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Analysing Multi-party Competition
in Plurality Rule Elections

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Abstract: The study of general election outcomes can be helped by finding better approaches for visualizing large quantities of information and asking questions about its patterning. We review the Nagayama or 'all possibilities' triangle display, and show that it can only be legitimately used to show an overall 'field' of results that is logically feasible, called the effective space of competition, which varies with the number of observable parties. We apply this reductionist view to analyse outcomes in three leading plurality rule systems (the USA, India and Great Britain), focusing on evidence of the Duvergerian psychological effect acting on voters during campaign periods. The ECS view illuminates some key differences across countries, and variations with rising numbers of parties competing. We next consider a more holistic approach, the 'crown' diagram, which links electoral district outcomes more closely to the most important politico-ideological dimension in each country. Both views suggest some tentative evolutionary hypotheses for the variegated development of plurality rule systems over time. Britain is a highly nationalized party system, but one that has moved substantially away from Duvergerian predictions of two-party focusing, and towards multi-party politics. The USA seems to be a case of 'stunted development'. And India shows a partial Duvergerian conformity, yet combined with a substantial vertical scatter of non-Duvergerian results. Applications to over-time and regional analysis within countries are also sketched.

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In the comparative analysis of elections and party systems we have yet to develop logically acceptable ways to chart the district-level outcomes of multi-party elections, and to assess the clustering or patterning of outcomes in systematic ways. In this paper we show that the Nagayama or 'all possibilities' triangle has major defects, but can be reformulated and re-applied in two new ways. The first captures the 'layer cake' character of general election outcomes in a reductionist fashion, showing how the number of observable parties competing for votes at district level influences outcomes. An alternative variant (the crown diagram) gives a more holistic picture of outcomes, shifting attention to the performance of the top two parties or blocs linked to the predominant political-ideological dimension in a political system. We link this second innovation to a tentative logic of evolutionary development applicable to plurality rule election systems.

1. Graphic representations of multi-party competition

In his discussion of 'paradigms' in science, Thomas Kuhn (1996) emphasized that the term is meant to cover not just very basic or fundamental ideas that may lie at the heart of salient encompassing theory changes, but also to whole congeries (or 'swarms') of complementing ideas, methods, and practices, many of which concern instrumentation, measurement, schemas, and the analysis, representation and visualization of data (Buchanan, 2000, p. 233-4). There are still widespread problems of defective instrumentation in political science, especially in electoral studies, where there has tended to be an over-reliance on a few key statistics with known inadequacies, such as the ENP score (Dunleavy and Boucek, 2003; Golosov, 2010). We also repeatedly admonish each other to make more use of visualizations so as to deepen our intuitive grip on complex data patterns (Tufte, 2001; 1997; 1990). And we know that the capacity for useful abstraction and visualization are closely linked (Arnheim, 1969). Yet in fact no clear set of charting tools has emerged for general election outcomes or secured regular usage in analysing multi-party elections.

In his leading text *The Geometry of Voting* (1994), a range of influential papers, and some more accessible treatments like *Chaotic Elections* (2001) Donald Saari popularized the use of an equilateral triangle simplex to display the results of a three party election. Figure 1 shows the basic set up here, with each of the vertices

assigned to represent 100 per cent for one player and zero for the other two, and with the votes decreasing for a particular player the further one moves away from their vertex. For example, along the whole AC axis here, B gets zero votes, but B's vote share increases the nearer the election outcome gets to the B vertex. The mid point of the triangle represents an even three-way division of the votes.

Figure 1: The simplex representation of a three-party election
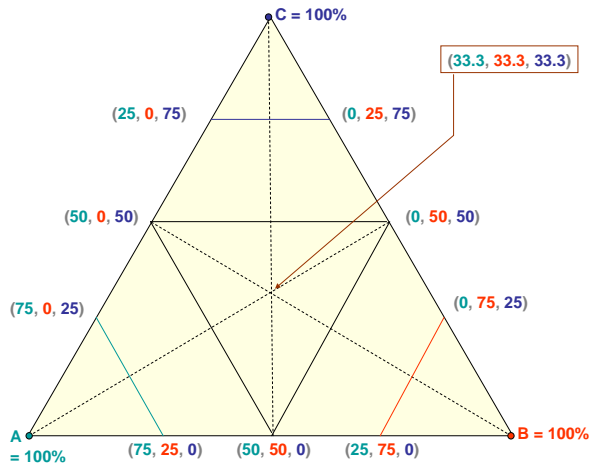
Notes: All co-ordinates show vote shares as (A, B, C)

Influential authors have argued for the use of a simplex representation to display general election results (Grofman et al, 2004), and yet this approach has in fact been relatively little employed for two reasons. In terms of practicalities, it seems to be not all that intuitive to locate particular voting outcomes on the simplex field. In addition, the political space tends to appear compressed as you get closer to an even three-way competition. This limitation is evident if we use the triangle to show aggregate election results. Normally the most populated central parts of the triangle just disappear behind a mass of overlapping constituency outcomes, rendering pattern-finding difficult.

Second, although Saari enthusiastically advocated an expanded (fold-out) simplex version for analysing four-party contests, this idea has not been picked up at all outside his own work, because the four-dimension representation is additionally complex to understand and use. Of course, at five parties and above this form of diagrammatic representation becomes completely infeasible. So Saari's suggested route also seems to be one that is inherently incapable of handling multi-party

systems.[1] Yet multi-party systems are everywhere the coming trend in liberal democracies, outside the USA.

An alternative 'all possibilities' triangle display (or APT) shows electoral district outcomes in general election results. It was pioneered by Nagayama[2] and has been energetically promoted by Grofman et al (2004) and Taagepera (2004, 2007). As Figure 2 shows the display seems to be simplicity itself, the horizontal axis showing V_1, the vote share of the largest party (from 0 to 100 per cent), and the vertical axis V_2, the vote share of the second largest party (from 0 to 50 per cent). The triangle is defined by the horizontal axis and the two boundary lines here, $V_1=V_2$ and $V_1 + V_2 = 100$. They intersect where V_2 equals 50, the logical maximum for the second largest party under any pattern of competition. A considerable problem of the APT is that V_1 and V_2 can both win votes from, or lose votes to, other parties whose support is not shown explicitly. None the less Taagepera (2004, 2007) has argued that these displays can also be used to chart the distribution of vote shares for the third-, fourth- or fifth-largest parties (peaking at 33.33, 25 or 20 per cent respectively), where

Figure 2: The 'all possibilities' triangle display (after Grofman et al)
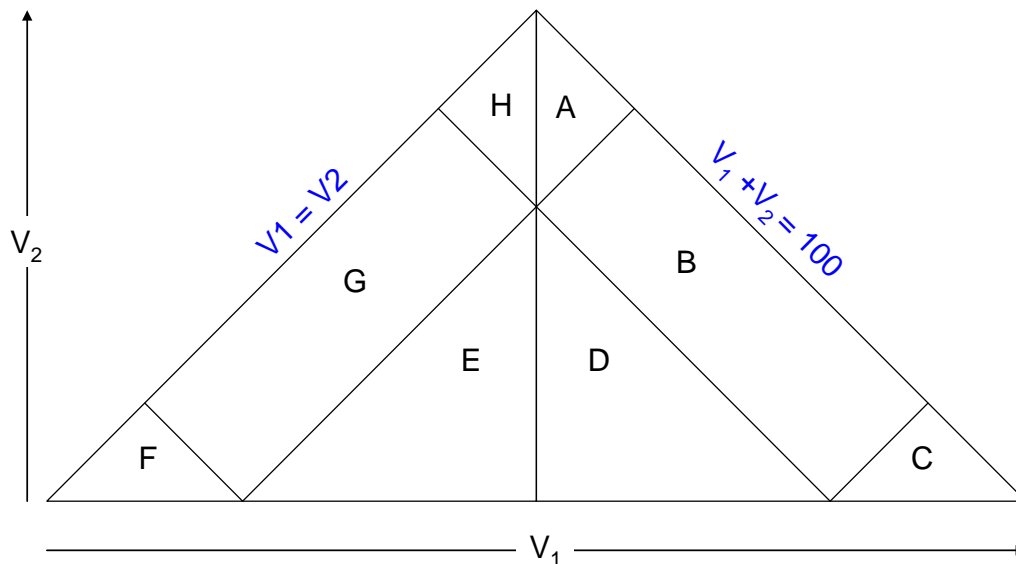
these are particular foci of interest.[3]

