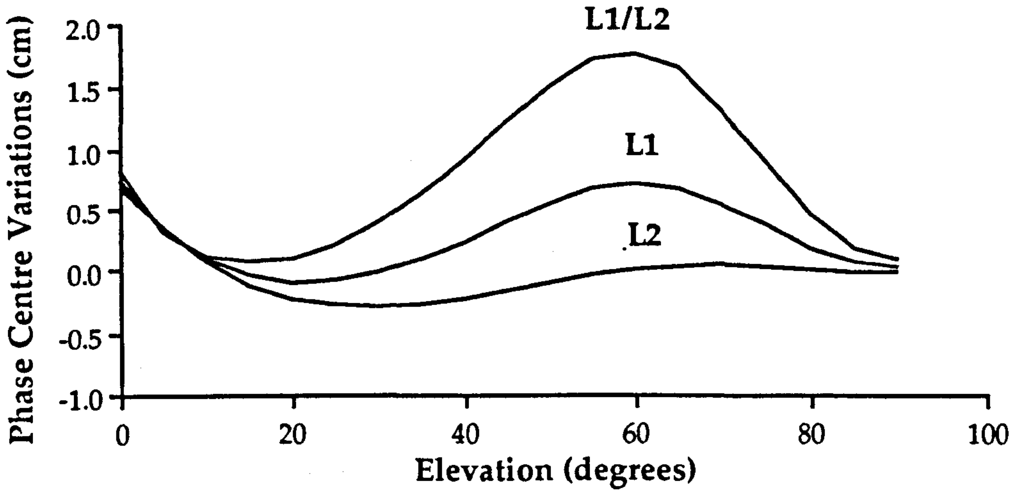


Figure 5.7 The L1, L2 and L1/L2 Elevation Angle Dependent Antenna Height Component Phase Centre Variations for the Trimble 4000 SST.

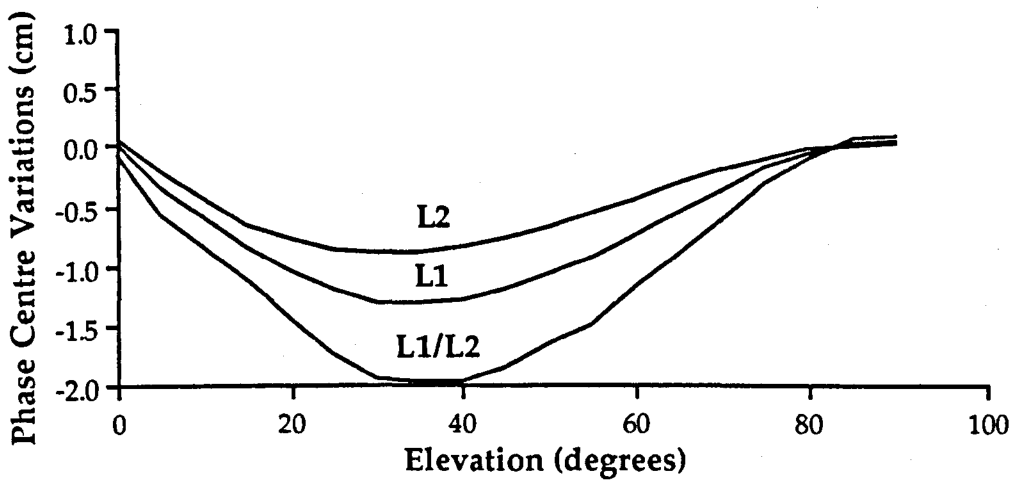


Figure 5.8 The L1, L2 and L1/L2 Elevation Angle Dependent Antenna Height Component Phase Centre Variations for the Rogue Dorne-Margolin.

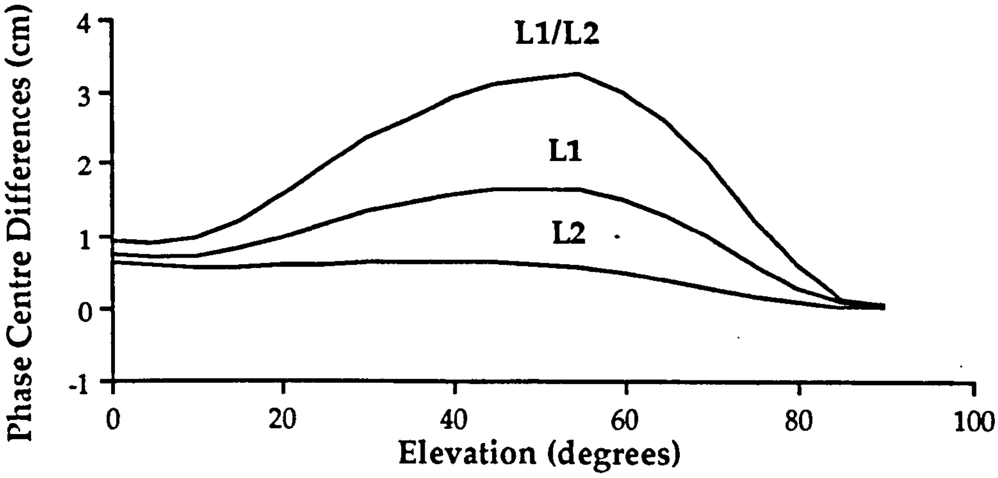


Figure 5.9 The L1, L2 and L1/L2 Differences in Antenna Phase Centre Variations between the Trimble 4000 SST and Rogue Dorne-Margolin Antenna.

Figure 5.9 shows the differences between these two antenna types, which can reach 3.5 cm at an elevation angle of approximately 55 degrees.

Table 5.8 shows the effect on Baseline 1 of applying an antenna phase centre model. There is clearly a dramatic improvement, particularly for the height component and tropospheric scale factor for the L1/L2 TSF solution. Baseline 2 showed no difference when the antenna phase centre model was applied.

Frequency	No TSF	TSF
L1	0	3 (1.64)
L2	8	5 (0.32)
L1/L2	14	-7 (2.57)

Table 5.8 The Difference in the Height Component (mm) Between the Various Solutions, when using Antenna Phase Centre Modelling, and Assuming the L1 No TSF Solution to be 'Correct'.

(The values in brackets are the Tropospheric Scale Factors).

The values when a Tropospheric Scale Factor is applied are now at the same level as those in Table 5.6. The results clearly highlight the danger of estimating tropospheric scale factors, without proper antenna phase centre modelling.

UK Gauge 91

The poor recovery of the Herstmonceux coordinates, (shown in section 5.4), when using the Trimble receiver and the improvement seen when using the Rogue receiver was attributed to the problem of combining different receiver types. Tests were performed, using the fixed fiducial stations of TOWM, to determine the difference in the recovery of the known coordinates of Herstmonceux, when using either the Rogue or Trimble receivers with and without antenna phase centre modelling. These results are shown in Table 5.9.

Receiver Type	Phase Centre Model Applied	ΔN	ΔE	ΔH
Trimble SST	✗	2	18	35
	✓	-2	9	-4
Rogue SNR8	✗	2	9	14
	✓	1	8	3

Table 5.9 Recovery of the JPL IGS92 Coordinates of Herstmonceux (mm) using the Rogue and Trimble Receivers, With and Without Antenna Phase Centre Modelling.

Without antenna phase centre modelling the height of the Trimble receiver is recovered to an accuracy two and a half times worse than that of the Rogue receiver, and the east component to an accuracy two times worse. However, with antenna phase centre modelling, both receivers coordinates are recovered to the same sub-centimetre accuracy. This clearly shows that when Rogue and Trimble receivers are mixed, in a fiducial GPS adjustment, without allowing for the different antenna phase centre variations, this can cause errors of up to 4 cm in height. It is obvious that this correction is crucial to the success of using GPS to determine high accuracy heights.

The reference framework tests, described in section 5.4 were re-processed with antenna phase centre modelling. The full results, for both the plan and height components are tabulated in Appendix C. They are summarised in Tables 5.10 and 5.11 which only show the height component. When compared with the corresponding table in section 5.4 (Table 5.4), table 5.10 shows a significant improvement in the recovery of the Herstmonceux Trimble 4000 SST. Furthermore, Table 5.10 also shows an improvement in the recovery of the Rogue station heights. This suggests that over baselines of 1000 to 1500 km, the constellation differs sufficiently to cause different phase centre variations even with the same receiver types. Most significant is the improved recovery of the height of Madrid when using the fixed fiducial stations of TWH (originally -74 mm). This is quite remarkable considering how far this station lies outside the region defined by the fixed fiducial stations and gives an indication of the quality and stability of the orbits.

Global Ref Frameworks	Test Reference Frames					Stations
	TOWM	OWM	TWM	TOM	TWH	
JPL IGS92 (pure GPS)	•	•	•	22	•	Wettzell^R
	•	•	-4	•	-1	Onsala^R
	•	•	•	•	-17	Madrid^R
	•	-7	•	•	•	Tromsoe^R
	-4	2	-1	-6	•	Herstm^T

Table 5.10 Recovery of the Height Component of the Free Fiducial Stations (mm) When Using Antenna Phase Centre Modelling.
(• Fixed station, ^{T} Trimble receiver, ^{R} Rogue receiver)

Global Reference Frameworks	Test Reference Frames				
	TOWM-OWM	TOWM-TWM	TOWM-TOM	TOWM-TWH	TOWM-TOWHM
JPL IGS92	7 ±1	4 ±1	-10 ±7	5 ±2	2 ±0

Table 5.11 Mean Height Differences and Corresponding Standard Errors (mm) Derived by Fixing Different Subsets of Fiducial Stations, When Using Antenna Phase Centre Modellling.
(Herstmonceux Trimble receiver used)

The differences in the UK coordinates when using fixed fiducial stations TOWM, with and without antenna phase centre modelling are shown in Figure 5.10. This shows that, with the exception of Newlyn, Bartinney and Brest, a systematic error of 3 to 4 cm in height can be caused by simply mixing receiver types without correcting for the different movements of the antenna phase centres. Investigation into the apparent anomaly in South West England showed that the tropospheric scale factors were consistently higher, on every day, for these three stations when compared with the other UK stations. This suggested that either larger variations in tropospheric delay were present, or that these tropospheric scale factors were absorbing some other unmodelled systematic errors. This is discussed further in Section 5.6.

These results clearly show that a constant antenna phase centre offset, determined by a pre-campaign calibration, is not adequate for high accuracy GPS involving mixed receiver types. Furthermore, antenna phase centre modelling must also be applied when using two receivers of the same kind on long baselines, where the constellation will differ considerably at both receivers. It is the author's opinion that this antenna phase centre information should be supplied by the manufacturer with each antenna.

5.6 Tropospheric Modelling

Tropospheric scale factors are included in a fiducial GPS adjustment in an attempt to absorb any unmodelled tropospheric delays. They are solved for as part of the least squares solution and allow the zenithal delay to deviate from its estimated (modelled) value. The magnitude of this deviation is usually less than ± 20 % and any unrealistic values can be used as an indication of a problem data set. Their inclusion has not evolved through theoretical rigour, but because they improve the short-term, day to day, repeatabilities. Therefore, as was demonstrated in section 5.5, they must be used with great care since they will attempt to absorb any unmodelled errors, not just tropospheric delays.

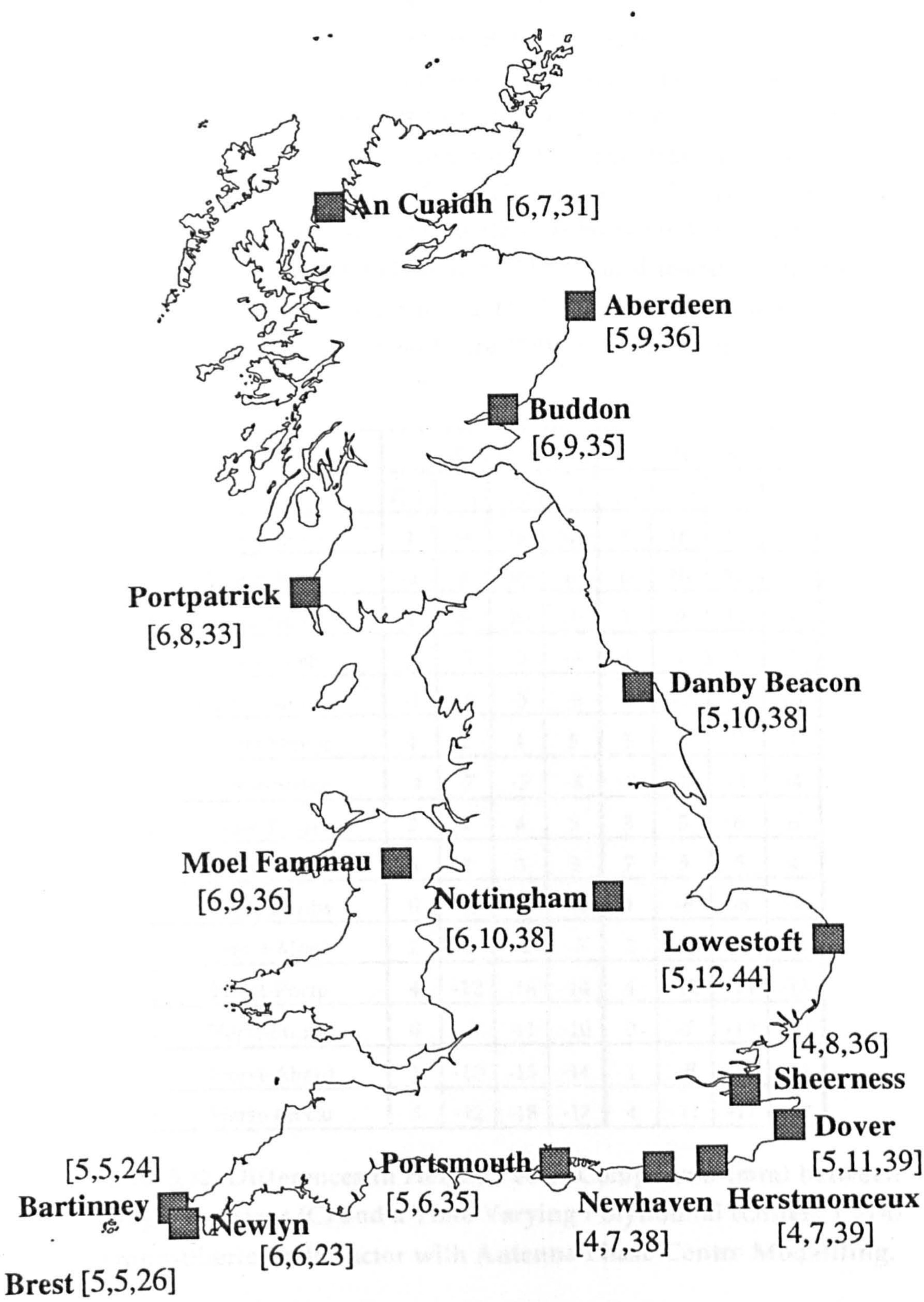


Figure 5.10 Map showing [Northing, Easting, Height] (mm) Differences Between the UK Coordinates from Fixed Fiducial Network TOWM, With & Without Antenna Phase Centre Modelling.

It is conventional in a fiducial GPS adjustment to solve for a constant tropospheric scale factor, per station, per session. However, this cannot truly represent variations in atmospheric conditions over several hours. In an attempt to improve the tropospheric modelling, tests were performed by solving for time varying polynomial tropospheric scale factors. The differences in the recovery of the heights of the regional stations relative to Herstmonceux, when using fixed fiducial stations TOWHM and TOWM, and using a constant or a time varying polynomial (orders 1 to 4) tropospheric scale factor are shown in Table 5.12. Full results are shown in Appendix D.

Baseline	TOWHM				TOWM			
	C-1	C-2	C-3	C-4	C-1	C-2	C-3	C-4
Herst-Brest	4	14	13	12	5	16	16	14
Herst-Barti	-1	9	10	6	0	10	10	8
Herst-Newly	1	8	10	7	1	9	13	9
Herst-Ports	2	-3	-3	-3	3	1	1	2
Herst-Newha	-1	-6	-5	-6	0	-2	-2	-6
Herst-Dover	4	2	4	5	5	6	7	7
Herst-Sheer	-4	-7	-5	-4	-3	-3	-1	-4
Herst-Lowes	2	1	4	5	3	5	6	6
Herst-Notti	6	2	3	3	7	5	5	4
Herst-Danby	0	-8	-10	-10	1	-6	-8	-9
Herst-Moel	2	-5	-4	-6	2	-3	-1	-5
Herst-Portp	4	-12	-16	-14	4	-12	-14	-13
Herst-Buddo	0	-8	-11	-10	0	-7	-10	-10
Herst-Aberd	1	-10	-15	-14	1	-8	-13	-14
Herst-An Cu	4	-12	-18	-17	4	-11	-17	-17

Table 5.12 Differences in Height Vector Component (mm) between using a Constant (C) and a Time Varying Polynomial (Orders 1 to 4) Tropospheric Scale Factor with Antenna Phase Centre Modellling.