The 'all possibilities' triangle has mostly been used visually to offer intuitive explanations. Thus a cluster of seats outcomes close to the left-hand boundary in Figure 2 indicates multi-party competition, while a bunching of outcomes close to the

right-hand boundary of the APT shows a polarized party system, with just two leading contenders (see Reed, 1990, 2001, 2003; and Diwakar, 2006). Grofman et al (2004) advocate sub-dividing the triangle display and counting the distribution of electoral district outcomes across the partitions thus created is a useful additional means of generating quantitative data for comparisons across elections. Figure 2 shows their recommended sub-divisions, created by lines set in from the two sloping triangle boundary lines by 20 per cent and by a vertical line through $V_1 = 50$. Grofman et al frankly admit that these internal partitions are completely *ad hoc*. Yet they suggest that the percentage of electoral district results falling into the different compartments should be used as a new variable in further quantitative analysis. In their view, the percentage of results in compartments ABC in Figure 2 above can be taken as indicating bipolarized results, while the proportion in FGH indicates multi-party results. This suggestion has been taken up by a few authors (for instance, Likhtenchtein and Yarmgomskaya (2005) and Diwakar (2007).

The problem here lies in interpreting the APT boundaries in absolute terms, as a fixed framework applicable in an unchanging way across very different competitive contexts. In Grofman et al, and other authors following their lead (such as Taagepera, 2004, 2007) there seems to be a background assumption that although a uniform distribution of districts across the whole triangle space is highly unlikely, it is none the less logically feasible. When analysts present a scatterplot of general election outcomes data situated only within the APT frame, the empirical patterns of seats is implicitly being compared with a potentially uniform distribution.

Yet in fact, we use the 'all possibilities triangle' label partly as a warning sign, because *for any single contest* large areas of the triangle necessarily cannot be populated with results.[4] For *a whole set of contests*, for instance across a display of all districts at a general election, a somewhat larger area of the APT will be logically feasible (because competition outcomes are more diverse). But even here, the feasible area can only ever encompass a fraction of the whole APT area, depending on the number of parties competing in the election. For these reasons fixed compartments such as those in Figure 2 are in fact particularly inappropriate. They will necessarily confuse some technical or mathematical effects within multi-party competition with substantive empirical patterns. Put another way, within the all possibilities triangle display there is no way of discriminating between feasible but empirically

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unpopulated positions or slots, and those V_1V_2 slots that are just not logically feasible, given the configuration of parties competing.

2. Effective competition space

The problems above can be completely avoided by introducing the concept of effective competition space (ECS), defined here as 'the set of all possible V_1V_2 outcomes that are logically possible in a given election situation'. The key insight needed is that the patterns associated with elections and party competition are not measured against an unchanging canvass. Instead, we have to see the shape of the outcomes as produced only within a specific field of possibilities, a field that itself changes with the number of parties competing for votes.

To make any progress at all in analysing this field, we also need to make a simplifying assumption about choosing a measuring instrument or grid. The problem here is analogous to Mandelbrot's famous (1967) paper on measuring the English coastline, where the length we come up with responds to the granularity of our measuring stick. We necessarily must have a unit of measurement, and this choice conditions our answer in some degree, hence we can only chart 'effective' competition space given that choice. To keep things simple, we assume here a 'positive integer universe', that is, one where all 'observable parties' have vote shares that are natural numbers of 1 or more, and where there are no decimal vote shares. This useful simplifying approach has already been pioneered in the context of legislatures by Laver and Benoit (2003) and Benoit and Laver (2005). Hence we require by definition that any 'observable' party competing in elections must get at least 1 per cent of the votes (if it gets less it is definitionally not 'observable').

Thus the number of observable parties (hereafter N_{op}) is the number of parties with 1 per cent or more of the votes. This move is essential if we are to be able to economically trace determinant interactions between party numbers and vote shares on the one hand and changes in competition space on the other hand. In empirical analysis the approach can be simply adapted to cope with sub-1 per cent parties or candidates by adding a composite 'other' term to hold these vote shares. However, in what follows it remains important for readers not to lose sight of the fact that the 'effective' E in ECS signals an important limitation.

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We begin by re-picturing the APT display above, but adding in two other measures that it also shows, but which have not been noted by previous authors (Figure 3a). First, within the integer universe assumption, we can only draw the triangle at all on the assumption that there are three or more parties in competition. Hence on the top right hand boundary of the APT the amount of votes going to the third and subsequent parties is 1 per cent (assuming three-party competition), and this increases as we move down and to the left, reaching a maximum of 98 per cent at the far left-hand point of the triangle (assuming 100 parties in competition). Clearly then the APT display incorporates all possibilities, *including* of course all shifts in the number of parties in competition.
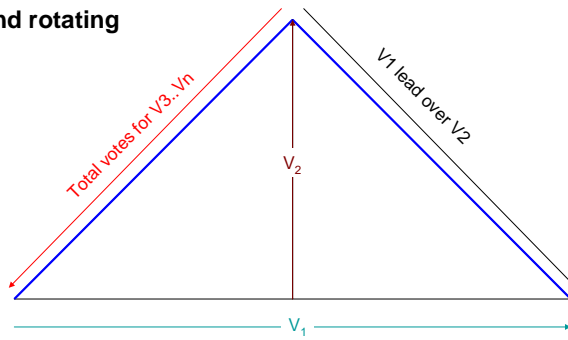
The second dimension shown in Figure 3a runs downwards from the top left-hand boundary, along which the largest and second largest parties get the same vote shares (and hence V_1 minus V_2 is zero). As we move down to the right, so the largest party's lead over its chief competitor increases, and the V_1-V_2 gap reaches a maximum (of 98 per cent) at the bottom right-hand point of the APT. Note that the measuring scales on the two hypotenuses here are compatible with each other. But they are different from the scales for V_1 and V_2 (which are also compatible with each other).

For a number of technical and substantive reasons (that will become clearer below), it is useful to re-present the APT making the 'V_1 lead over V_2' dimension into the horizontal axis, and the 'Total votes for V_3..V_N' into the vertical axis. As Figures 3b and 3c show, this entails first flipping the conventional APT downwards along its horizontal axis, and then rotating the flipped triangle clockwise by 45 degrees. This produces a right angle triangle, where the hypotenuse measures V_1 downwards (from 1 at the top to 98 at the bottom), and where V_2 is measured as the normal from the hypotenuse (where V_2 is a minimum 1 per cent) to a maximum (of 49 per cent) at the bottom right corner.