It can be seen from Table 5.12 that the different orders of polynomial tropospheric scale factors change the height of the cluster of stations in the South East (Portsmouth, Newhaven, Dover, Sheerness, Lowestoft), Nottingham and Moel Fammau by a few millimetres only.

Whereas, the differences at the Northern (Danby Beacon, Portpatrick, Buddon, Aberdeen, An Cuaidh), and South Western (Newlyn, Brest, Bartinney) regional stations are up to 20 mm. At these regional stations, a first order (gradient) polynomial showed no significant improvement when compared to a constant tropospheric scale factor. Similarly, a 2nd order or higher polynomial had virtually the same effect, suggesting that they were modelling the same variations. However, the differences between the 1st and 2nd order polynomials were significant. The plan coordinates showed differences of up to 10 mm, but with an apparently random trend.

The conclusions that can be drawn from Table 5.12 are either that the tropospheric delays for the short baseline cluster in the South East England are being correctly modelled, or that there is a further unmodelled error source, which is geographical in nature.

The JPL IGS92 test reference frames were re-processed using both 1st and 2nd order polynomial tropospheric scale factors, per station, per session. The full results are shown in Appendix D and summarised below in Tables 5.13 and 5.14. These shows that for Herstmonceux a 1st order polynomial produces the best results, with the gradient of this scale factor agreeing with the tropospheric variations indicated by the surface pressure readings recorded at this station. The recovery of the other free fiducial stations show only slightly improvement when using a 1st order polynomial, as opposed to a constant. Furthermore, a second order polynomial seemed to give worse recovery.

The problem of using polynomial tropospheric scale factors is selecting which order of polynomial to use. Tables 5.13 and 5.14 suggest the use of a first order polynomial, which is consistent with the surface pressure readings recorded at all regional stations. However, Table 5.12 suggests that a 1st order scale factor might not be adequate for the Northern and South Western regional stations due to the apparent geographical anomaly, which is now discussed further in Section 5.7.

Polynomial Scale Factor	Test Reference Frames					Stations
	TOWM	OWM	TWM	TOM	TWH	
1st Order	•	•	•	22	•	Wettzell^R
	•	•	-6	•	-3	Onsala^R
	•	•	•	•	-16	Madrid^R
	•	3	•	•	•	Tromsoe^R
	0	-2	-2	-7	•	Herstm^T
2nd Order	•	•	•	18	•	Wettzell^R
	•	•	1	•	3	Onsala^R
	•	•	•	•	-45	Madrid^R
	•	-12	•	•	•	Tromsoe^R
	10	13	13	-1	•	Herstm^T

Table 5.13 Recovery of the Height Component of the Free Fiducial Stations (mm), when using Antenna Phase Centre Modelling and Solving for Polynomial Tropospheric Scale Factors.

(• Fixed station, ^{T} Trimble receiver, ^{R} Rogue receiver)

Polynomial Scale Factor	Test Reference Frames				
	TOWM-OWM	TOWM-TWM	TOWM-TOM	TOWM-TWH	TOWM-TOWHM
1st Order	5 ±1	-2 ±1	-14 ±6	0 ±2	-1 ±0
2nd Order	3 ±1	3 ±1	8 ±4	-11 ±4	-10 ±1

Table 5.14 Mean Height Differences and Corresponding Standard Errors (mm) Derived by Fixing Different Subsets of Fiducial Stations when using Antenna Phase Centre Modelling and Polynomial Tropospheric Scale Factors.

(Herstmonceux Trimble receiver used)

5.7 Ocean Tide Loading

In order to further analyse the large differences in height apparent at certain regional stations when a constant or a polynomial tropospheric scale factor is used, a comparison was performed between the shape of the 2nd order polynomial tropospheric scale factors on baselines from Herstmonceux to Newlyn, Herstmonceux to Danby Beacon and Newlyn to Brest. There was no clear trend between Newlyn and Brest. On the other baselines, however, the tropospheric scale factors for

Newlyn and Danby Beacon seemed to be absorbing an unmodelled error, which was sinusoidal in nature, and had a repeat period of approximately 24 hours. Similar trends, with different magnitudes, were also observed between Herstmonceux and any other regional station in the North or South West of the UK. The magnitude of these polynomials in microgal's (1/2 μgal ≈ 1 cm), relative to Herstmonceux, are shown in Figure 5.11, along with contours of the Ocean Tide Loading due to the M2 Ocean Tide *[Baker, 1984]*, which is a result of the periodic surface loading due to the weight of the ocean tides.

As can be seen, the unmodelled error source being absorbed by the 2nd order polynomial tropospheric scale factors seems to have a very strong correlation with the effect of ocean tide loading due to the M2 ocean tide. At the current stage of analysis, therefore, the 2nd order polynomial tropospheric scale factors have effectively removed the majority of the ocean loading effect. This successful elimination being largely due to the calm, slowly varying troposphere present in 1991.

For further analysis, an ocean tide loading model should be incorporated into the adjustment, enabling the tropospheric scale factors to be used to "only" model tropospheric delays, and not to absorb all unmodelled error sources. Improved modelling of ocean tide loading is currently being researched at the IESSG, funded by the Proudman Oceanographic Laboratory and NERC.

5.8 Earth Body Tides

The solid Earth deforms due to the gravitational attraction of the moon and sun that generate the ocean tides. These are known as Earth Body Tides or Solid Earth Tides and have a range of over 40 cm in Europe. They can be computed to an accuracy of a few millimetres at any point using Love Numbers *[Baker, 1984]* . Tests were carried out using the UK Gauge 91 data set and the Wahr earth body tide model *[Wahr, 1979]*. The difference in the unknown station coordinates between applying, and not applying, the Earth Tide Model was one or two millimetres. Clearly, the effect of Earth Body Tides is significant, if millimetric accuracies are required.

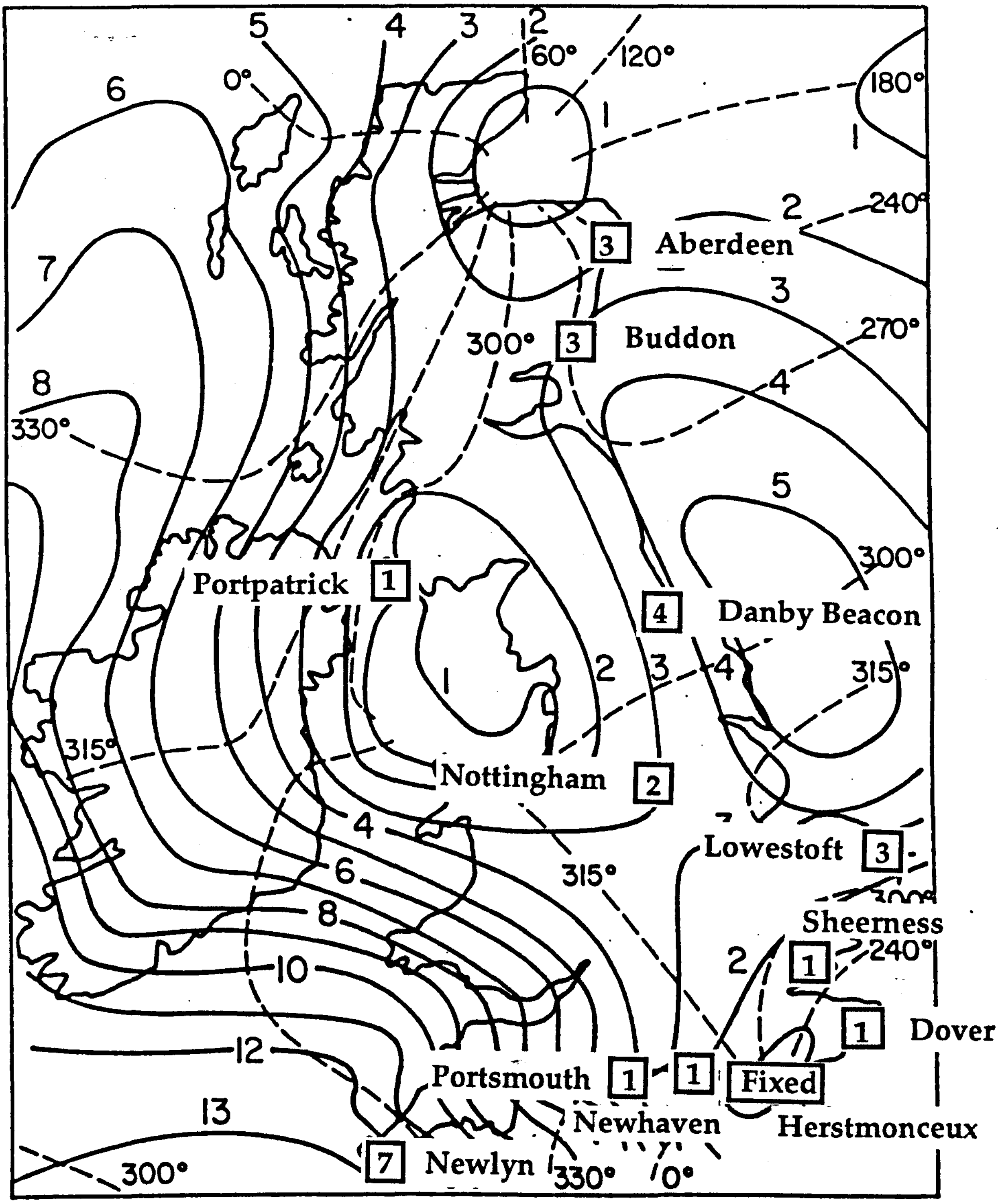


Figure 5.11 Magnitudes [μgal] of the 2nd Order Polynomial Tropospheric Scale factors and Contours of the Ocean Tide Loading due to the M2 Ocean Tide (μgal).

5.9 Ambiguity Resolution

It is generally accepted that both the precision and accuracy of the horizontal components of a GPS baseline can be improved by a factor of 2 to 3 times by fixing the carrier phase integer ambiguities to their correct 'true' values, with the vertical component being unaffected. In the reference frame tests described in section 5.4 integer fixing was attempted using the JPL IGS92 reference frame, antenna phase centre modelling, a 2nd order polynomial scale factor and the Wahr Earth Tide model.

The integer fixing process is a two stage process (see section 2.2.5). Firstly, the widelane integers (86cm) are determined and then these are input into a L1/W solution to solve for the L1 integers (19cm). On average, within the UK, only 80% of the widelane integers could be determined, and only a further 80% of the L1 integers could be determined from the widelane integers. This resulted in only about 60% of the integers being fixed. The integer fixed results showed that with a five hour observation session and good satellite constellation that no significant improvements in either the horizontal or vertical components were obtained. This did not alter the conclusions based on the integer free results and, therefore, it was considered 'safer' not to fix ambiguities, in order to avoid any potential problems of fixing them to the wrong values.

5.10 Comparison of Coordinates from UK Gauge 91 and UK Gauge 92 (preliminary solution).

The success of a fiducial GPS campaign is not only dependent on the careful design of the network, selection of the stations and general organisation, but also on the GPS operators setting up the receiver correctly and accurately measuring the relevant antenna heights. If the antenna is only set up once for the duration of the campaign then a blunder in the antenna height measurement will go undetected until the site is re-occuppied on a subsequent campaign. It is for this reason that the author compared his final results for UK Gauge 91, with the preliminary results from UK Gauge 92, processed holding Herstmonceux fixed and using the NGS precise ephemeris.

The comparison was performed on a baseline by baseline basis relative to Herstmonceux. The cluster of stations in South East England showed differences of a few centimetres, which gradually increased up to 8 cm in Northern Scotland. This effect was attributed to the quality of the NGS precise ephemeris, and since none of the stations deviated from this trend it was concluded that no gross blunders had been detected in the UK Gauge 91 coordinates.

5.11 High Accuracy Fiducial GPS Processing of UK Gauge 1991

Following the improvements made in the fiducial GPS technique, as described in the previous sections, a final coordinate set has now been produced from a fiducial GPS adjustment carried out by;

(i) Holding the fiducial stations of Tromsoe, Onsala, Wettzell, Herstmonceux and Madrid fixed to their JPL IGS92 coordinate values.
(ii) Using the ionospherically free observable with ambiguities free.
(iii) Modelling antenna phase centre variations.
(iv) Modelling the tropospheric delay (and ocean loading) using the MAGNET tropospheric model and solving for a 2nd order polynomial scale factor, per station, per session.
(v) Solving for 6 orbital parameters per satellite per station.
(vi) Modelling for Earth Body Tides using the Wahr model.

The session to session differences in baseline components from a weighted average are shown in Figure 5.11. The horizontal line on each graph represents the 2σ confidence level, ie 95% of the points are contained in the area below this line. The reader is urged to compare Figure 5.11 with Figure 5.5, the corresponding plot for the UK Gauge 91 processing using the conventional fiducial GPS technique. It can be clearly seen that there is a factor of two improvement in Northing, Easting and Length, and a factor of three improvement in Height. The precision of these results is now better than the one centimetre level for all components.

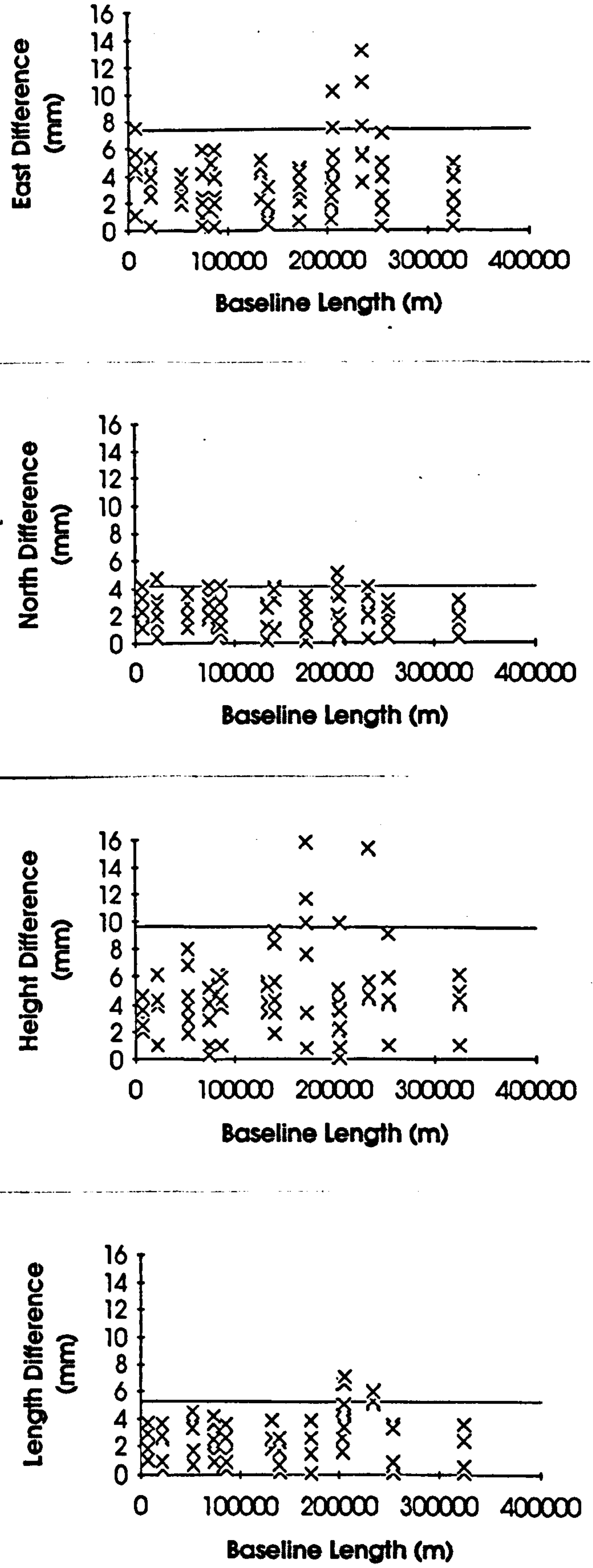


Figure 5.11 The Session to Session Differences (mm) in Baseline Component from a Weighted Mean (Final Results).