Figure 3: Flipping and rotating the APT

(a) The original APT

Total votes for V3..Vn

V1 lead over V2

V_2

V_1

(b) the APT mirrored in the horizontal axis

V_1

Total for V3..Vn

V_2

V1 lead over V2
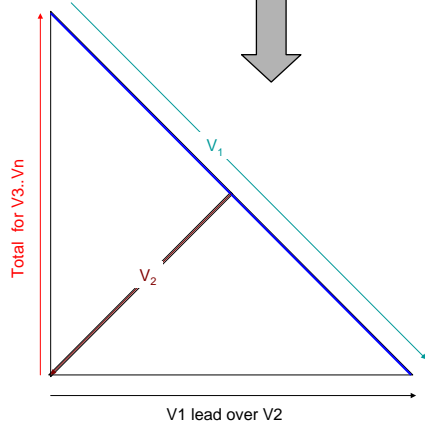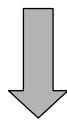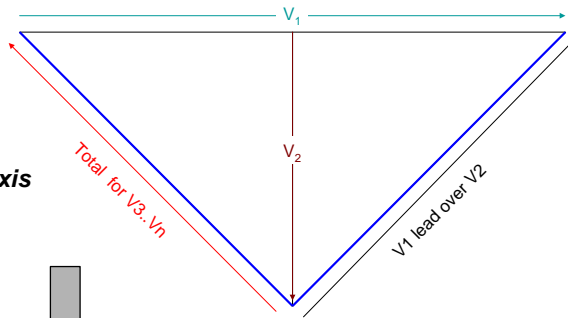
(c) the flipped APT rotated Clockwise by 45 degrees

Total for V3..Vn

V_1

V_2

V1 lead over V2

Within this re-presented APT space we can now define the empirical boundaries of the effective competition space shown in Figure 4 for any given number of parties in competition and contrast them with the APT itself as follows:

	Effective competition space (ECS)	All possibilities triangle (APT)
Left boundary (I to II)	$V_2 = V_1$ (part)	$V_2 = V_1$ (whole)
Top right boundary (I to III)	$V_2 = (100-V_1)/(N-1)$	$V_2 = 0$
Bottom boundary(II to III)	$V_2 = 100 - V_1 - (N-2)$	$V_2 = 100 - V_1$

Figure 4: The size and shape of the ECS area at 3, 10 and 50 observable parties
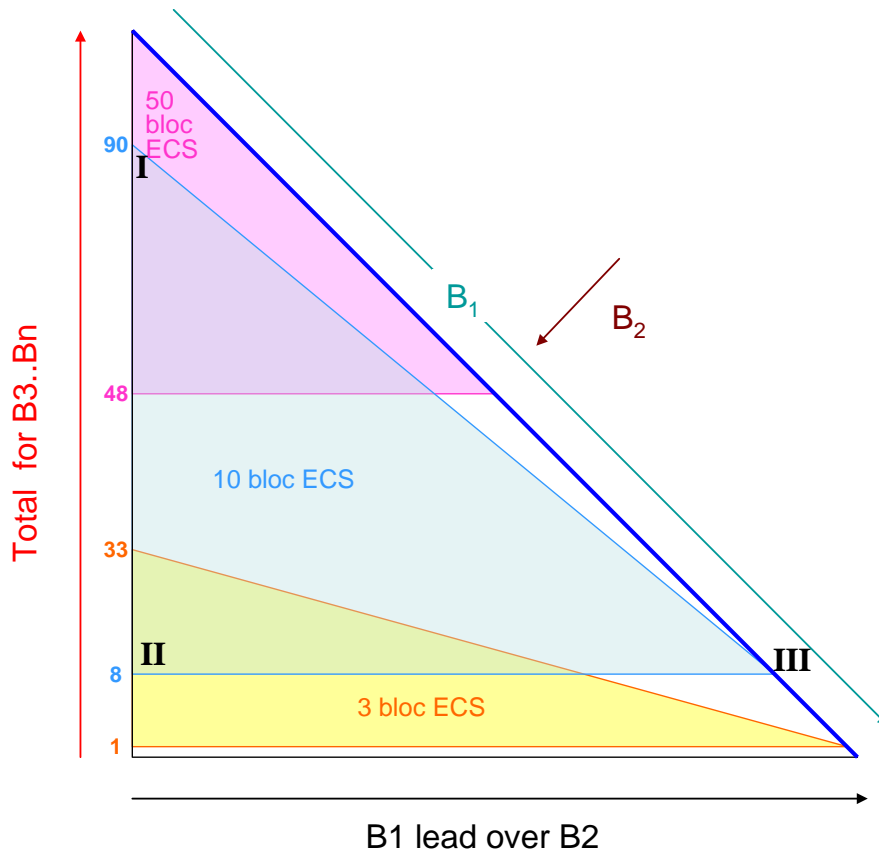
Note: The ECS as proportion of the $V_1 V_2$ slots in the APT area is at its maximum at 10 and 11 observable parties.

While the APT display remains completely invariant, the effective competition space for a single contest is always necessarily a sub-set of the APT, and its shape can vary greatly. With only two parties in competition, the ECS is just the bottom boundary of the APT. But it becomes a triangular space as soon as 3 parties compete, shown by the yellow shaded area in Figure 4. The size of competition space (measured in terms of integer V_1-V_2 slots) then increases to reach a maximum extent with 10 or 11 parties competing (shown shaded blue in Figure 4). From there the ECS gradually decreases

in size as the incrementally-rising bottom boundary increasingly squeezes the feasible space from below. The ECS takes up only around a third of the APT at 50 parties (shown shaded pink in Figure 4. And by 100 parties (the maximum in our integer universe) the competition space shrinks to a single dot at the top of the APT.

A clear implication of Figure 4 is that we need to establish how many parties are competing in different districts, before the pattern of election results can be sensibly mapped onto the background competition space. For example, in a three-party race the maximum size of the third party vote is 33 per cent, and there is only a single chance (out of 833 possible V_1-V_2 slots with three-party competition) that a district outcome can end up at this point. With four parties, the maximum on the vertical axis is 50 per cent (if all four parties get 25 per cent), then 60 per cent with five parties, 80 per cent with ten parties etc. In addition, the bottom of the ECS moves upwards by one slot with each extra party competing. With three observable parties the lower ECS boundary is at 1 per cent, rising to 8 per cent at 10 parties, and 48 per cent at 50 parties.

A key context here is Maurice Duverger's (1951) influential discussion of the 'mechanical' and 'psychological' effects operating on voters, party elites and potential counter-elites so as to sustain two-party competition in plurality rule systems. Table 1 shows an expanded version of this argument, which leads to twin predictions

A: that when an election campaign starts, voters will have few (perhaps just two) choices of candidates to choose from; and

B: that when voters cast their actual ballots they will assign little or no support to smaller parties. Hence, the number of observable parties (passing the 1 per cent level) is small and the aggregate level of support for parties ranked third or subsequently in each district is diminutive. (These effects automatically imply that ENP_{votes} is also small, close to or below 2).[5]

Notice that prediction B rests solely on the campaign psychological effect acting on voters alone, and not on the behaviours of leaders, funders or activists. By this stage, the decisions of elites or counter-elites to stay loyal, break away or stand candidates against the major parties are by now all made, and so they cannot further affect campaign outcomes. Prediction B is the focus of most of the analysis below.

Table 1: The mechanical and psychological impacts of plurality rule elections on party competition, according to Duverger

Impacts from plurality rule elections	Major party elites	Potential counter-elites	Voters
Mechanical effect - after election 1	Major parties are disproportionately rewarded with seats, given their vote shares	Smaller parties are radically under-represented in the legislature, given their vote shares - winning no or few seats	Votes cast for smaller parties are recognized by voters as ineffective, failing to convert into representation
Psychological effect in the election run-up period - prior to the start of election 2	Dissenting sub-leaders remain inside the ranks of major parties, fearing that breakaway parties would be electoral suicide	Counter-elites fear that efforts to start-up new parties or back existing smaller parties will fail – so candidates and finance are hard for such parties to attract. Hence smaller parties stand few (effective or competitive) candidates.	Voters fear that smaller parties are going to be ineffective. Voters fail to express support for these parties in opinion polls, by-elections, or 'secondary' elections
Prediction A	The number of parties standing candidates in each local electoral district is small (perhaps just 2)		
Psychological effect during the campaign period for election 2			Voters fear that smaller parties present on the ballot are going to be ineffective, and so fail to vote for them
Prediction B	The Number of observable parties (N_{op}) is small, with little support for third and subsequent parties. (Hence ENP_{votes} is automatically low, close to or below 2).		
Mechanical effect - after election 2	As before	As before	As before

Unfortunately some methods of analysis risk confounding together the separate stages in Table 1, and take no account of changes in competition space. Calculating effective number of party scores without paying attention to the feasible competition space is potentially misleading. For instance, Chhibber and Kolman (2004, p. 48-9) count any ENP score of 2.5 or less as evidence supportive of Duverger's (1951) 'Law'. But it should be clear that if only two (observable) parties contest seats (as in many US Congressional districts), no seat can have an ENP score exceeding 2. Even with three observable parties, the odds of a district outcome

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achieving a 2.5 ENP score are necessarily very slender indeed, and the chances of achieving less than this level are very high. So the problems here are fundamental ones. Essentially, we cannot disentangle the campaign psychological effect acting on voters alone without taking account of ECS variations.