Since all five reference stations were fixed in this final solution no accuracy tests could be performed. However, the recoveries of the free reference stations shown in Table 5.13 and Appendix D, which were processed using the same options, give an indication of the accuracy of this final solution. They suggest an accuracy of one centimetre in all three components.

These results show that the accuracy and precision of the vertical component is now comparable with those of the plan component making GPS a truly a three-dimensional system. This final solution is the UK Gauge 91 coordinate set which has been used for the comparisons described in section 5.12 and the tests performed in Chapter 6.

5.12 Comparison of Coordinates from UK Gauge 91 and EUREF 89.

This section details the comparison performed between the final EUREF 89 coordinates, adopted by the EUREF Subcommission, and the final coordinates obtained by the author for UK Gauge 91. As EUREF 89 was based on the ETRF 89 reference frame and the JPL IGS92 solution was 'loosely' constrained to ITRF 91, any systematic biases between these two reference frames had to be removed before a comparison could be performed.

Tests showed that only the three translations of the origin, and a Z-axis rotation were significant between any combination of the permanent VLBI and SLR stations present in both reference frames. A bias was considered significant if it was 2 to 3 times larger than its variance. Since the biases varied only slightly (1 cm) when using different combinations of stations, the biases determined using Wettzell, Onsala, Madrid, Matera and Kootwijk, were adopted. The RMS differences at these stations, once biases had been removed, in latitude, longitude and height were 10, 33 and 10 mm respectively.

The biases determined were then applied to the UK Gauge 91 coordinates and the comparison with the adopted EUREF 89 coordinates is shown in Table 5.15. Since Herstmonceux was not used to determine the biases, it can be used to give an indication of the quality of this comparison in the UK.

Station	dN	dE	dH	Comment
Herst	-4	7	2	Permanent SLR
Brest	8	-39	-14	EUREF 89
Buddon	4	-15	-103	Mobile VLBI (1989)
Bartinney	23	59	-15	EUREF 89
Moel	19	15	-83	Adopted
An Cuaidh	11	40	-48	Coordinates

Table 5.15 Differences UK Gauge 91 (ETRF 89) minus EUREF 89 Coordinates (mm).

The conclusions that can be drawn from Table 5.15 are as follows

- The differences computed at Herstmonceux, ie less than 1 cm, give an indication of the quality of the transformation from JPL IGS92 (ITRF 91) to ETRF 89.

- The EUREF 89 GPS processing highlighted a potential error in either the Mobile VLBI or GPS antenna height measurement at Brest. These differences show that there is no problem with the Mobile VLBI height of this station, and suggest that there was a gross error in the GPS antenna height for EUREF 89.

- The height difference at Buddon shows a difference of 10 cm. This cannot be due to movement of the station over this short period of time (~4cm/year). It is more likely to be a gross error in the Mobile VLBI height, which is confirmed to some extent by the absence of a gross error in UK Gauge 91 (when compared on day to day and with UK Gauge 92).

- The coordinates of Bartinney, Moel Fammau and An Cuaidh are consistent with the 4 to 6 cm accuracy of the adopted EUREF 89 coordinates.

During the Nottingham processing of EUREF 89 the coordinates of both Buddon and Brest were fixed to their ETRF 89 values. Neither of these stations were fixed in the Berne Group solution and their Mobile

VLBI values were substituted in to the final adopted coordinates. Clearly, the fixing of the Buddon MVLBI values in the Nottingham processing will have adversely affected the heights of the other UK EUREF stations, and accounts for the differences between the two processing group solutions (see section 4.6).

5.13 Conclusions

1. The UK Gauge 91 data set has proved to be of extremely high quality.

2. The conventional fiducial GPS Processing of UK Gauge 91 produced coordinates with a precision of little better than 4 cm, and an accuracy of only 10 cm.

3. The choice of reference frame, or the assigning of highly consistent time tagged coordinate values to the fiducial stations, is crucial to the success of the fiducial GPS technique. The tests showed that a pure GPS based global reference frame (JPL IGS92) provided the highest 3-d positional accuracies.

4. For geophysical deformation monitoring projects (eg crustal dynamics or tide gauge heights), it is essential to time-tag the reference framework coordinates to the epoch of the GPS fiducial campaign. The IGS could provide coordinates at a regular interval, from their global tracking GPS network, for use by the geophysical community in regional GPS campaigns, such as the UK Gauge project.

5. The careful modelling of antenna phase centre variations, tropospheric errors and ocean tide loading, which predominately affect the vertical component, has produced station heights with an accuracy of 1 cm, comparable with the plan component.

6. These high accuracies have been achieved without determination of the integer ambiguities.

7. This improvement in accuracy when using the high accuracy fiducial GPS technique will reduce the observation interval needed in order to detect a deformation of sufficient magnitude.

8. The UK Gauge 91 coordinates will be further improved based on current IESSG research into ocean tide loading effects and tropospheric modelling. In addition, the use of an ambiguity search routine could successfully determine the correct integer values and further improve the plan component.

CHAPTER 6

National GPS and Terrestrial Geodetic Networks

As described in section 3.2, the horizontal and vertical terrestrial networks for Great Britain, as for most other countries, have always been treated separately. However, the development of GPS as a three-dimensional surveying tool enables the definition of three-dimensional datums. This led to the establishment of EUREF (see chapter 4) which produced three-dimensional coordinates for a number of primary triangulation stations, within each country. It was then the responsibility of the National Survey Organisations to densify the EUREF network to meet national requirements, such as mapping and surveying. This has resulted in the old terrestrial networks being replaced by new networks based entirely on GPS.

This chapter describes tests that have been performed to assess the quality of the horizontal and vertical terrestrial networks of Great Britain, by comparison with the GPS coordinates resulting from the EUREF 89 and UK Gauge 91 GPS campaigns. Section 6.1 compares the OS(SN)80 coordinates with the GPS coordinates and then describes re-adjustments of the OS(SN)80 terrestrial network whilst including these GPS coordinates. In section 6.2 the GPS ellipsoidal heights are combined with several precise geoids, and compared with the Third Geodetic Levelling of Great Britain, in an attempt to solve the British Sea Slope Anomaly. Section 6.3 describes the densification of EUREF in the UK and simulated adjustments are carried out to assess the effect of combining the terrestrial observations from the OS(SN)80 network with a national network of GPS observations. The chapter is concluded in section 6.4.

6.1 GPS and the Horizontal Control of Great Britain

In order to remove the large systematic biases inherent in the OSGB70(SN) adjustment (see section 3.2.1.3), independent observations were required which were free from such biases. In the early 1980's the obvious choice was the Transit system, which could produce positions, using the Precise Ephemeris, to an accuracy of 1 metre (see section 2.3). The OS(SN)80 adjustment included a total of eleven Transit Doppler positions which had been corrected by -0.4 ppm in scale and 0.8 arc-seconds in longitude. At the time these were the accepted corrections to the Transit Doppler System to make the scale consistent with VLBI and the orientation consistent with the BIH zero meridian.

The inclusion of these Transit positions allowed systematic biases in the terrestrial observations to be solved for as bias parameters in the least squares adjustment. One bias parameter was assigned to each of the EDM instruments (Geodimeter lightwave and Tellurometer microwave) and one to the Laplace Azimuths. The adjustment results showed that the Tellurometer distances were 3.2 ppm too short, while the Geodimeter distances and Laplace azimuths did not contain any significant systematic biases (see section 3.2.1.4).

However, the values used to correct the Transit System were different to those used in the realisation of WGS84. Therefore, it is possible that the Transit positions included in OS(SN)80 (and WGS84) are still contaminated with systematic errors. If this is the case, then clearly the terrestrial bias parameters will also be in error by a similar amount. It is, therefore, necessary to use a higher order system to determine the systematic biases of the Transit system. The UK EUREF station coordinates determined as part of the EUREF 89 GPS Campaign (10 cm) and the UK Gauge 91 GPS Campaign (1-2 cm) are a one to two orders of magnitude improvement over the Transit Doppler positions.

A direct comparison was performed between the UK Gauge 91 coordinates and the OS(SN)80 coordinates. Once a systematic translation of the origin had been removed, the plan coordinates differed by an average of 22 cm. However, the standard errors for the translation of the origin were as large as 1 metre and this comparison cannot, therefore, give a true indication of the quality of the OS(SN)80 adjustment.

Consequently, the only way to assess the quality of OS(SN)80 is by re-adjustment of the original observations, including the GPS positions, to model not only the biases in the terrestrial observations but also possible biases in the Transit positions.

6.1.1 Re-adjustment of OS(SN)80

The re-adjustment involved using the terrestrial observations included in the OS(SN)80 network, the eleven original Transit positions, eight EUREF 89 GPS positions (Herstmonceux, Bartinney, Moel Fammau, Collier Law, An Cuaidh, Crockinacoe, Carrigaderragh and Carrigfadda) and four UK Gauge 91 GPS positions (Herstmonceux, Bartinney, Moel Fammau and An Cuaidh). Unfortunately, the primary EUREF stations of Danby Beacon and Buddon were not part of the primary triangulation, and hence not part of the OS(SN)80 terrestrial network.

Each test involved using the terrestrial observations with different combinations of the space derived positions. These test networks are detailed in Table 6.1, showing the number of each type of observation used and an ID to identify each test network. The a-priori standard errors for the terrestrial observations and Transit positions were the same as those used in the original OS(SN)80 adjustment. For the GPS positions, 10 cm and 2 cm were used for the EUREF 89 and UK Gauge 91 coordinates respectively. The SN 80 adjustment was identical to the OS(SN)80 adjustment and used to test the software and validate that the author was using it correctly.

ID	Direct'ns	Distances	Laplace	Transit	GPS (EUREF+UK Gauge)
SN 80	2244	239**	26*	11	-
SN 91 A	2244	239**	26*	11	8+0
SN 91 B	2244	239**	26*	11	8+4
SN 91 C	2244	239**	26*	11*	8+4
SN 91 D	2244	239**	26*	-	8+4
SN 91 E	2244	239**	26*	-	0+4

Table 6.1 Observations Included in Test Network Adjustments (* Bias parameter determined, ** Two bias parameters determined)

The test networks were adjusted using the Nottingham network adjustment program COSTNET. This program, which was originally written for the adjustment of OS(SN)80, accepts terrestrial observations (horizontal angles, directions, distances, azimuths, absolute or relative latitudes and longitudes) as well cartesian positions or position differences. COSTNET performs a combined least squares adjustment of the 2-d and 3-d observations. The program can determine biases for each type of observation and operate in either a real data or simulation mode. Further details of COSTNET including the program options and adjustment models can be found in *[Turney, 1988]*.

The adjustments were performed identically to OS(SN)80. Firstly the GPS coordinates were transformed from the earth-centred datum (ETRF 89 for EUREF 89 and ITRF 91 for UK Gauge 91) to the local geocentric datum (Airy) using the transformation parameters, dX, dY and dZ, between the existing geodetic coordinates at Herstmonceux and the corresponding GPS coordinates. After the adjustment was performed, the resulting coordinate set was transformed as a single entity, to bring the Herstmonceux coordinates back to their original values (see section 3.2.1.4).

The results form the SN80 adjustment were identical to those from the original Nottingham OS(SN)80 adjustment, which confirmed that the software was working correctly.

6.1.2 Bias Parameters

Table 6.2 shows the bias parameters and their corresponding standard errors determined during each test adjustment.

	Azimuths (arc-seconds)	Distances (ppm) Tellurometer	Geodimeter	Transit (ppm)
SN80	-0.074 ±0.23	3.247 ±0.80	0.423 ±0.91	-
SN 91 A	-0.189 ±0.17	2.722 ±0.35	-0.185 ±0.55	-
SN 91 B	-0.188 ±0.15	2.616 ±0.26	-0.221 ±0.46	-
SN 91 C	-0.188 ±0.15	2.617 ±0.25	-0.221 ±0.46	-0.322 ±0.12
SN 91 D	-0.183 ±0.16	2.594 ±0.26	-0.240 ±0.47	-
SN 91 E	-0.170 ±0.16	2.540 ±0.28	-0.314 ±0.48	-

Table 6.2 Bias Parameters Determined During Network Adjustments.

The following are the conclusions that can be drawn from Table 6.2.

(1) The SN 91 C adjustment shows that there is a significant scale bias of -0.3 ± 0.1 ppm for the Transit positions, ie the Transit scale is too large by 0.3 ppm. This bias has clearly affected the determination of the terrestrial biases in the SN 80 adjustment. No significant orientation biases were found in the Transit positions.

(2) The inclusion of different subsets of GPS positions has little significant effect on the determination of the terrestrial biases.

(3) The SN 91 adjustments agree with the OS(SN)80 adjustment that only the Tellurometer EDM has a significant bias. The new value of this bias, 2.6 ± 0.3 ppm, agrees well with the value of 2.6 ± 0.4 ppm determined by the Ordnance Survey from measuring multiple baselines using different types of EDM *[Williams, 1979]*.

(4) Between the SN 80 and SN 91 adjustments the value of the Geodimeter bias has changed from 0.42 to -0.20 ppm. This 0.5 ppm decrease is insignificant when considered with the corresponding standard errors.

These results show that the Transit positions included in the OS(SN)80 should have been corrected for a scale error of -0.7 ppm instead of -0.4 ppm. This new value is in closer agreement with the value used for the realisation of WGS84, ie -0.6 ± 0.1 ppm.

6.1.3 Residuals

The residuals for the Transit positions are plotted in Figure 6.1. These confirm that the correct a priori standard error of 1 metre was applied.

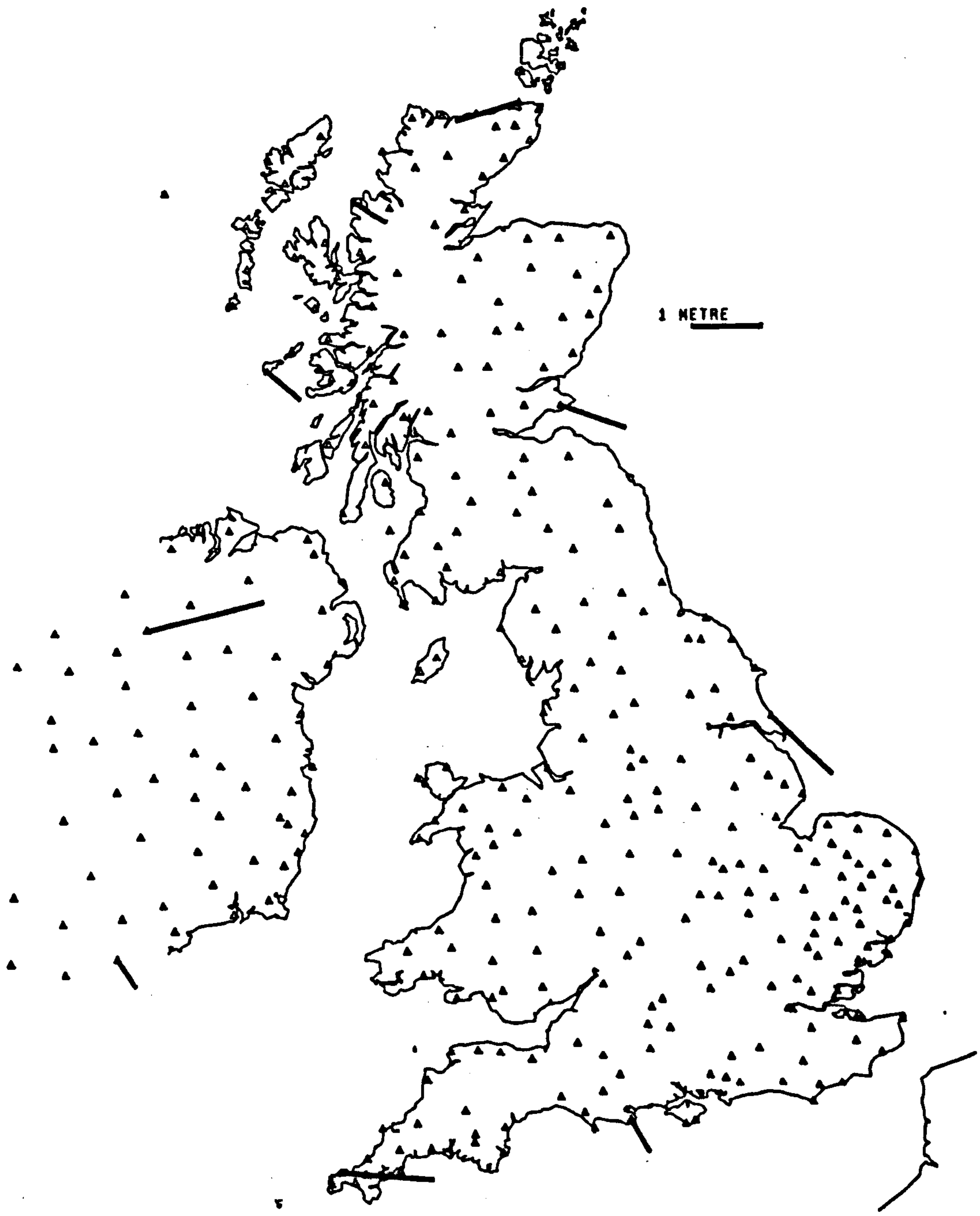


Figure 6.1 Vector Residuals for the Transit Positions.