In this analysis we restrict attention to three of the world's leading plurality rule electoral systems, widely thought to share a Duvergerian tendency for two-party predominance – the USA (now almost the only perfect two-party system still in being), and India and Britain. Plurality rule electoral systems also have other have important continuities in terms of how the election system shapes competition (Shugart, 2005; Cox, 1997). All three countries use plurality rule in single-member local districts, with 435 districts in the US House of Representatives; 534 seats in Indian lower house, the Lok Sabha; and 628 seats in the House of Commons for Great Britain (but excluding Northern Ireland, which has its own distinct party system).

We focus on outcomes in 2005 or 2006 general elections. Table 2 shows the distribution of the number of 'observable' parties contesting districts. In the USA the number of parties or candidates with 1 per cent of the local vote ranged across districts from 2 to 5 candidates (although two-party predominance remained pervasive in terms of vote shares and seat wins), in India from 2 to 11 parties, and in Britain from 3 to 8 parties (with no two-party contests at all). Because even a 1 per cent vote share may need to be built up over time, these results already show quite different outcomes in terms of prediction A and a part of prediction B in Table 1 above. The patterns suggest strong disincentives for new or small parties in the USA, only a weak effect in India, and virtually no effect in Great Britain.

Table 2: The percent of all election districts with different number of 'observable' parties in UK, Indian and US elections in the mid 2000s.

Election districts with a given N_{op} (ie parties receiving 1% or more of local votes)	United States, House of Representatives 2006	Indian general election 2004	Great Britain, general election 2005
One	7.8	0	0
Two	52.6	3.87	0
Three	29.0	13.81	3.50
Four	9.2	23.39	32.32
Five	0.7	26.70	41.08
Six	0.2	18.42	17.52
Seven	0.2	8.66	4.94
Eight	0.2	3.13	0.64
Nine or more	0	2.03	0
Total	*100%*	*100%*	*100%*
No of cases	*435*	*546*	*628*

The patterning of party competition in the USA is the most straightforward to represent, as Figure 5 shows. In numerous districts there were only Democrat and Republican candidates and here the competition space is restricted to the bottom boundary of the APT. Some districts with NOP scores of two actually do have one or two small parties or individual candidates getting less than 1 per cent of the vote, slightly lifting the third and subsequent party vote share off the boundary. Most of the remaining outcomes have three observable parties, and all but three results (including the few for four or more party districts) are within the competition space feasible with three observable parties. In Figure 5 there are just 11 districts where the smaller candidates or parties in aggregate achieved more than 10 per cent of the vote. Some US districts are so rock-solid that no contested election occurred at all. In 28 districts Democratic ascendancy was so guaranteed that no Republican candidate stood against them in 2006, and there are 5 reciprocal cases where Democrats did not stand in Republican seats. Finally, some apparently larger $V_3..V_N$ vote shares here actually reflect multiple Democratic or Republican party candidates competing against each other in exceptional election conditions. The highest $V_3..V_N$ total occurred when an open election with 4 Democratic Party, 2 Republican and several other candidates was

Figure 5: The 2006 US House of Representatives election outcomes at district level
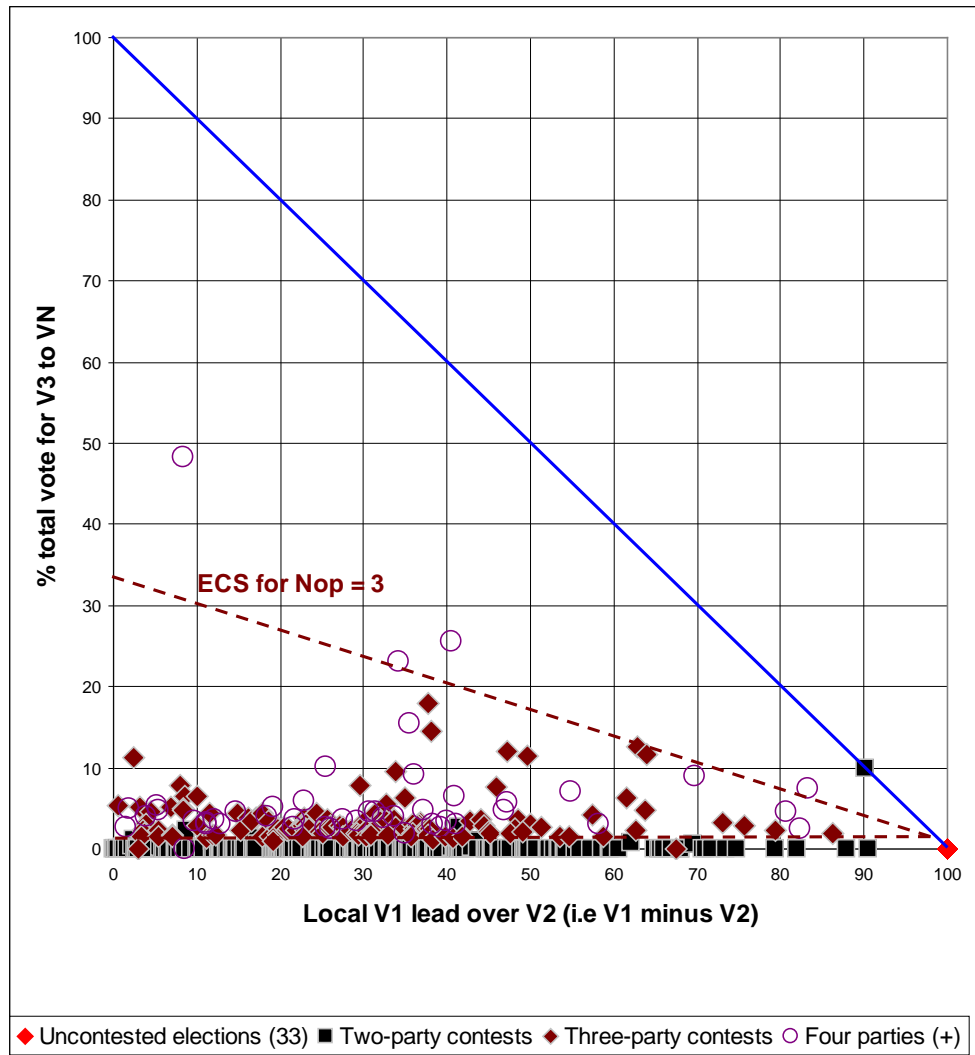
held in a Louisiana district, following the incumbent Democrat representative being charged with corruption close to the election. (He none the less topped the poll, and subsequently won a run-off election).