6.1.4 Coordinate Differences

Figure 6.2 shows the vector differences between OS(SN)80 and SN 91 coordinates at 30 stations throughout the UK. The reader is urged to compare Figure 6.2 with Figures 3.2, 3.3 and 3.4 to see the improvement in the coordinates of the primary triangulation of Great

Britain, over the past two hundred years. The effect of the more accurately determined systematic biases in SN 91 can clearly be seen in Figure 6.2, as the difference between the two adjustments forms a radial pattern with increasing magnitude from the origin, Herstmonceux. The differences between OSGB36, OSGB70(SN), OS(SN)80 and SN 91 at Saxavord, in the Shetlands and Slievemore, in Ireland, on the northern and western edges of the network respectively are shown in Table 6.3.

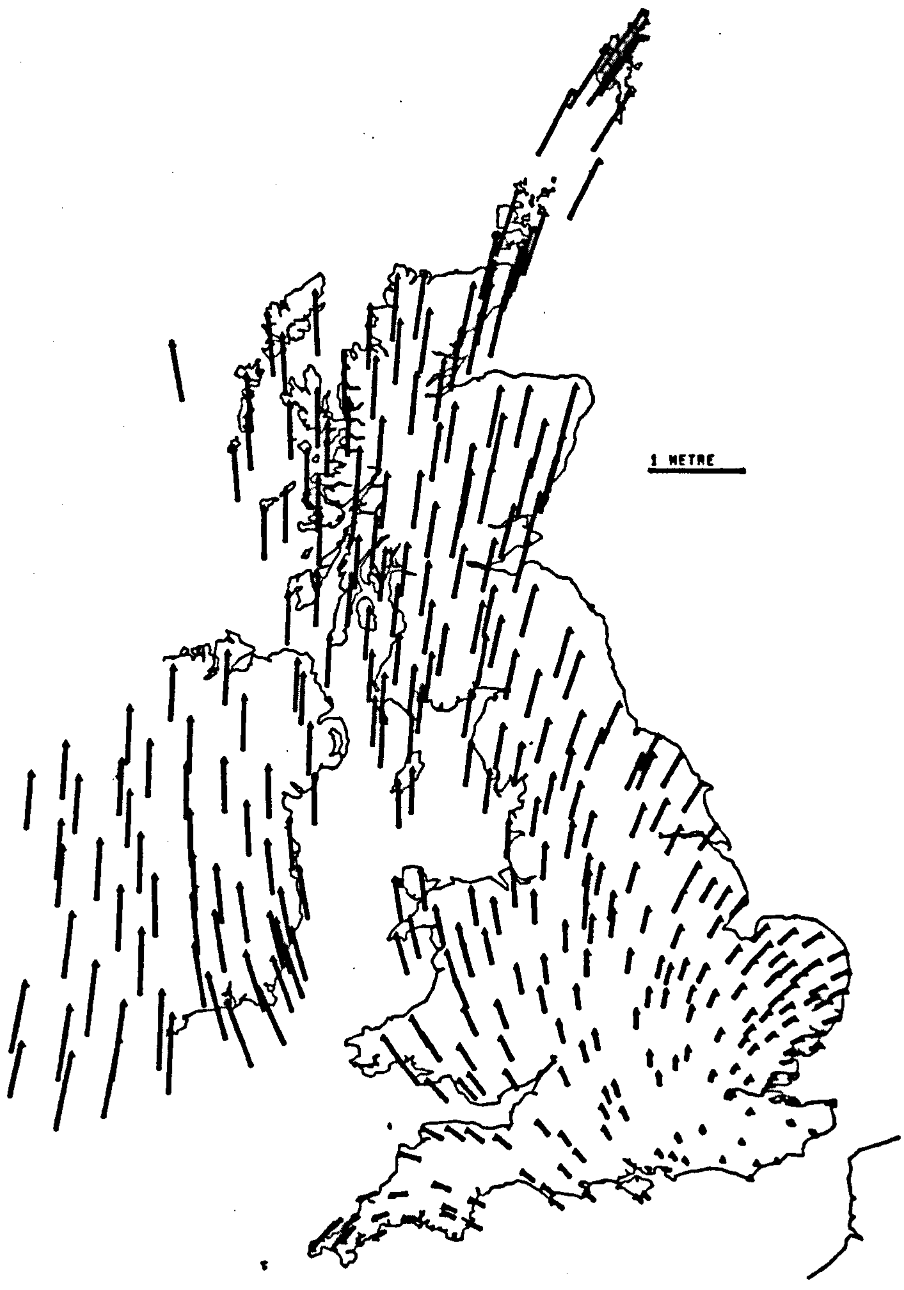


Figure 6.2 Differences at Selected Stations Between OS(SN)80 and SN 91.

	OSGB70(SN)- OSGB36		OS(SN)80- OSGB70(SN)		SN 91- OS(SN)80	
	dE	dN	dE	dN	dE	dN
Saxavord	-3.622	-23.360	-1.084	4.125	-0.057	-0.457
Slievemore	15.032	-3.923	-1.147	1.092	-0.889	-0.343

Table 6.3 Difference (m) Between Various Adjustments of the Primary Triangulation on the Northern and Western Edges of the Network.

Table 6.3 illustrates how the length of the UK has changed between the various adjustments. The 23.36 metre decrease in the length of the British Isles between OSGB36 and OSGB70(SN) was due to the varying scale of OSGB36. The 4.13 metre increase between OSGB70(SN) and OS(SN)80 was due to the Tellurometer EDM scale bias. Finally, the length of the British Isles has shrunk by 0.45 metres due to the removal of the scale bias in the Transit positions, and the correct modelling of the Tellurometer scale bias. This order of magnitude improvement in scale between each adjustment suggests that over the length of the country, the SN 91 adjustment has a scale bias of the order of a few centimetres.

6.1.5 Recovery of Known Station Coordinates

Tests were performed to assess the accuracy of the SN 91 adjustments. These involved using test network SN 91 E, which included the four UK Gauge 91 GPS positions with a-priori standard errors of 2 cm, and releasing one GPS position at a time. The recovered coordinates of the free station, based on the terrestrial observations, were then compared with the GPS coordinates. These recoveries are shown in table 6.4.

The recoveries of the known station coordinates in Table 6.4 suggests that, when two or three GPS stations are included and well distributed geographically, the accuracy of the adjustment is between 10 - 50 cm. This is quite remarkable considering that many of the terrestrial observations were observed as long ago as 1936.

	Herstm'cx		Bartinney		Moel Fammau		An Cuaidh	
	E	N	E	N	E	N	E	N
SN 91 E1	•	•	•	•	12	23	•	•
SN 91 E2	•	•	8	20	7	17	•	•
SN 91 E3	20	23	•	•	23	24	•	•
SN 91 E4	•	•	•	•	40	35	53	32
SN 91 E5	•	•	15	24	•	•	41	27
SN 91 E6	•	•	12	21	•	•	•	•

Table 6.4 Recoveries (cm) of Known Coordinates (• fixed station).

These tests have shown that, when systematic biases are correctly modelled, the OS(SN)80 terrestrial observations are of a high quality. Their contribution to the future mapping control network of Great Britain is discussed further in section 6.3.

6.2 GPS and the Geodetic Levelling of Great Britain

The monitoring of mean sea level on any large geographical scale requires the tide gauge benchmarks to be related to a common datum. Conventionally, this has been achieved in two ways. Firstly, using geodetic levelling to relate the individual tide gauge benchmarks to a single datum point, usually mean sea level for a particular epoch at a single tide gauge, or secondly using oceanographic levelling techniques. However, obvious problems occur if these two methods do not agree.

As was described in section 3.2.2, the Second Geodetic Levelling of Great Britain suggested that a sea slope might exist, and the Third Geodetic Levelling clearly indicated the presence of a slope of 5.3 cm per degree of latitude between Southern England and Scotland (see figure 3.5). This slope was considered difficult to explain by oceanographers, and conflicted with their results from three independent oceanographic levelling techniques, which agreed to within 6 cm and suggested a net sea slope of zero *[Thompson, 1980]*. This discrepancy is known as the 'British Sea Slope Anomaly' and it has been suggested that it is due to a systematic error in the geodetic levelling, but to date (September 1993) this has not been proven.

From 1987 to 1990, the University of Nottingham in conjunction with the Ordnance Survey of Great Britain, Proudman Oceanographic Laboratory and the University of Edinburgh collaborated on a project to attempt to resolve the British Sea Slope Anomaly *[Ashkenazi et al, 1990]*. This involved using GPS and a precise relative geoid to provide an independent determination of orthometric height differences between five tide gauges situated along the East coast, from Lowestoft to Leith. The final results from a GPS campaign observed in June 1988 proved to be inconclusive, mainly due to insufficient satellites, the lack of fiducial stations in Europe and because high accuracy fiducial GPS technology was yet to be developed.

The processing of the UK Gauge 91 GPS Campaign, as described in Chapter 5, overcame the problems of the 1988 GPS Campaign, and produced ellipsoidal heights accurate to 1-2 centimetres. This section describes how these ellipsoidal heights have been combined with several different precise relative geoids to produce relative orthometric heights at the tide gauge stations, in an attempt to solve the British Sea Slope Anomaly.

6.2.1 Precise Relative Geoids

At present, the only available geoids for the UK are relative geoids, which have been calculated during independent research projects at the Universities of Nottingham *[Gerrard, 1990]*, Edinburgh *[Stewart, 1990]*, and Oxford *[Featherstone, 1992]*. The current Nottingham Geoid only covers England and Wales, and the other two cover the whole of the UK. Details of the computational techniques used to calculate these geoids is beyond the scope of this thesis and the reader is referred to the above references.

6.2.2 Solving the British Sea Slope Anomaly

In order to solve the British Sea Slope anomaly, the GPS ellipsoidal heights of the tide gauge stations have been combined with precise geoid heights and used to correct the Third Geodetic Levelling orthometric heights. At Whitby and Dunbar the correction was taken from the nearest UK Gauge regional station, namely Danby Beacon and Buddon respectively. The procedure employed was as follows,

(i) The GPS ellipsoidal height differences along the following baselines from southern England to Scotland; Dover - Sheerness - Lowestoft - Danby - Portpatrick - Buddon - Aberdeen, were computed.

(ii) The ellipsoidal height differences from (i) were combined with each geoid to produce orthometric height differences, along each baseline.

(iii) Assuming the orthometric height at Dover to be correct, the GPS orthometric height differences were used to determine new orthometric heights for the six other tide gauges.

(iv) The new orthometric heights were then differenced from the Third Geodetic Levelling orthometric heights, and this difference subtracted from the values of mean sea level (1960-75), shown in Figure 3.5, to produce new mean sea level values.

Figure 6.3 shows a long section along the baselines from Dover to Aberdeen for mean sea level determined by combining GPS with the Nottingham, Oxford and Edinburgh Geoids as well as from the Third Geodetic Levelling.

The conclusions that can be drawn from Figure 6.3 are,

(i) Between Dover and Danby Beacon (400 km) all three geoids agree within 100 mm suggesting an accuracy of 0.3 ppm. However, North of Danby Beacon (Whitby), the Oxford and Edinburgh geoids differ by 400 mm. Similar problems were also found with the Nottingham Geoid in this area, and this is the reason why this Geoid currently only covers England and Wales. The differences between the geoids from Southern England to Scotland is the same size as the magnitude of the British Sea Slope Anomaly.

(ii) The Nottingham geoid appears to solve the British Sea Slope Anomaly. From Dover to Whitby the Third Geodetic Levelling difference of 226 mm has been reduced to 15 mm, and the local bulge at Sheerness has also been flattened (see Figure 6.4). This geoid produces the results that the author was expecting, ie that the British Sea Slope Anomaly was caused by a systematic error in the Third Geodetic Levelling, but to make this conclusion based on one result, which could be a 'coincidence', is not correct. Therefore, it is safer to conclude that at the current level of geoid accuracy, it is not possible to solve the British Sea Slope Anomaly.

Although the British Sea Slope Anomaly has puzzled geodesists and oceanographers for many years, todays requirements for sea level monitoring have changed. It is no longer adequate to just monitor mean sea level relative to a tide gauge benchmark, and then connect these tide gauges using a single epoch geodetic measurement eg geodetic levelling. Due to effects such as post glacial rebound, it is now necessary to simultaneously monitor the vertical movement of the land at tide gauge sites. The solution of the British Sea Slope Anomaly is only important in order to resolve the differences between oceanographic and geodetic levelling techniques. The simultaneous monitoring of vertical movement using fiducial GPS and mean sea level at tide gauges will enable the monitoring of changes in absolute sea level. By combining the measurements from several tide gauges, global changes in absolute sea level can be monitored, without the application of a gravimetric geoid.

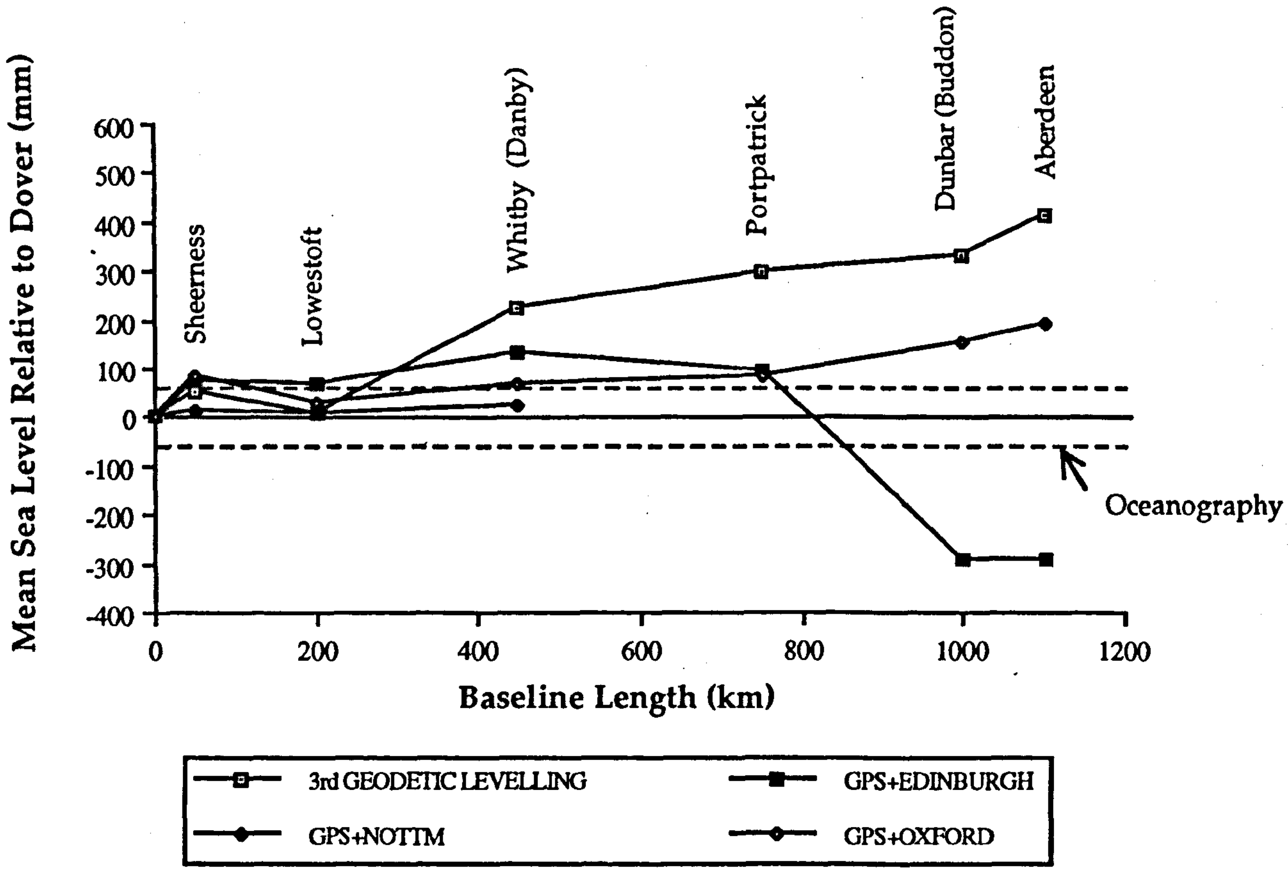


Figure 6.3 A Long Section along the Baselines from Dover to Aberdeen for Mean Sea Level Determined by Combining GPS with the Nottingham, Oxford and Edinburgh Geoids as well as the Third Geodetic Levelling.