For India we show the range of outcomes in Figure 6, each sub-graph here covering districts with different numbers of parties competing. Here the key parties accounting for most of the V_1 and V_2 positions are the Congress bloc and the Bharatiya Janata Party/Jan Sangh (BJP for short), or their state or regional allies. There

Figure 6: The 2004 Indian general election outcomes for districts with different numbers of observable parties

(a) Districts with 2 or 3 observable parties
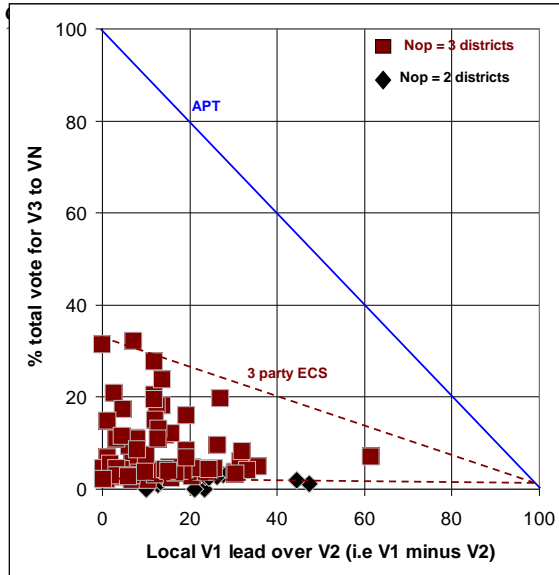
(b) Districts with 4 observable parties
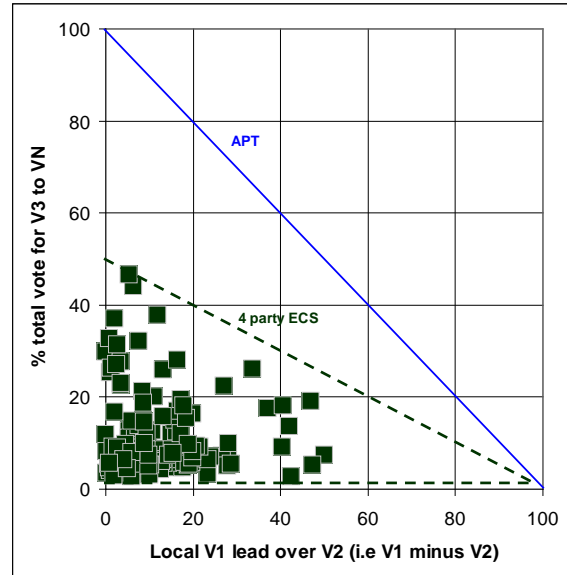
(c) Districts with 5 observable parties

(d) Districts with 6 observable parties

Figure 6 continued

(e) Districts with 7 or more observable parties

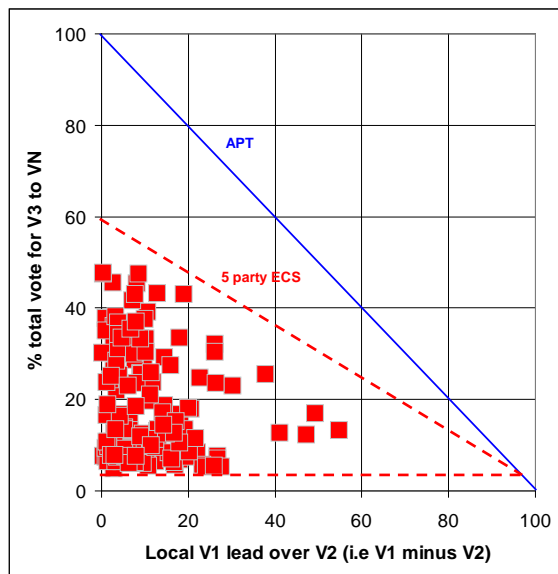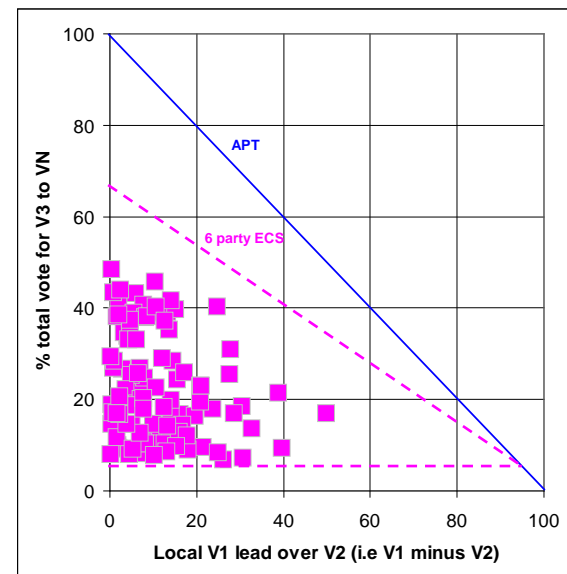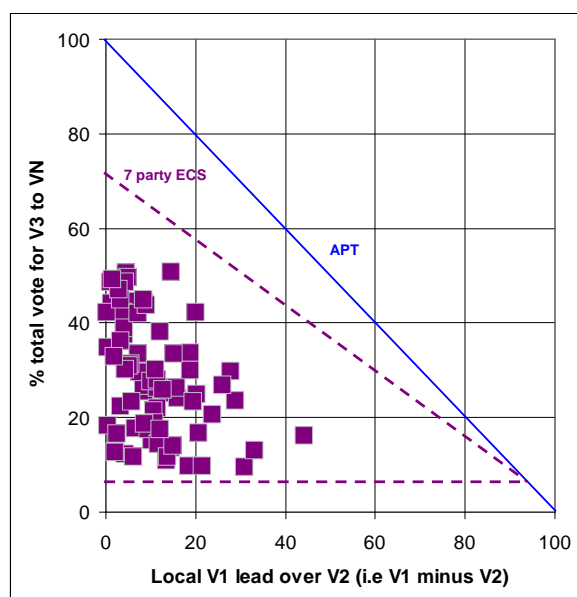are also substantial numbers of Communist victors and state or regional party winners who are not aligned with the leading party blocs.

Clearly in Figure 6a the few two-party districts, and most of the larger number of three-party ones, closely conform to Duverger's law predictions, with the top two local parties attracting all but a handful of votes. But a third of the three-party districts are much more widely scattered upwards across the feasible ECS space. This vertical scattering away from two-party races clearly increases for the four party districts, about half of which have 10 per cent or more of the votes going to third or subsequent parties.

The pattern of vertical scattering is clearly stronger in those districts with larger numbers of observable parties contesting the election. The five-party districts in Figure 6c still show a clustering at the lowest feasible levels of $V_3..V_N$ votes shares, but now more than half of the outcomes are scattered across the ECS above the 10 per cent level for the locally third and subsequent parties. With six observable parties the bottom ECS boundary clearly lifts further and the scattering increases. And with 7 or more observable parties competing the outcomes are relatively evenly distributed vertically, within the leftmost half of the feasible ECS area.

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Turning next to the British case, Figure 7 shows a pattern that is clearly distinct again from the other two countries, starting with the complete absence of two-horse races, and the very small numbers of three-way contests. [6] There is also clearly a single national pattern that applies in a similar way whatever the number of observable parties and the associated shape of the ECS area. There is a strong horizontal banding of the results whatever the level of V_3 to V_N aggregate votes, although there is a slight upwards drift of votes shares from three to seven plus observable parties, most visible in Figure 7d. But this change is modest, with the smaller parties always getting from 10 to 31 per cent of all local votes between them. Similarly there are some indications that in tight V_1-V_2 races locally the total vote for third and fourth parties tends to be squeezed down a little, but this is not a strong or clear-cut effect.

To sum up so far, we have shown that ECS analysis effectively characterizes the USA, India and UK as completely distinct party systems, and uncovers interesting patterns of V_1V_2 outcomes across constituencies, generally showing a slight tendency across all three systems for results to scatter more as the number of observable parties increases. A clear next step will be to find ways of statistically characterizing the patterns shown in Figures 5 to 7. This is not a straightforward topic and lies outside the scope of this paper for two main reasons. First, our focus here is solely on the counting and patterning of V_1V_2 slots to characterize the ECS area, but there are other possible methods of counting outcome slots that challenge this approach. At a limit, we might count all 'non-equivalent distributions' of the vote across multiple parties as different outcomes, using an equi-probability assumption - an approach that has its own advantages and disadvantages. This is a larger meta-

Figure 7: The 2005 general election outcomes in Great Britain for districts with numbers of observable parties

(a) Seats with 4 observable parties
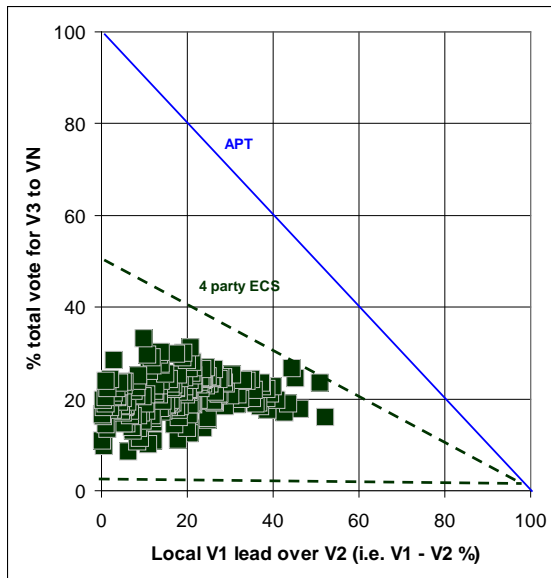
(b) Seats with 5 observable parties
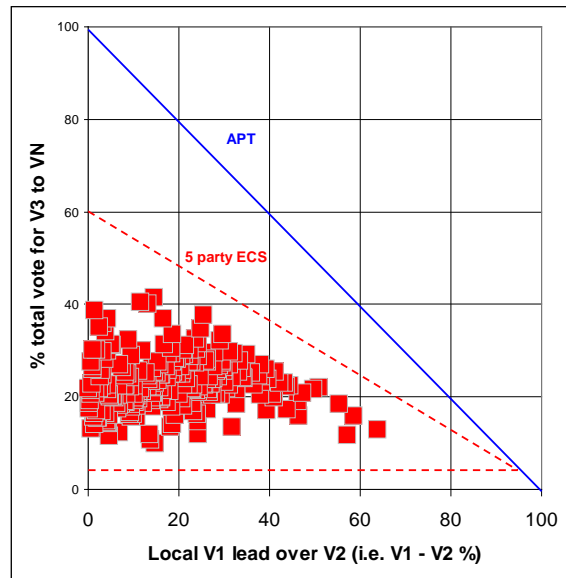
(c) Seats with 6 observable parties
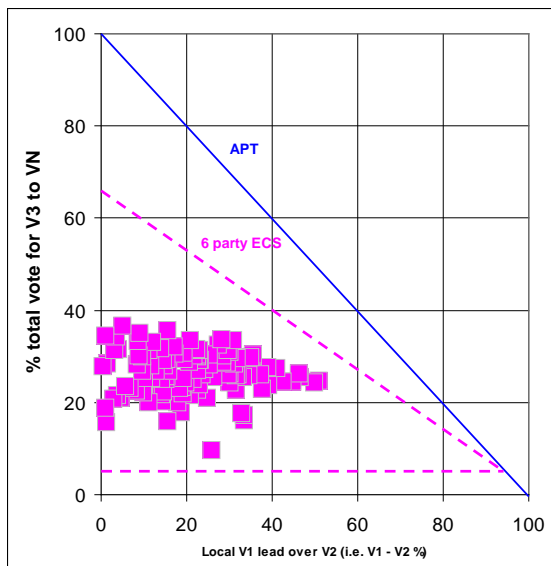
(d) Seats with 3 and 7 observable parties
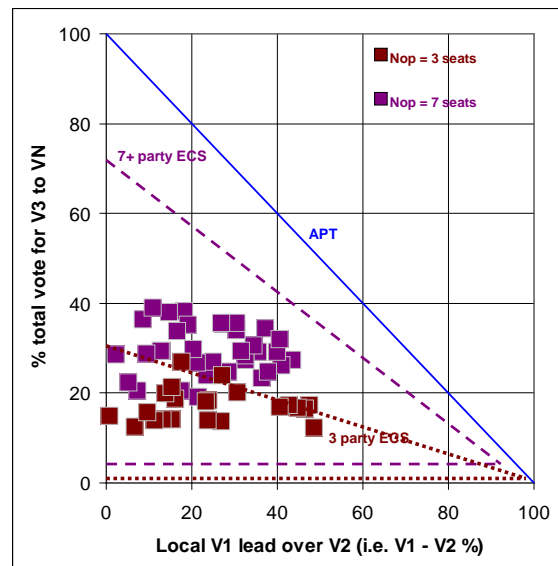
theory topic, explored in a separate paper. Second, even if could accept the $V_1 V_2$ focus here as the optimal basis for analysis, the statistical analysis of outcomes patterns within ECS areas is a little complex, although developments in cluster and classification techniques from applied mathematics and the biological sciences offer useful suggestions here (Gan et al, 2007; Fielding, 2007).

3. An alternative approach – the 'crown' diagram

For the comparative analysis of elections, the ECS approach above has some disadvantages or drawbacks. It is a reductionist way of proceeding, one that requires analysts to break down aggregate election outcomes into different layers, locating outcomes against competition spaces that vary a lot from 1 to 9 observable parties in competition, the levels most commonly found within electoral districts in liberal democracies. Accurately fixing the number of observable parties (down to 1 per cent) is not a trivial problems in many countries. Even in the USA, many official records of district level results are restricted to Democrat, Republican and 'other' vote shares, and some states (like New York) allow the same candidate to be registered under multiple different party labels. In other countries too 'other party' vote shares are used in most media and academic datasets, and so one needs to go to original official records to construct data on observable parties. These difficulties compound the tendency noted above, encouraging analysts to construct a raw APT picture from V_1 and V_2 data that risks confusing blank spaces created by logical infeasibilities with empirical patternings.

Although valuable in ways set out above, the ECS focus on $V_1 V_2$ competition within each local area also suppresses a lot of information that could flow from knowing precisely which party occupies these roles. For instance, in the British context the component parts of Figure 7 do not discriminate between those seats where the Conservatives and Labour compete in the top two slots and those with other patterns. Of course, different icons can be helpfully used in APT and ECS charts to indicate this additional kind of information (see Likhtenchtein and Yarmgomskaya, 2005). But in our three countries the normal concentration of general election outcomes still makes visually distinguishing different competitive cases difficult.

In addition, the traditional Duvergerian analysis of plurality rule systems has stressed that the rationale for expecting two-party competition is based not just on forces limiting competition within each electoral district, but also on the strong elite and mass incentives sustaining an overall nationalization of party politics (Cox, 1997). It would be useful to gain a more holistic picture of whole elections by linking the visual representation of district outcomes to this second theme.

In terms of non-Duvergerian dynamics, Dickson and Scheve (2007) advance what we term an 'invulnerable majority' conjecture, grounded on the rational choice principle that (if at all possible) individuals and groups prefer to be represented by a party closer to their own ideological optimum points. In plurality rule systems even if a large majority group in a locality splits evenly down the middle, they know that they are so numerous that one of their factions will still top the poll and defeat the opposition. They are invulnerable to defeat if they split support between just two parties or candidates – thereby enhancing the proximity of the winner to the 'majority of the majority' grouping. Thus large majority groups, most commonly linked to the top two parties in a polity, can afford the 'luxury' of intra-group competition (see also Rabinowitz et al, 1991). By contrast, narrow majority groups (accounting for only 50 to 66 per cent of local voters) cannot risk a split. Hence we should see local districts with up to two thirds of the votes being cast for the largest party, but none above this level. This pattern should be especially evident in conditions where social groups in local areas are regularly and enduringly politically aligned with the top two political parties - as with castes and ethnic groups in India, and social classes in the UK.

Dickson and Scheve also raise a second, more general conjecture opposed to Duvergerian expectations, which we term 'imitative fragmentation'. Suppose that the main local *minority* Y grouping in a district splits its support across two or more different parties or factions, in a way that is predictable, or that re-occurs with a reasonably high level of certainty from one election to the next. Under plurality rule, the local majority X grouping is now able to split its support further across parties as well (should it wish to do so, or have an opportunity to do so) – but only so long as X's largest factional party clearly remains larger than Y's leading factional party. We might be inclined to take the 'imitative fragmentation' conjecture most seriously wherever the main Dickson-Scheve 'invulnerable majority' hypothesis seems to be borne out.