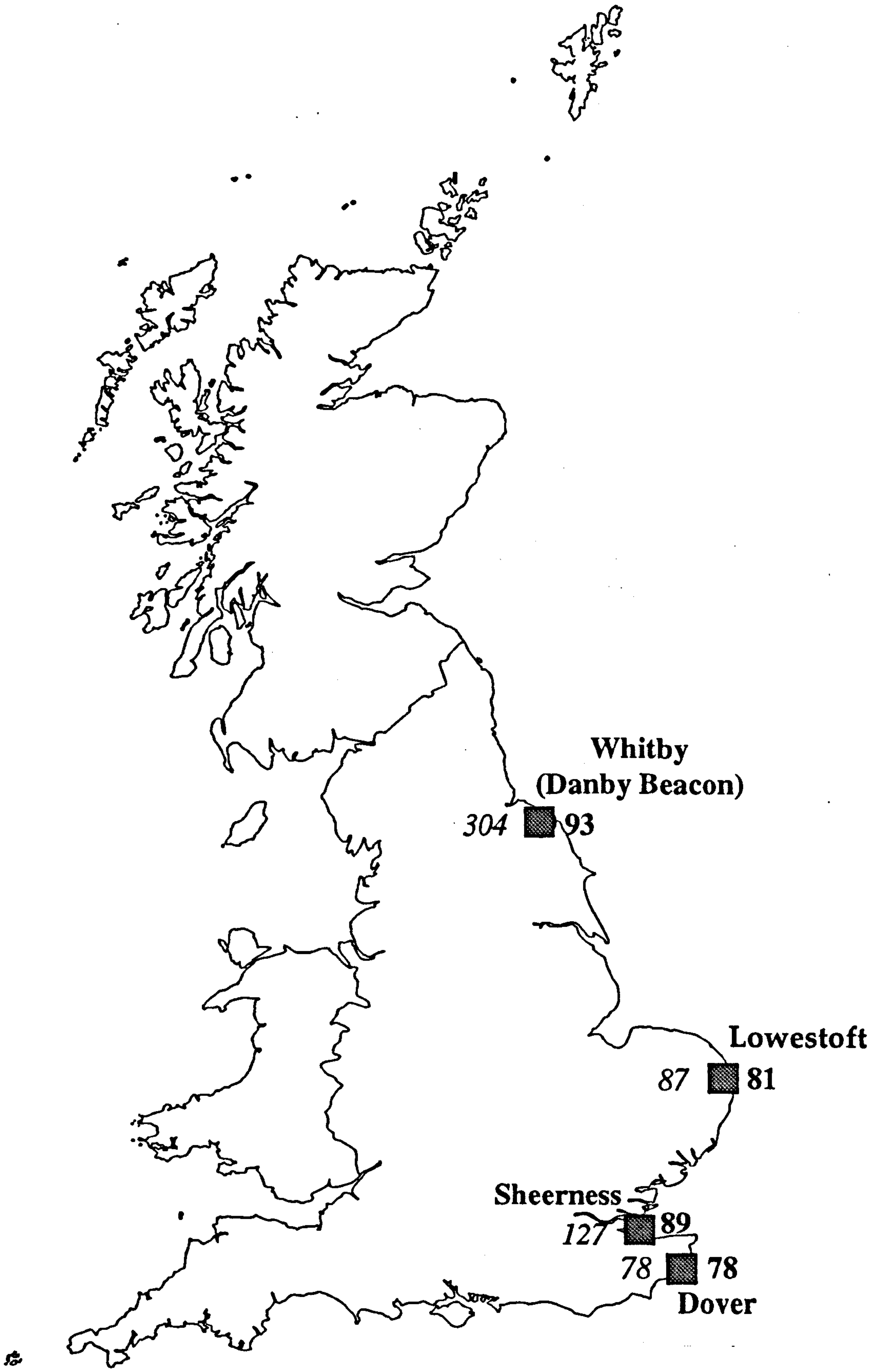


Figure 6.4 Mean Sea Level (mm) 1960-75 Relative to Dover
[*Third Geodetic Levelling*, UK Gauge 91 GPS + Nottm geoid]

6.3 Future Geodetic Networks for Great Britain

In 1987 the Ordnance Survey of Great Britain started using GPS for mapping control. Since the national mapping is based upon OSGB36, all GPS measurements have to be adjusted (scaled) to fit the existing control. The nature of the OSGB36 adjustment resulted in large distortions across the country and, therefore, no single transformation can be applied to transform GPS vectors (WGS84) into OSGB36. Furthermore, the inaccessibility of many of the original triangulation pillars, located on mountain tops for intervisibility, and the high cost of maintaining over 6000 triangulation pillars led the Ordnance Survey to review its requirements for mapping control. The main conclusion was that a new mapping control network should be established using GPS *[Christie, 1991]*.

The aim of this National GPS network was to make the use of GPS more efficient. The station interval was selected as 20-25 km in urban areas, and up to 50 km in rural areas. New stations were established that were suitable for GPS, and many of the primary triangulation pillars were included in order to allow transformation parameters to be computed. The network consists of 500 stations and was observed between March 1989 and May 1991, using single and dual frequency receivers, and being processed using the Broadcast Ephemeris.

The six primary UK EUREF stations will provide the 'zero order' control upon which the National GPS Network will be based. This will ensure that the new mapping control for Great Britain is compatible with other European countries. However, the UK EUREF stations are placed at intervals of approximately 300 - 500 km and Ordnance Survey believed that it was necessary to create an intermediate scientific network of some 10 - 15 stations spaced at approximately 100 - 150 km intervals. This was observed using GPS in October 1992 and is known as the Ordnance Survey Scientific Network or Sci Net. The three levels of GPS control networks, EUREF, Sci Net and National will effectively replace the existing triangulation networks. The results from section 6.1 suggested that when the terrestrial observations included in the OS(SN)80 network were controlled by several high accuracy GPS positions, the adjusted network could have an accuracy of 10 - 20 cm. This is clearly approaching the accuracy achievable from the National GPS network.

A series of simulated adjustments were carried out to assess the effect of including terrestrial observations (distances, directions, and azimuths) in a National GPS network of positions and position differences. The simulated network consisted of the terrestrial observations from the OS(SN)80 network, twenty GPS positions (5 cm accuracy) and 900 position differences (1-2 ppm). The tests were performed using the network adjustment program COSTNET (see section 6.1) and are described in Table 6.5.

ID	Direct'ns	Distances	Laplace	GPS Posn's	GPS Posn's Diffs
SIM A	2244	239**	26*	20	-
SIM B	-	-	-	20	900 (2ppm)
SIM C	2244	239**	26*	20	900 (2ppm)
SIM D	-	-	-	20	900 (1ppm)
SIM E	2244	239**	26*	20	900 (1ppm)

Table 6.5 Observations Included in Simulated Adjustments
(* Bias parameter determined, ** Two bias parameters determined).

In order to allow a direct comparison between successive simulations, a set of 30 independent baselines were selected for which the a-posteriori RMS errors (variance-covariance) were calculated. These lines were well distributed and divided into three groups depending on length, 15 short lines (30 - 50 km), 10 medium lines (200 - 300 km) and 5 long lines (500 - 600 km). Table 6.6 shows the RMS errors for the selected baselines from the simulation adjustments.

		Length		Orientation
		(m)	(PPM)	(secs)
SIM A	Short	0.084	1.813	0.35
	Medium	0.101	0.417	0.09
	Long	0.200	0.221	0.07
	Mean	0.109	1.083	0.22
SIM B	Short	0.060	1.329	0.27
	Medium	0.069	0.286	0.06
	Long	0.100	0.127	0.03
	Mean	0.070	0.781	0.16
SIM C	Short	0.045	0.991	0.20
	Medium	0.054	0.222	0.05
	Long	0.076	0.096	0.02
	Mean	0.053	0.586	0.12
SIM D	Short	0.030	0.666	0.14
	Medium	0.036	0.149	0.03
	Long	0.052	0.067	0.01
	Mean	0.036	0.394	0.08
SIM E	Short	0.027	0.596	0.12
	Medium	0.033	0.136	0.03
	Long	0.047	0.060	0.01
	Mean	0.032	0.354	0.07

Table 6.6 The RMS Errors for Selected Baselines From the Simulated Adjustments.

Table 6.6 shows that the effect of the terrestrial observations is dependent upon the quality of the GPS observations. By comparing SIM B and SIM C, it can be seen that for an a-priori standard error of 2 ppm or worse, the inclusion of terrestrial observations has a significant effect. However, by comparing SIM D and SIM E it can be seen that for a-priori errors less than 2 ppm, terrestrial observations do not have a significant effect.

Clearly, the inclusion of the terrestrial observations into the adjustment of the National GPS network will provide a method of quality assessment as well as possibly improving the strength of the network.

6.4 Conclusions

(1) The Transit positions included in the OS(SN)80 adjustment have a scale bias of -0.3 ± 0.1 ppm, ie too small by 0.3 ppm, which has effected the OS(SN)80 adjustment.

(2) The Tellurometer EDM scale bias from the SN 91 adjustments compares exactly with those from independent tests performed by the Ordnance Survey from measuring multiple bases using different EDM's.

(3) The OS(SN)80 terrestrial observations, when adjusted with four UK Gauge 91 GPS positions which have been well distributed geographically, have an accuracy of 10 - 20 cm.

(4) Using the latest high precision geoids it is not possible to solve the British Sea Slope Anomaly. This is because the accuracy of the geoids over the length of Great Britain is the same order of magnitude as the anomaly itself.

(5) The monitoring of mean sea level using tide gauges requires the monitoring of the vertical land movement at the tide gauge site. This cannot be achieved using precise spirit levelling and hence, for this purpose, the solution of the British Sea Slope Anomaly is not necessary. Instead, high accuracy fiducial GPS can be used to monitor changes in the ellipsoidal height at the tide gauge sites.

(6) The terrestrial observations from OS(SN)80 should be included with the National GPS network adjustment to provide a method of assessing the quality of the GPS, as well as to strengthen the network.

CHAPTER 7

Conclusions and Future Work

7.1 Conclusions

EUREF and the European Terrestrial Reference Frame

1. The EUREF 89 GPS Campaign was a success, despite receiver hardware problems in the UK (loss of L2 data), the long baselines observed and high ionospheric activity.

2. The adopted UK coordinates are of an accuracy of better than 10 cm, which is sufficient for national mapping control, navigation and the determination of transformation parameters between national or European coordinate systems and WGS84.

3. The resulting coordinates are not of a comparable accuracy with VLBI and SLR and, therefore, are not suitable for a national geodetic network for engineering applications.

The UK Gauge Project and High Accuracy Fiducial GPS

1. For a fiducial GPS adjustment the choice of reference framework, or more precisely the allocation of highly consistent time-tagged coordinate values to the fiducial stations, significantly affects the accuracy of both the horizontal and vertical coordinate components.

2. Using the UK Gauge 91 Data Set, tests clearly showed that the 'best' reference framework was based upon global GPS, rather than VLBI, SLR or a combination of these two.

3. Global GPS is as accurate as VLBI and SLR, but has the further advantage that it does not require the use of local offset measurements and antenna heights. Furthermore, global GPS networks have the potential to provide time-tagged fiducial

station coordinates at the observation epoch, hence, removing the need to use plate motion models.

4. In a fiducial GPS adjustment, unmodelled errors due to the atmosphere, ocean tide loading effects and antenna phase centre variations degrade the vertical coordinate component. This mismodelling in conventional fiducial GPS adjustments has led to the wisdom that height accuracies are two to three times worse than the corresponding horizontal accuracies.

5. The results in Chapter 5 showed that by modelling the antenna phase centre variations and solving for a polynomial tropospheric scale factor, to absorb the variations in the troposphere and ocean tide loading, accuracies of 1 cm in height can be achieved. These are comparable with the horizontal coordinate components, making fiducial GPS a truly three-dimensional system.

6. The results in this thesis have serious implications for the geophysical community. The order of magnitude improvement in accuracy between the Conventional Fiducial GPS Technique (10 cm) and the High Accuracy Fiducial GPS Technique (1 cm) means that shorter monitoring periods will be required to detect a deformation of significant magnitude.

7. The coordinates of the UK EUREF stations obtained from high accuracy processing of the UK Gauge 91 Campaign are comparable with VLBI and SLR and are, therefore, suitable to be used for control of a national geodetic network for engineering applications.

Terrestrial Geodetic Networks and GPS

1. The re-adjustment of the OS(SN)80 terrestrial network, controlled by high accuracy GPS, has shown that the Transit Doppler positions have a scale error of -0.7 ppm, and not the -0.4 ppm used in the original OS(SN)80 adjustment. This scale bias in the Transit Positions affected the determination of the Tellurometer EDM scale bias by 0.3 ppm and the scale of the original OS(SN)80 adjustment by a similar amount.

2. When controlled by high accuracy GPS, the re-adjustment of the OS(SN)80 network has an accuracy from 10 to 40 centimetres.

3. The solution of the British Sea Slope Anomaly is not possible using GPS ellipsoidal heights (1-2 centimetres) and the latest precise relative geoids. This is because, over the length of Great Britain, the accuracy of these geoids is of the same order of magnitude as that of the anomaly itself.

4. The geodetic levelling of Great Britain will never be re-observed and the solution of this anomaly has become of academic interest only.

5. The High Accuracy Fiducial GPS Technique can be used to connect all UK Tide Gauges at regular intervals in order to monitor changes in vertical land movement, and hence monitor changes in mean sea level.

6. The re-adjustment of the National GPS network of Great Britain should include the OS(SN)80 terrestrial observations to strengthen the network and allow improved quality assessment of the GPS measurements.

7.2 Future Work

1. The re-adjustment the UK Gauge 91 Data Set including an Ocean Tide Loading Model. This will allow tropospheric scale factors to be solved which 'only' absorb the effects due to the troposphere, and enable further research into improved tropospheric modelling.

2. The accuracy and repeatability of the plan component could possibly be further improved by fixing the carrier phase ambiguities to their correct integer values. This could be achieved using ambiguity search routines.

3. Compare the results from all three UK Gauge GPS campaigns processed using the High Accuracy Fiducial Technique. This will give information about the long-term repeatability of GPS

and possibly the vertical movement, over three years, of the land at the tide gauge sites.

4. Establish a reference frame for global GPS solutions in a similar way to those for VLBI and SLR.

5. Re-process all UK Gauge GPS campaigns using IGS orbits and compare the results with those from the High Accuracy Fiducial GPS Technique. This will give an indication of the accuracy of IGS orbits and enable the investigation of the use of IGS orbits to improve, extend and densify the original EUREF network.

6 Establish a permanent network of GPS receivers at UK tide gauge sites to enable more regular solutions. This will give more information about seasonal effects on GPS observations, as well as eliminating antenna set-up errors and problems due to failure of one, or more, fiducial station GPS receivers, which are common to short annual campaigns.

APPENDIX A

WGS 84 Single Point Positioning

This Appendix describes work done by the author as part of the "WGS 84 Pilot Survey" for Eurocontrol Experimental Centre, France [*Ashkenazi et al, 1992b*]. This project compared different techniques for determining the WGS 84 coordinates of a point, one of which is described below.

Single point positioning using TRANSIT over 3 to 5 days is capable of achieving accuracies of 1 to 2 metres with the precise ephemeris. Prior to the implementation of Selective Availability (SA) similarly accuracies could be achieved using GPS pseudo-ranges over several hours with the broadcast ephemeris. However, with SA it was expected that this accuracy would be degraded to 50 metres or worse.