To help explore these varied effects, we introduce a modified variant of the APT chart, called the 'crown' diagram (for reasons that will be clear later on). We first mirror our re-presented APT in its vertical axis, so as to create a double-APT chart, shown in Figure 8. We then use the horizontal axis of the double APT to plot all district outcomes in terms of the local vote for the *top two nationally leading parties* (or blocs) over their opponents. Seats where national party (or bloc) A leads the other leading national party (or bloc) B are shown on the right hand side of the graph,

arranged in order of A-B vote shares; and vice versa, seats where party B is ahead of A are shown on the left-hand side, going out in order of negative B-A vote shares. The A and B labels here simply denote the most important basis for political polarization or differentiation between the top two parties or blocs, whatever it is that this dimension involve. [7] Where a polity has no clear top or overall dimension organizing political competition then this approach may become problematic. But this is clearly not the case with our three large plurality-rule systems. On the vertical axis we chart support for all other parties (i.e not A and not B) as an aggregate.

Figure 8 shows how this basic set-up operates. For instance, if party A leads party B by 20 per cent, this district's outcome will be located at + 20 on the horizontal axis, and if B leads party A by 10 per cent this will be located at -10. This location system applied *whether or not* parties A and B are locally the top two parties in the district being charted, or alternatively one of more of these positions goes to (say) the national V_3 or national V_4. Figure 8 also includes shaded triangle areas showing those seats where the top two parties have local majority (50 per cent plus 1) support. Within these areas it also shows the Dickson-Scheve 67 per cent limit as a dashed line.

Where the top two national parties are also the local V_1 and V_2, then the district outcome will tend to occur lower down on the crown diagram; where the top two national parties are lower down the rankings in a district (say, the local V_3 and V_4), then the district outcome will be higher up the chart. Notice that shifting *away* from showing the *local* V_1 and V_2 support (as in Part 2 above) fundamentally alters the diagram. Across electoral districts we are now charting in each district on the horizontal axis just the difference in the vote shares there for the nationally leading top two parties (aligned along the polity's main dimension of competition), against the combined local vote share for the national third and subsequent parties, shown on the vertical axis.

Figure 8: The 'crown' within the double APT diagram
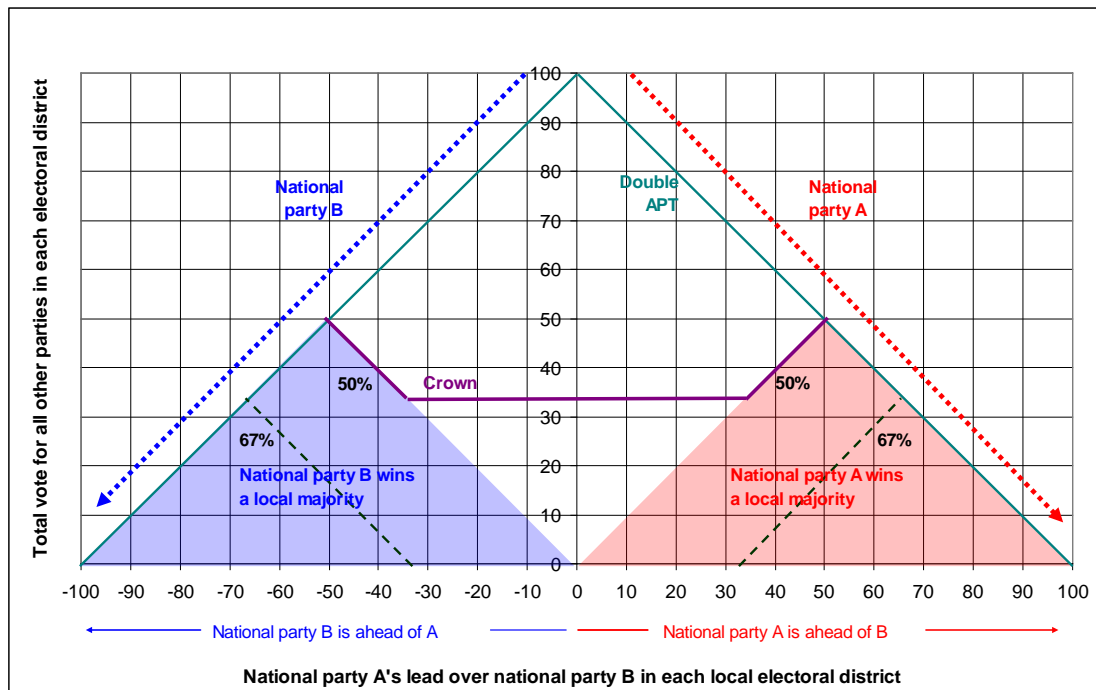
The 'crown' area in Figure 8 shows seats outcomes where the third or subsequent parties have some *potential* to win a given district, which requires that:

-   neither of the top two national parties has a vote share of 50 per cent or more; and

-   the combined $V_3…V_N$ vote share must reach or exceed 33.34 per cent.

No third or subsequent party can win a seat outside the crown. But because the vertical axis only shows the aggregate $V_3..V_N$ vote, which may be fragmented between several or many parties, seats inside the crown can still be won by one of the top two parties nationally, especially along the bottom edge of the crown area.

The distribution of seats outcomes within the crown area also responds strongly to one of the leading blocs nationally not contesting a given local district. If the top two parties nationally are A and B, but B does not stand in a given safe seat for A, or does very poorly in winning support there, then the local race is between party A and other parties (C, D, E etc) that are nationally ranked third or lower. If the V_3 to V_N vote is split several ways, A can still win; whereas if one of the V_3 to V_N parties is locally dominant there can be a two-horse race, but one that is different from the national picture. In practice the latter outcome is more common in plurality rule

countries. Here the outcome may be on or close to the top APT boundaries of the crown area, on one side or the other.

Outcomes may also occur on or close to the double APT boundary lower down, outside the crown area and in zones where party A or B has local majority support. Those outcomes actually on the APT boundary itself always show that one of the top two national parties has not contested that district. Outcomes close to but not on the lower APT boundaries may suggest only a 'no hope' or nominal local campaign by one of the top two parties (A or B) in its rival's safe seats.

Applying this approach, Figure 9 shows the 2006 US House of Representatives election outcomes. The crown display spreads out and makes more visible the vast bulk of outcomes that occur either on or close to the horizontal axis (when the number of observable parties is two), or with a V_3 to V_N total vote that is less than 10 per cent. In line with Duvergerian expectations, these outcomes also spread far out along the bottom axis, into 171 districts (two fifths of the total) where the largest party gained more than two thirds local majority support in the 2006 election. So the USA shows no support at all for Dickson and Scheve's 'invulnerable majority' prediction.

Elsewhere, the US pattern exhibits a kind of super-Duvergerian phenomenon with clusters of outcomes located on the lower left and right hypotenuses, because one or other major party does not stand a candidate against an incumbent (as discussed above, page 14). In such seats all the votes not going to the majority party necessarily go to smaller parties standing locally, especially to Independents and in some states the Greens or the Libertarian Alliance – all ranked third or lower in national vote shares. Thus the US display 'curls up' somewhat at the extreme edges. (The only result in the crown area is the highly unusual Louisiana open election contest mentioned previously, with second and subsequent Democratic and Republican party candidates here counting within the $V_3..V_N$ total).