Tests were performed to assess the effect of SA using data collected over 3 days by the Rogue receiver at Herstmonceux, which is operated as part of the CIGNET tracking network. The data was processed using PSEUDO in hourly segments, using the MAGNET tropospheric model, and solving for X, Y, Z coordinates of the point and a receiver clock offset per epoch. The hourly coordinates were then combined using their covariance matrices in CARNET to produce an accumulated solution. Both single frequency and dual frequency data was used.

Figures A.1 and A.2 show the scatter of the individual hourly solutions about the 'known absolute' WGS 84 coordinate of Herstmonceux (this known coordinate was taken from ITRF91 [*Boucher et al, 1992*] which for all practical purposes is identical to WGS 84). The single frequency solution shows a scatter of 50 to 60 metres in plan and 100 metres in height whereas the dual frequency solution has a scatter of 40 metres in plan and 60 metres in height.

Figures A.3 and A.4 show how the accumulated solutions slowly converge after several days and the discrepancy between the 'known' and accumulated WGS 84 coordinates for Herstmonceux. The results indicate that single frequency pseudorange data alone can lead to an accuracy of 2 m in Latitude and Longitude and 12 m in height. However, using dual frequency data, which eliminates nearly all ionospheric effects, achieves accuracies of 0.5 m in Latitude and Longitude and 1 m in height. This is quite remarkable considering that WGS 84 was realised using the adopted positions of 1591 Transit positions which are only accurate to ± 1 to 2 metres. Clearly, most of the effects due to SA are eliminated by averaging over several days.

Further tests were performed to confirm the validity of these preliminary results. Dual frequency data was used from two more CIGNET tracking stations, namely Onsala in Sweden and Wettzell in Germany. Figures A.5 and A.6 show the accumulated solutions and the final position discrepancies for all three CIGNET sites are summarised below in Table A.1.

CIGNET site	Δ Latitude	Δ Longitude	Δ Height
Herstmonceux	-0.55 m	0.64 m	-0.93 m
Wettzell	1.37 m	-0.48 m	0.16 m
Onsala	1.05 m	-0.42 m	0.67 m

Table A.1 Time Averaged Dual Frequency Pseudo-range Position Errors for Three CIGNET Sites.

With the threat of Anti Spoofing, which would result in the loss of the L2 pseudorange observable, tests were performed using single frequency data and the Klobuchar Ionospheric Model (Klobuchar terms taken from the Broadcast Ephemeris) *[Klobuchar, 1982]*. The results showed no improvement over the single frequency solutions. However, this problem of obtaining dual frequency pseudoranges has been solved by the new generation of receivers, such as the Trimble 4000 SSE, which can measure an L2 pseudorange, even when AS is on, using the cross corelation technique (see section 2.1.1.3).

Single point positioning using dual frequency pseudoranges over several days is a very quick and simple method of obtaining the absolute WGS 84 coordinates of a point anywhere in the world.

However, it must be remembered that the WGS 84 coordinates of a point will differ depending upon the technique used to obtain them. Furthermore, the amount of data required will vary as the characteristics of SA are changed (ie period, amplitude etc).

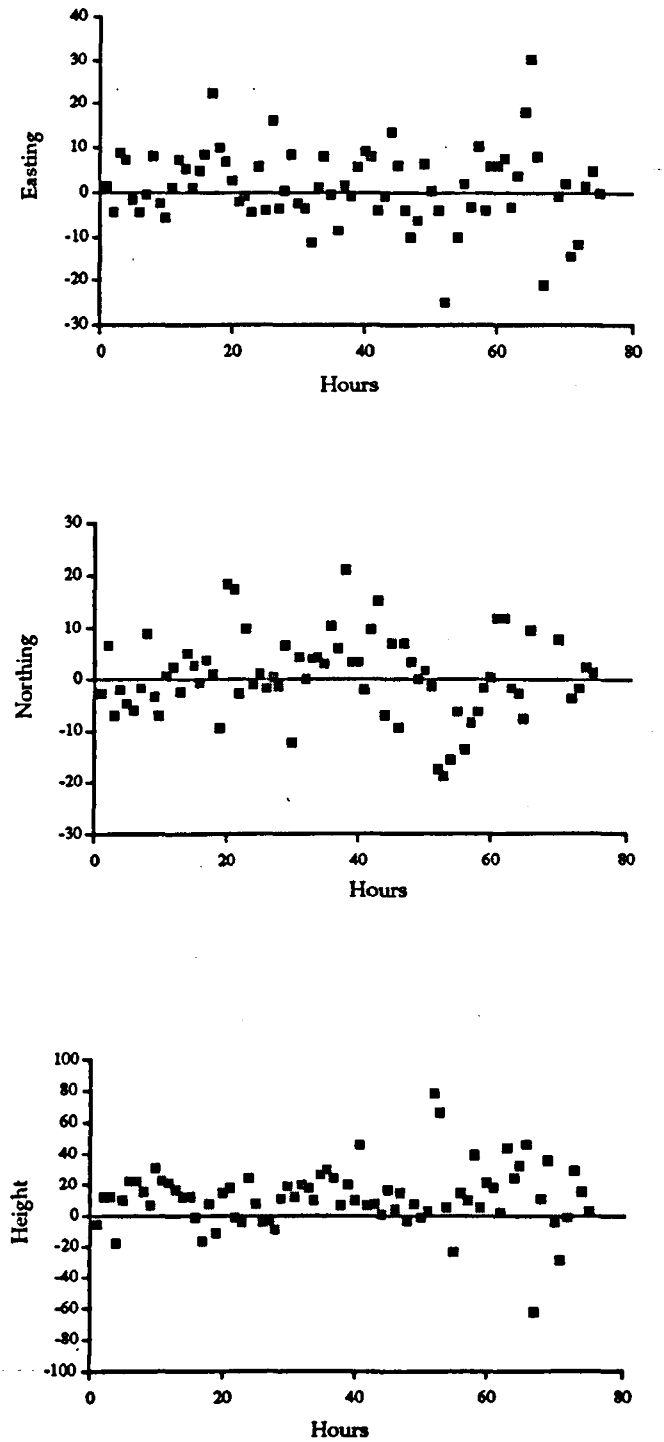


Figure A.1 Single Frequency Pseudo-Range Point Positioning Errors (m) for Herstmonceux, 6-8th February 1992.

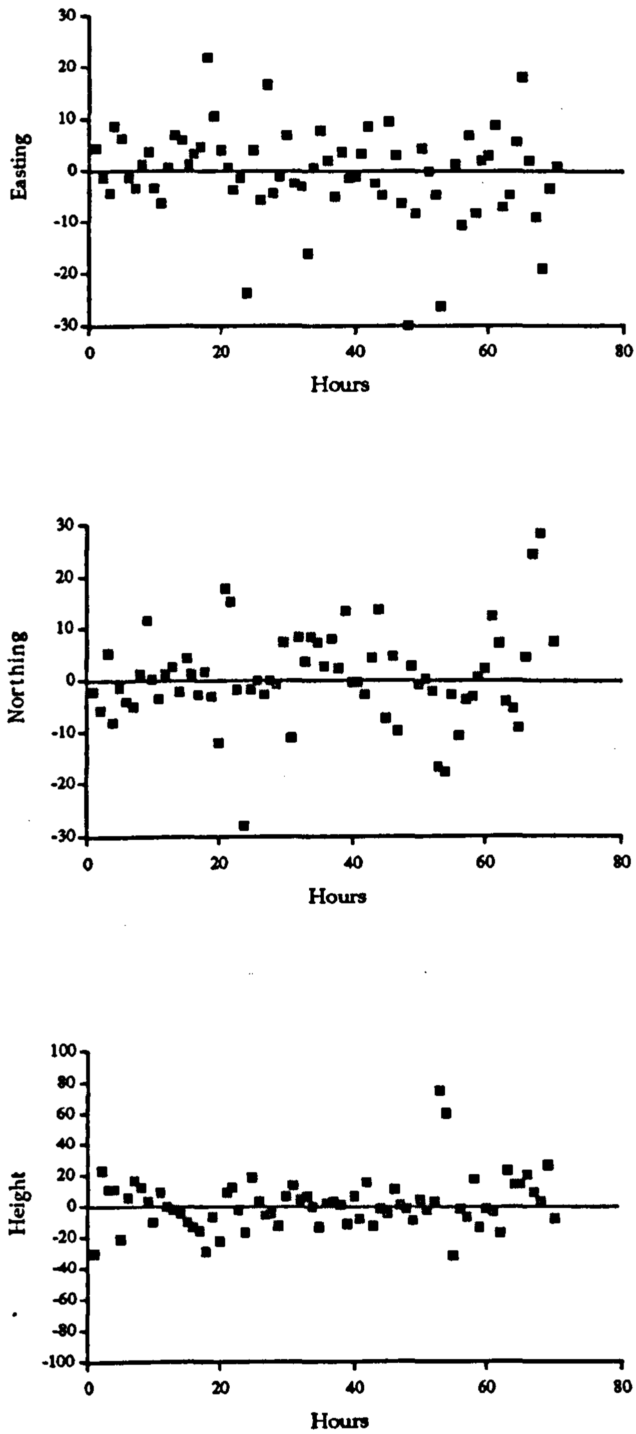
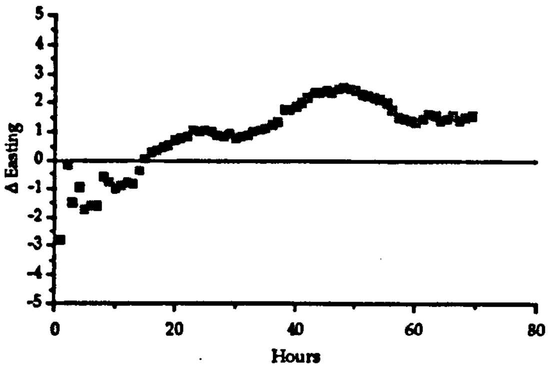
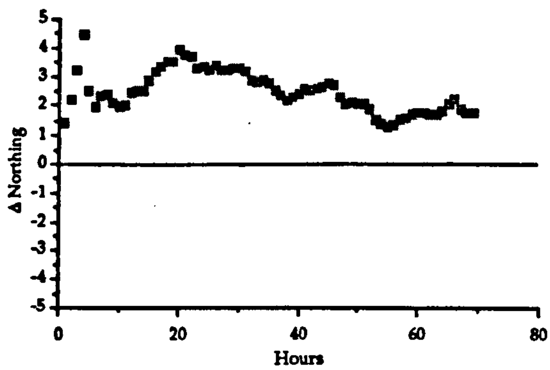


Figure A.2 Dual Frequency Pseudo-Range Point Positioning Errors (m) for Herstmonceux, 6-8th February 1992.





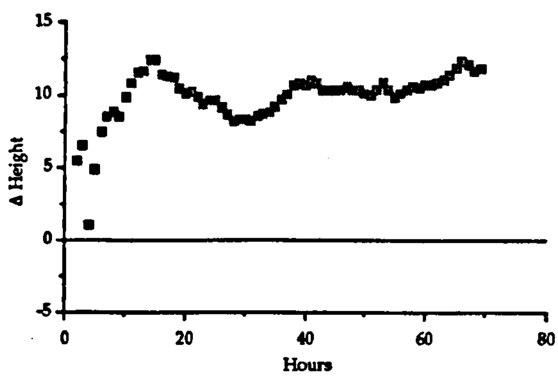
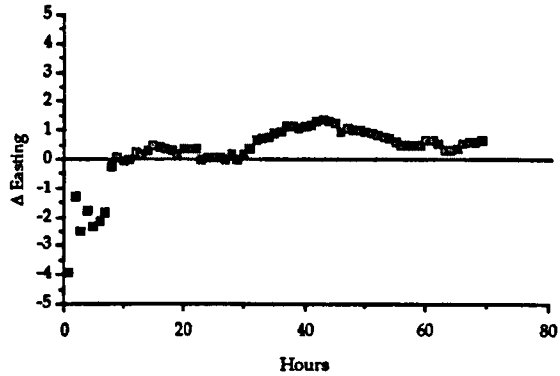
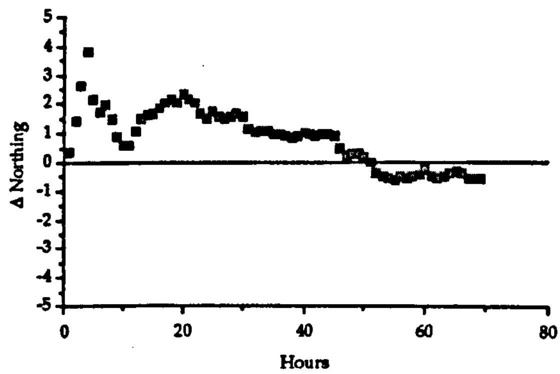


Figure A.3 Single Frequency Time Averaged Pseudo-Range Positioning Errors (m) for Herstmonceux, 6-8th February 1992.





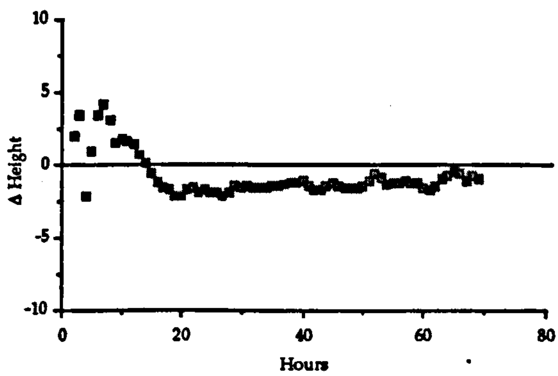


Figure A.4 Dual Frequency Time Averaged Pseudo-Range Positioning Errors for (m) Herstmonceux, 6-8th February 1992.

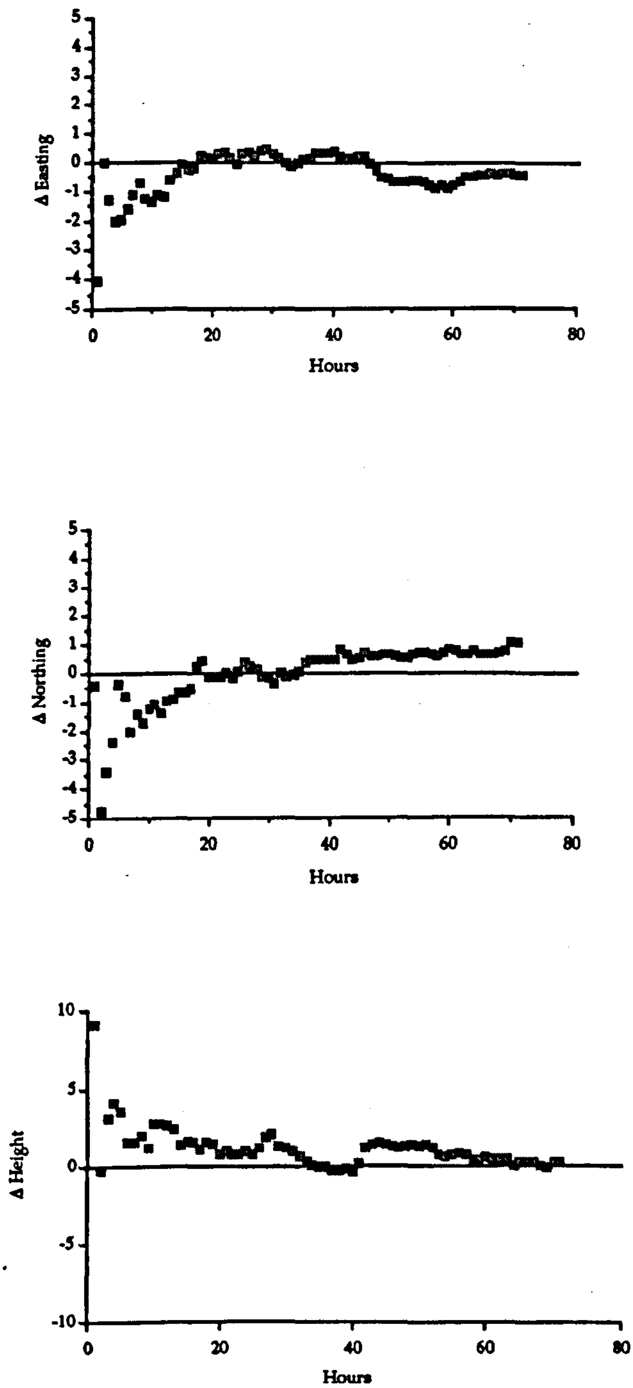


Figure A.5 Dual Frequency Time Averaged Pseudo-Range Positioning Errors (m) for Onsala, 10-12th March 1992.

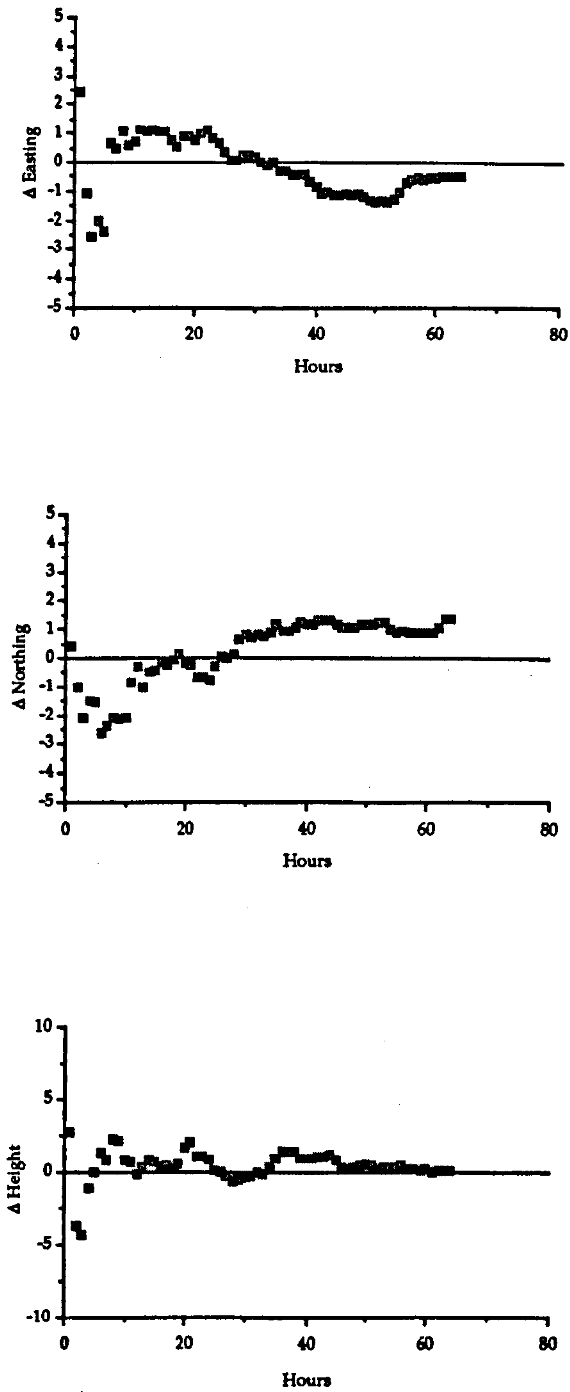


Figure A.6 Dual Frequency Time Averaged Pseudo-Range Positioning Errors (m) for Wettzell, 10-12th March 1992.

APPENDIX B

Reference Frame Definition Tests

Global Reference Frameworks	Test Frameworks																					Stations
	TOWM			OWM			TWM			TOW			TOM			THW			TOWHM			
	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	
	•	•	•	•	•	•	•	•	•	•	•	•	4	-35	24	•	•	•	•	•	•	Wettzell^R
	•	•	•	•	•	•	1	11	-1	•	•	•	•	•	•	5	-7	-11	•	•	•	Onsala^R
ITRF91N	•	•	•	•	•	•	•	•	•	-69	55	-104	•	•	•	-21	-24	-169	•	•	•	Madrid^R
(combined	•	•	•	3	11	-37	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	Tromsoe^R
VLBI & SLR)	7	11	-10	11	12	-46	9	24	-7	-19	25	-57	5	44	-4	14	-33	-85	10	-35	-78	Buddon^T
	0	-5	40	0	-6	13	1	4	41	-48	26	-13	-3	18	62	-4	-48	-62	0	-40	-15	Brest^T
	-3	32	72	-2	33	61	-2	42	73	-33	47	36	-5	46	89	•	•	•	•	•	•	Herstm^T
	•	•	•	•	•	•	•	•	•	•	•	•	7	-28	6	•	•	•	•	•	•	Wettzell^R
	•	•	•	•	•	•	-1	5	7	•	•	•	•	•	•	3	-11	-5	•	•	•	Onsala^R
ITRF91A	•	•	•	•	•	•	•	•	•	-61	79	-123	•	•	•	-29	4	-177	•	•	•	Madrid^R
(combined	•	•	•	5	28	-51	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	Tromsoe^R
VLBI & SLR)	14	9	-7	18	9	-50	13	17	1	-8	38	-66	8	40	10	14	-24	-87	10	-23	-71	Buddon^T
	9	-8	48	8	-11	18	8	-3	54	-33	41	-21	3	14	78	-4	-32	-59	2	-32	-1	Brest^T
	5	21	73	5	20	61	3	26	80	-22	51	27	-1	35	94	•	•	•	•	•	•	Herstm^T

Table B.1 Recovery of Free Fiducial Station Coordinates (mm) When Fixing ITRF91N and ITRF91A Coordinate Values
(• Fixed station, ^{T} Trimble receiver, ^{R} Rogue receiver)

Global Reference Frameworks	Test Frameworks																					Stations
	TOWM			OWM			TWM			TOW			TOM			THW			TOWHM			
	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	
GSFC (pure VLBI)	•	•	•	•	•	•	•	•	•	•	•	•	2	-37	23	-	-	-	-	-	-	Wettzell^{R}
	•	•	•	•	•	•	1	17	-11	•	•	•	•	•	•	-	-	-	-	-	-	Onsala^{R}
	•	•	•	•	•	•	•	•	•	-74	29	-39	•	•	•	-	-	-	-	-	-	Madrid^{R}
	•	•	•	4	-8	-17	•	•	•	•	•	•	•	•	•	-	-	-	-	-	-	Tromsoe^{R}
	10	6	3	14	9	-23	12	27	1	-20	3	-26	14	30	-19	-	-	-	-	-	-	Buddon^{T}
	6	-10	55	6	-6	33	7	6	54	-47	-1	26	7	6	43	-	-	-	-	-	-	Brest^{T}
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Herstm^{T}
NOAA (pure VLBI)	•	•	•	•	•	•	•	•	•	•	•	•	3	-36	23	-	-	-	-	-	-	Wettzell^{R}
	•	•	•	•	•	•	2	12	-10	•	•	•	•	•	•	-	-	-	-	-	-	Onsala^{R}
	•	•	•	•	•	•	•	•	•	-81	50	-78	•	•	•	-	-	-	-	-	-	Madrid^{R}
	•	•	•	1	8	-22	•	•	•	•	•	•	•	•	•	-	-	-	-	-	-	Tromsoe^{R}
	13	9	22	17	10	-10	16	22	17	-21	19	-13	14	36	17	-	-	-	-	-	-	Buddon^{T}
	2	3	34	3	3	8	5	13	30	-56	30	2	1	3	34	-	-	-	-	-	-	Brest^{T}
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Herstm^{T}

Table B.2 Recovery of Free Fiducial Station Coordinates (mm) When Fixing GSFC and NOAA Coordinate Values

(• Fixed station, ^{T} Trimble receiver, ^{R} Rogue receiver)

Global Reference Frameworks	TOWM			OWM			TWM			TOW			TOM			THW			TOWHM			Stations
	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	
IGS92 (pure GPS)	•	•	•	•	•	•	•	•	•	•	•	•	9	-18	20	•	•	•	•	•	•	Wettzell[R]
	•	•	•	•	•	•	-5	12	-4	•	•	•	•	•	•	-2	0	-8	•	•	•	Onsala[R]
	•	•	•	•	•	•	•	•	•	-15	13	-101	•	•	•	-7	-27	-74	•	•	•	Madrid[R]
	•	•	•	10	-17	-7	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	Tromsoe[R]
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Buddon[T]
	-	-	-	-	-	-	-	-	-	-	-	-	-	-		-	-	-	-	-	-	Brest[T]
	2	18	35	-1	24	41	-5	25	39	-1	19	-14	-2	24	36	-	-	-	-	-	-	Herstm[T]
IGS92	2	9	14	-1	12	20	-5	12	18	2	16	-36	-1	16	15	•	•	•	•	•	•	Herstm[R]

Table B.3 Recovery of Free Fiducial Station Coordinates (mm) When Fixing JPL IGS 92 Coordinate Values

(• Fixed station, [T] Trimble receiver, [R] Rogue receiver)

Station	Test Reference Frames																	
	TOWM-OWM			TOWM-TWM			TOWM-TOW			TOWM-TOM			TOWM-TWH			TOWM-TOWHM		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Newly	1	-3	-28	1	11	3	-46	29	-57	-3	29	17	-1	-48	103	0	-42	-64
Ports	0	-1	-9	1	11	2	-34	19	-41	-3	19	17	2	-35	-80	2	-32	-54
Newha	0	1	-11	1	10	1	-34	17	-38	-3	15	17	2	-32	-75	2	-29	-51
Dover	1	0	-9	1	10	1	-28	14	-34	-3	13	17	3	-29	-68	2	-26	-48
Sheer	1	1	-8	1	11	1	-29	15	-36	-3	15	15	3	-31	-70	2	-28	-50
Lowes	1	-3	-14	1	11	0	-25	11	-33	-3	14	13	4	-29	-64	3	-27	-49
Portp	3	-1	-36	2	13	3	-34	21	-55	-2	36	8	5	-50	-88	2	-50	-72
Aberd	5	2	-34	2	13	3	-24	12	-45	-2	32	5	7	-42	-71	3	-45	-66
Notti	2	-2	-23	1	12	2	-30	16	-42	-3	23	12	4	-38	-76	2	-36	-59
Danby	3	-1	-29	1	12	2	-27	13	-41	-3	25	10	6	-38	-72	3	-38	-61
Bartin	1	-3	-26	1	11	3	-46	30	-57	-3	29	17	-2	-48	-103	0	-43	-64
Moel	-4	2	-21	1	12	2	-34	19	-49	-3	29	11	4	-44	-85	2	-43	-66
AnCu	2	-1	-31	2	13	4	-28	18	-58	-1	40	3	7	-52	-85	2	-55	-74
Brest	0	-1	-27	1	9	2	-48	31	-52	-3	23	23	-3	-42	-102	0	-35	-54
Buddo	4	0	-36	2	13	3	-26	14	-47	-2	33	6	7	-44	-75	3	-46	-68
Mean	1	-1	-22	1	11	2	-33	18	-45	-3	24	13	3	-39	-81	3	-38	-61
Stan Dev	2	2	10	0	1	1	7	6	8	1	8	5	3	7	12	2	8	9

Table B.4 Differences Between Station Coordinates (mm) derived by Fixing Different Subsets of Fiducial Stations to their ITRF91N Coordinate Values (Herstmonceux Rogue receiver used).

Station	Test Reference Frames																	
	TOWM-OWM			TOWM-TWM			TOWM-TOW			TOWM-TOM			TOWM-TWH			TOWM-TOWHM		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Newly	0	-5	-32	-1	6	7	-39	48	-74	-6	29	28	-10	-29	-109	-8	-29	-58
Ports	-1	-3	-11	-1	6	7	-30	34	-53	-6	18	23	-6	-23	-82	-5	-22	-52
Newha	-1	-2	-13	-1	5	6	-28	31	-48	-5	15	21	-5	-21	-76	-4	-20	-49
Dover	0	-2	-10	-1	5	6	-25	26	-42	-5	12	19	-4	-20	-68	-4	-18	-46
Sheer	1	-1	-10	-1	6	6	-25	28	-45	-5	14	19	-4	-21	-71	-4	-19	-48
Lowes	1	-5	-16	-1	6	6	-22	23	-41	-5	14	17	-2	-21	-64	-3	-19	-47
Portp	3	-2	-43	-1	8	8	-28	37	-70	-6	35	20	-3	-35	-95	-6	-35	-67
Aberd	5	1	-41	-1	8	8	-20	26	-56	-6	31	15	0	-32	-75	-4	-31	-64
Notti	2	-4	-27	-1	6	7	-26	30	-53	-6	22	20	-3	-27	-79	-5	-25	-55
Danby	3	-2	-33	-1	7	7	-23	27	-52	-6	24	18	-1	-28	-74	-4	-26	-58
Bartin	0	-5	-30	-1	6	7	-39	49	-74	-6	29	28	-10	-29	-110	-8	-29	-58
Moel	-4	0	-26	-1	7	8	-29	35	-62	-6	28	21	-4	-30	-89	-6	-29	-61
AnCu	3	-1	-41	-1	8	8	-22	35	-72	-6	40	17	0	-37	-92	-5	-38	-70
Brest	0	-3	-29	-1	5	7	-41	50	-69	-6	22	31	-12	-24	-107	-7	-23	-49
Buddon	4	0	-43	-1	8	8	-22	29	-59	-6	31	17	0	-33	-80	-4	-32	-64
Mean	1	-2	-26	-1	6	7	-28	34	-57	-6	24	21	-4	-27	-84	-5	-26	-57
Stan Dev	2	2	12	0	1	1	6	8	11	0	8	4	4	5	14	1	6	8

Table B.5 Differences Between Station Coordinates (mm) derived by Fixing Different Subsets of Fiducial Stations to their ITRF91A Coordinate Values (Herstmonceux Rogue receiver used).

Station	Test Reference Frames											
	TOWM-OWM			TOWM-TWM			TOWM-TOW			TOWM-TOM		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Newly	0	1	-22	2	18	-1	-52	8	-32	2	20	-19
Ports	0	4	-5	1	18	-3	-38	1	-25	1	14	-8
Newha	-1	5	-9	0	17	-4	-34	1	-23	0	11	-4
Dover	0	5	-7	1	17	-5	-30	-1	-21	0	9	-2
Sheer	1	6	-5	1	18	-5	-31	0	-22	0	11	-5
Lowes	1	1	-11	1	18	-5	-27	-2	-20	0	11	-4
Portp	3	2	-26	2	22	-2	-39	1	-33	4	26	-26
Aberd	4	4	-24	2	22	-3	-28	-3	-28	4	24	-21
Notti	1	2	-18	1	20	-4	-34	-1	-25	2	18	-12
Danby	3	3	-22	1	21	-4	-30	-3	-25	2	19	-14
Bartin	1	1	-20	2	18	-1	-52	8	-32	2	20	-19
Moel	-5	6	-14	2	20	-3	-38	1	-29	2	21	-18
An Cu	1	1	-17	3	22	-1	-34	0	-36	6	29	-31
Brest	0	3	-22	1	16	-1	-53	9	-30	1	16	-12
Buddon	3	3	-26	2	22	-2	-31	-2	-29	4	25	-22
Mean	1	3	-17	1	19	-3	-37	1	-27	2	18	-14
Stan Dev	2	2	7	1	2	2	9	4	5	2	6	9

Table B.6 Differences Between Station Coordinates (mm) derived by Fixing Different Subsets of Fiducial Stations to their GSFC Coordinate Values .

Station	Test Reference Frames											
	TOWM-OWM			TOWM-TWM			TOWM-TOW			TOWM-TOM		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Newly	1	-3	-27	3	10	-4	-55	25	-38	0	24	8
Ports	0	0	-9	3	11	-6	-41	14	-28	-1	16	12
Newha	0	1	-11	2	11	-6	-38	12	-25	-1	13	13
Dover	1	1	-10	2	11	-6	-33	10	-23	-1	11	14
Sheer	2	2	-8	3	11	-6	-34	10	-25	-1	13	11
Lowes	2	-2	-14	3	12	-7	-30	8	-23	-1	13	10
Portp	4	-1	-33	4	12	-5	-42	17	-39	1	30	-4
Aberd	5	2	-30	4	13	-5	-30	8	-33	1	27	-5
Notti	3	-2	-22	3	12	-6	-37	11	-29	-1	20	5
Danby	4	0	-27	3	13	-6	-33	9	-30	0	22	2
Bartin	2	-2	-25	3	10	-4	-56	26	-38	0	25	8
Moel	-4	3	-19	4	12	-5	-41	15	-34	0	25	2
AnCu	2	1	-25	4	13	-4	-36	15	-43	3	34	-11
Brest	1	0	-27	3	9	-4	-57	26	-33	-1	20	16
Herst		2	-12	2	11	-6	-37	12	-25	-1	13	13
Buddon	4	1	-32	4	13	-5	-33	10	-35	1	28	-5
Mean	2	0	-21	3	12	-5	-40	14	-31	0	20	6
Stan Dev	2	2	8	1	1	1	9	6	6	1	7	8

Table B.7 Differences Between Station Coordinates (mm) derived by Fixing Different Subsets of Fiducial Stations to their NOAA Coordinate Values.

Station	Test Reference Frames											
	TOWM-OWM			TOWM-TWM			TOWM-TOW			TOWM-TOM		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Newly	-3	6	7	-7	8	6	-5	-1	-74	-7	15	-6
Ports	-4	6	6	-7	6	4	-4	1	-54	-5	9	0
Newha	1	6	6	-7	6	4	-4	1	-50	-4	8	2
Dover	-4	6	5	-7	5	3	-3	1	-44	-4	6	3
Sheer	-3	6	6	-7	6	4	-3	1	-46	-4	8	2
Lowes	-3	6	5	-7	6	3	-2	1	-41	-4	8	1
Portp	-3	5	6	-8	12	4	-2	-3	-63	-7	20	-12
Aberd	-2	3	5	-7	12	3	0	-2	-49	-6	18	-11
Notti	-3	6	6	-7	9	4	-3	0	-52	-6	13	-4
Danby	-3	5	5	-8	10	3	-2	-1	-48	-6	14	-5
Bartin	-3	6	7	-7	8	6	-5	-1	-74	-7	15	-6
Moel	-3	6	6	-8	10	4	-3	-1	-59	-7	16	-7
AnCu	-1	2	5	-7	14	3	0	-4	-61	-6	23	-17
Brest	-3	6	7	-7	6	6	-7	1	-73	-6	12	-2
Buddon	-2	4	5	-7	12	3	-1	-2	-52	-6	18	-11
Mean	-3	5	6	-7	8	4	-3	-1	-56	-6	13	-5
Stan Dev	1	1	1	0	3	1	2	2	10	1	5	6

Table B.8 Differences Between Station Coordinates (mm) derived by Fixing Different Subsets of Fiducial Stations to their JPL IGS 92 Coordinate Values (Herstmonceux Rogue receiver used).

Station	Trimble						Rogue					
	TOWM-TWH			TOWM-TOWHM			TOWM-TWH			TOWM-TOWHM		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Newly	-4	-29	-50	-4	-23	-31	-4	-7	-6	-4	-2	-3
Ports	-3	-20	-39	-3	-18	-27	-3	-5	-6	-3	-1	-2
Newha	-2	-18	-36	-2	-17	-25	-2	-5	-5	-2	-1	-2
Dover	-2	-16	-33	-2	-15	-24	-2	-5	-4	-2	-1	-2
Sheer	-2	-17	-34	-2	-16	-24	-2	-5	-6	-2	-1	-2
Lowes	-2	-15	-31	-1	-15	-24	-2	-6	-5	-1	-1	-2
Portp	-3	-27	-42	-4	-26	-33	-3	-10	-5	-4	-2	-3
Aberd	-1	-21	-34	-2	-24	-30	-1	-11	-4	-2	-2	-3
Notti	-2	-20	-28	-2	-20	-40	-2	-8	-5	-2	-2	-2
Danby	-2	-19	-35	-2	-21	-28	-2	-8	-4	-2	-2	-2
Bartin	-5	-29	-50	-4	-23	-31	-5	-6	-5	-4	-2	-3
Moel	-3	-24	-41	-3	-23	-31	-3	-10	-5	-3	-2	-3
AnCu	-2	-27	-41	-3	-28	-33	-2	-11	-5	-3	-2	-3
Brest	-5	-27	-50	-4	-19	-28	-5	-6	-6	-4	-1	-2
Buddon	-1	-22	-36	-3	-24	-31	-2	-10	-5	-3	-2	-3
Mean	-3	-22	-39	-3	-21	-29	-3	-8	-5	-3	-2	-2
Stan Dev	1	5	7	1	4	4	1	2	1	1	1	1

Table B.9 Differences Between Station Coordinates (mm) derived by Fixing Different Subsets of Fiducial Stations to their JPL IGS 92 Coordinate Values .

APPENDIX C

Antenna Phase Centre Tests

Global Reference Frameworks	Test Frameworks																		Stations
	TOWM			OWM			TWM			TOM			THW			TOWHM			
	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	
JPL IGS 92 (pure GPS)	•	•	•	•	•	•	•	•	•	6	-14	22	•	•	•	•	•	•	Wettzell[R]
	•	•	•	•	•	•	-5	10	-4	•	•	•	-3	6	-1	•	•	•	Onsala[R]
	•	•	•	•	•	•	•	•	•	•	•	•	11	-36	-17	•	•	•	Madrid[R]
	•	•	•	11	-21	-7	•	•	•	•	•	•	•	•	•	•	•	•	Tromsoe[R]
	-2	9	-4	-6	17	2	-8	25	-1	-4	13	-6	•	•	•	•	•	•	Herstm[T]

Table C.1 Recovery of Free Fiducial Station Coordinates (mm) when using Antenna Phase Centre Modelling

(• Fixed station, [T] Trimble receiver, [R] Rogue receiver)

Station	Test Reference Frames														
	TOWM-OWM			TOWM-TWM			TOWM-TOM			TOWM-TWH			TOWM-TOWHM		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Newly	-4	7	8	-7	6	5	-4	7	-12	4	-20	5	1	-12	2
Ports	-4	7	7	-7	5	4	-3	4	-4	3	-13	3	2	-10	2
Newha	-4	7	7	-6	4	3	-2	4	-2	2	-11	3	2	-9	2
Dover	-4	6	6	-6	3	3	-2	3	-1	2	-9	2	2	-8	2
Sheer	-4	7	7	-7	4	3	-2	4	-2	2	-9	3	2	-9	2
Lowes	-4	6	6	-7	4	2	-2	4	-2	1	-7	3	2	-9	1
Portp	-3	5	8	-7	10	4	-3	9	-18	1	-13	8	1	-14	2
Aberd	-2	3	6	-7	9	2	-2	8	-16	0	-8	7	1	-13	2
Notti	-4	7	7	-7	7	3	-3	6	-8	1	-10	5	2	-11	2
Danby	-3	6	6	-7	8	3	-3	7	-10	1	-8	6	2	-11	2
Bartin	-4	7	8	-7	6	6	-4	7	-12	4	-20	6	1	-12	2
Moel	-4	7	7	-7	8	4	-3	7	-13	2	-13	6	2	-13	2
AnCu	-2	2	6	-7	11	3	-2	10	-23	1	-12	10	0	-14	3
Brest	-4	7	8	-6	4	5	-3	5	-7	4	-21	3	1	-11	2
Buddon	-2	4	6	-7	9	3	-2	9	-16	1	-9	8	1	-13	2
Mean	-4	6	7	-7	7	4	-3	6	-10	2	-12	5	2	-11	2
Stan Dev	1	2	1	0	2	1	1	2	7	1	4	2	1	2	0

Table C.2 Differences Between Station Coordinates (mm) derived by Fixing Different Subsets of Fiducial Stations when using JPL IGS 92 and Antenna Phase Centre Modelling.

APPENDIX D

Tropospheric Modelling Tests

Station	Order of Polynomial											
	Con - 1st			Con - 2nd			Con - 3rd			Con - 4th		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Herst-Brest	-1	-1	4	-4	-9	14	-5	-10	13	-4	-6	12
Herst-Barti	-1	3	-1	-3	-4	9	-4	-6	10	-3	0	6
Herst-Newly	-1	3	1	-3	-3	8	-5	-6	10	-3	2	7
Herst-Ports	1	4	2	1	4	-3	1	3	-3	2	6	-3
Herst-Newha	0	1	-1	0	0	-6	0	-1	-6	1	-2	-7
Herst-Dover	0	3	4	0	3	2	-1	3	4	-1	1	5
Herst-Sheer	0	-4	-4	0	-3	-7	-1	-4	-5	-1	-3	-4
Herst-Lowes	0	5	2	0	4	1	-1	4	4	-1	3	7
Herst-Notti	0	9	6	0	8	2	0	7	3	0	8	3
Herst-Danby	-1	4	0	0	3	-5	1	2	-10	1	6	-10
Herst-Moel	0	6	2	0	3	-5	-1	2	-4	1	10	-6
Herst-Portp	1	7	4	3	6	-12	4	4	-16	5	12	-14
Herst-Buddo	0	6	0	1	4	-8	1	3	-11	2	6	-10
Herst-Aberd	0	5	1	1	3	-9	2	2	-15	3	10	-14
Herst-An Cu	0	10	4	3	6	-10	4	5	-18	6	8	-17

Table D.1 Differences in Vector Components (mm) between using a Constant (Con) and a Time Varying Polynomial (Orders 1 to 4) Tropospheric Scale Factor, with Fixed Fiducial Stations TOWHM (JPL IGS 92) and Antenna Phase Centre Modelling

Station	Order of Polynomial											
	Con - 1st			Con - 2nd			Con - 3rd			Con - 4th		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Herst-Brest	-1	-1	5	-4	-8	16	-5	-9	16	-4	-6	14
Herst-Barti	-1	3	0	-3	-4	10	-5	-9	10	-4	-3	8
Herst-Newly	-1	3	1	-3	-3	9	-5	-7	13	-5	-1	9
Herst-Ports	1	4	3	1	4	1	1	3	1	2	6	2
Herst-Newha	0	1	0	0	0	-2	0	0	-2	0	2	-6
Herst-Dover	0	3	5	0	3	7	0	4	8	-1	2	10
Herst-Sheer	0	-4	-3	-1	-2	-3	-1	-3	-1	-1	-2	-4
Herst-Lowes	0	5	3	0	5	3	-1	5	8	-1	5	7
Herst-Notti	0	9	7	0	7	5	0	6	5	0	8	4
Herst-Danby	-1	4	1	-1	2	-6	0	0	-8	1	4	-9
Herst-Moel	0	6	2	0	2	-3	-1	-1	-1	0	6	-5
Herst-Portp	1	6	4	3	4	-12	3	-1	-14	3	5	-13
Herst-Buddo	0	5	0	1	2	-7	0	-1	-10	1	5	-10
Herst-Aberd	0	4	1	1	1	-8	1	-1	-13	2	5	-14
Herst-An Cu	0	9	4	2	4	-11	2	-1	-17	4	6	-17

Table D.2 Differences in Vector Components (mm) between using a Constant (Con) and a Time Varying Polynomial (Orders 1 to 4) Tropospheric Scale Factor with Fixed Fiducial Stations TOWM (JPL IGS 92) and Antenna Phase Centre Modelling

Global Reference Frameworks	Test Frameworks															Stations
	TOWM			OWM			TWM			TOM			THW			
	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	
JPL IGS 92 (pure GPS)	•	•	•	•	•	•	•	•	•	2	-12	22	•	•	•	Wettzell[R]
	•	•	•	•	•	•	-3	7	-6	•	•	•	-2	2	-3	Onsala[R]
	•	•	•	•	•	•	•	•	•	•	•	•	10	-27	-16	Madrid[R]
	•	•	•	8	-15	3	•	•	•	•	•	•	•	•	•	Tromsoe[R]
	-2	12	0	-5	18	-2	-6	20	-2	3	3	-7	•	•	•	Herstm[T]

Table D.3 Recovery of Free Fiducial Station Coordinates (mm) using Antenna Phase Centre Modelling and Solving for a 1st Order Polynomial Tropospheric Scale Factor (• Fixed station, [T] Trimble receiver, [R] Rogue receiver)

Global Reference Frameworks	Test Frameworks															Stations
	TOWM			OWM			TWM			TOM			THW			
	N	E	H	N	E	H	N	E	H	N	E	H	N	E	H	
JPL IGS 92 (pure GPS)	•	•	•	•	•	•	•	•	•	2	-11	18	•	•	•	Wettzell[R]
	•	•	•	•	•	•	-2	6	1	•	•	•	-1	-3	3	Onsala[R]
	•	•	•	•	•	•	•	•	•	•	•	•	4	-27	-45	Madrid[R]
	•	•	•	6	-12	-12	•	•	•	•	•	•	•	•	•	Tromsoe[R]
	-2	16	10	-4	20	13	-4	24	13	-2	20	-1	•	•	•	Herstm[T]

Table D.4 Recovery of Free Fiducial Station Coordinates (mm) using Antenna Phase Centre Modelling and Solving for a 2nd Order Polynomial Tropospheric Scale Factor (• Fixed station, [T] Trimble receiver, [R] Rogue receiver)

Station	Test Reference Frames														
	TOWM-OWM			TOWM-TWM			TOWM-TOM			TOWM-TWH			TOWM-TOWHM		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Newly	-2	6	6	-4	11	-1	1	4	-18	4	-19	-1	2	-14	-1
Ports	-2	6	5	-4	10	-1	1	3	-8	3	-13	-1	2	-11	-1
Newha	-3	6	4	-4	9	-1	1	3	-6	3	-12	-2	2	-10	-1
Dover	-3	6	4	-4	9	-1	1	2	-4	2	-10	-1	2	-10	-1
Sheer	-3	6	4	-4	9	-1	1	3	-6	2	-11	-1	2	-10	-1
Lowes	-2	6	4	-4	9	-2	1	3	-6	2	-9	-1	2	-10	-1
Portp	-2	6	5	-4	10	-2	2	5	-20	2	-16	2	2	-12	-2
Aberd	-1	5	5	-4	12	-3	3	5	-20	1	-11	2	2	-14	-1
Notti	-2	6	5	-4	11	-2	1	4	-13	2	-12	0	2	-12	-2
Danby	-2	6	5	-4	11	-2	2	5	-14	1	-11	1	2	-13	-2
Bartin	-2	6	5	-4	10	-1	1	4	-18	3	-19	-1	2	-13	-1
Moel	-2	6	5	-4	12	-2	1	5	-18	2	-14	0	2	-13	-2
AnCu	-1	5	6	-4	12	-3	4	5	-27	1	-15	2	2	-14	-1
Brest	-2	6	5	-3	9	-1	0	3	-13	3	-19	-3	2	-11	-2
Buddon	-1	5	6	-4	12	-2	3	5	-20	1	-12	2	2	-14	-1
Mean	-2	6	5	4	10	-2	2	4	-14	2	-14	0	2	-12	-1
Stan Dev	1	0	1	0	1	1	1	1	6	1	3	2	0	1	0

Table D.5 Differences Between Station Coordinates (mm) derived by Fixing Different Subsets of Fiducial Stations using JPL IGS 92, Antenna Phase Centre Modelling and Solving for a 1st Order Polynomial Tropospheric Scale Factor.

Station	Test Reference Frames														
	TOWM-OWM			TOWM-TWM			TOWM-TOM			TOWM-TWH			TOWM-TOWHM		
	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH	ΔN	ΔE	ΔH
Newly	-2	4	3	-3	9	4	0	6	-10	2	-26	-17	1	-19	-10
Ports	-2	4	3	-3	8	3	0	4	-5	2	-18	-13	2	-16	-9
Newha	-2	4	3	-3	8	3	0	3	-3	2	-17	-12	2	-15	-8
Dover	-2	4	3	-3	8	3	0	3	-2	2	-15	-10	2	-14	-8
Sheer	-2	4	3	-3	8	3	0	3	-3	2	-15	-10	2	-14	-8
Lowes	-2	3	3	-3	8	3	0	3	-3	2	-14	-8	2	-14	-8
Portp	-2	2	3	-3	11	4	1	7	-13	2	-23	-10	1	-22	-12
Aberd	-1	1	1	-3	11	3	1	7	-11	1	-18	-6	1	-21	-11
Notti	-2	4	3	-3	10	3	0	5	-7	2	-18	-10	2	-17	-10
Danby	-2	3	2	-3	10	3	1	6	-8	1	-17	-8	2	-18	-10
Bartin	-2	4	3	-3	9	4	0	6	-10	2	-26	-17	1	-19	-10
Moel	-2	3	3	-3	10	4	1	6	-10	2	-21	-11	1	-20	-11
AnCu	-1	0	1	-3	12	4	2	8	-16	1	-23	-8	1	-24	-12
Brest	-2	4	3	-3	8	4	0	5	-7	3	-25	-20	1	-16	-10
Buddon	-1	2	2	-3	11	4	1	7	-12	1	-19	-7	1	-21	-11
Mean	-2	3	3	-3	9	3	0	5	8	2	-20	-11	2	-18	-10
Stan Dev	0	1	1	0	1	1	1	2	4	1	4	4	1	3	1

Table D.6 Differences Between Station Coordinates (mm) derived by Fixing Different Subsets of Fiducial Stations using JPL IGS 92, Antenna Phase Centre Modelling and Solving for a 2nd Order Polynomial Tropospheric Scale Factor.

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