Figure 9: The seats outcomes in the US House of Representatives election, 2006
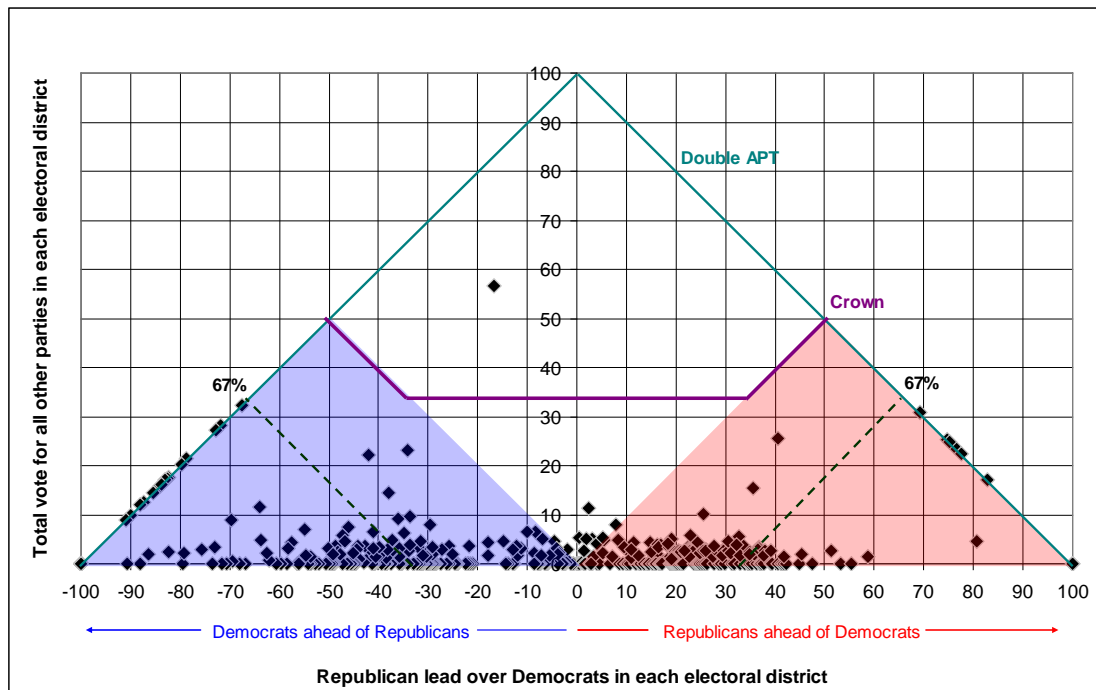
Turning to the Indian general election of 2004, Figure 10 shows a radically different party system. First, we should note that the horizontal axis scores here are based not on the top two parties alone, that is Congress and the Bharatiya Janata Party/Jan Sangh (or BJP for short), but on our analysis of the wider grouping of India's multiplicity of parties into the Congress bloc, the BJP bloc, or an 'other' category. The footnote here describes in detail the party composition of both the top two blocs.[8] Most state and regional parties are aligned to one of these main blocs and participate in their governing coalitions. The key 'other' parties here are the Communists, and unaligned state parties.

The two main blocs of Indian parties have both held power nationally for long terms in recent decades, overturning the previous pattern where Congress was a nationally dominant party, able to govern on its own for three decades after independence. Congress's position was essentially undermined by the growth of more and more state and local parties on the one hand, and by the emergence of the BJP as the core Hindu party, able to give coherence to anti-Congress forces, on the other.[7] In the modern period, the bloc partners normally agree not to contest those local seats

Figure 10: The seats outcomes in the Indian general election of 2004
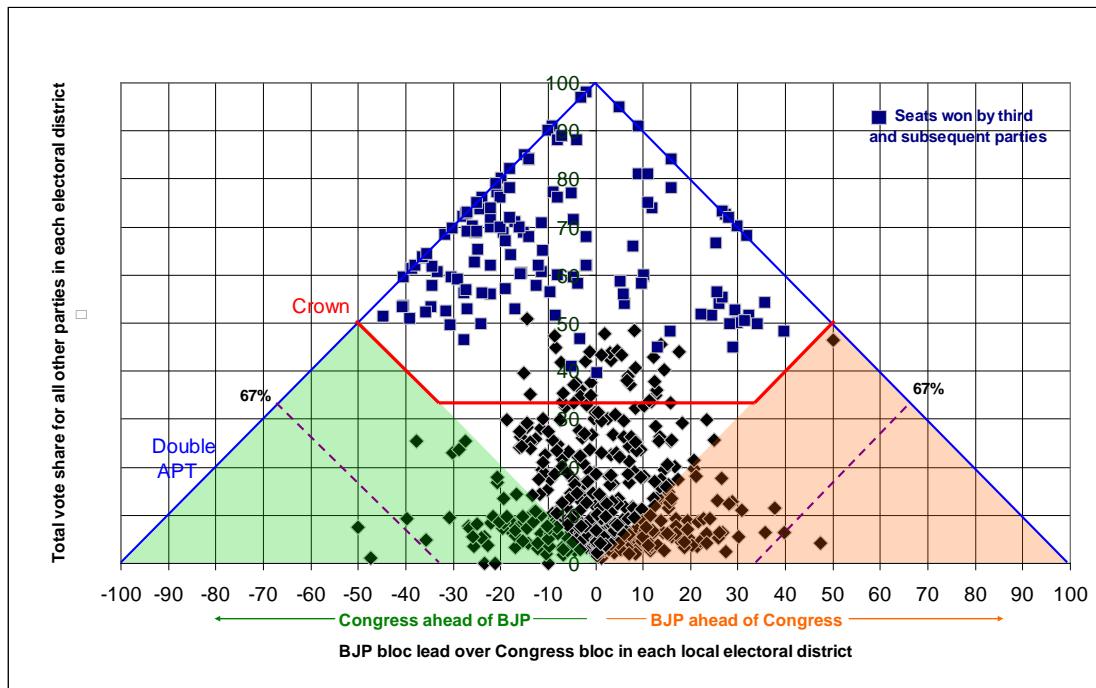
where another member of the bloc has a viable chance of winning, withdrawing candidacies in exchange for a similar quid pro quo in other areas. An agreed pattern of reciprocal withdrawals emerges from complex negotiations. These arrangements may not cover the full canvass of seats nationally (so that there are some cases of intra-bloc competition), but they usually suffice to form the basis for reasonably secure coalition government formation after the election.

The pattern in Figure 10 clearly shows a large mass of outcomes in districts where close BJP and Congress competition predominates, giving the strong clustering of seats evident at the bottom centre of the double APT here. Almost half (255) districts have a $V_3 \ldots V_N$ vote share under 15 per cent, rising to 299 districts if the cut off were 20 per cent. However, the horizontal spread of seats here is concentrated in a much narrower horizontal range than in the USA. Dickson and Scheve's 'invulnerable majority' prediction is strongly borne across all the Indian districts. In only three BJP versus Congress contests did the leading party attract support from more than 67 per cent of local voters (and then not by much). Leads of 30 per cent or more in competitions between the top two parties occurred in only 13 seats anywhere on the chart.

The other key pattern in Figure 10 is a strong and wide 'mushroom cloud' of results, straggling upwards from the base of two-party contests into the crown. Within the crown itself the scattering of results clearly increases, with many outcomes there occurring on or near to the hypotenuse boundaries of the double APT. This reflects especially decisions by the BJP not to stand candidates against (or not to campaign strongly against) state or ethnic parties that are competing with the Congress bloc. (As the former dominant party Congress is somewhat more active in 'no hope' seats). The pattern of district outcomes here, allied with the large number of districts with seven or more observable parties, suggests that 'imitative fragmentation' could have taken place in India, despite the impacts of the party-blocs in re-simplifying competition in most seats. Tracking given district outcomes over time could shed more light here (a task made easier by the freezing of most electoral boundaries since 1971).

If we re-presented Figure 10 using the bottom axis just to map the BJP and Congress core party votes, then the distribution would become even more vertically differentiated and 'top heavy' – because we would now be counting Congress-allied and BJP-allied parties as lower-ranked parties, forming part of the $V_3..V_N$ group, instead of in the top two blocs. Unfortunately because of the selective withdrawals way of creating coalitions, this would also mean increasing numbers of missing data cases for the core parties, piling up more cases on the APT boundaries, and creating substantial cases where neither core party is competing, so that we could not locate these districts on the crown diagram at all. Accordingly, the bloc representation in Figure 10 seems to be the correct one both analytically and substantively.

We turn next to the British general election of 2005 where a third, radically different pattern is evident. Figure 11 shows that there are no constituencies where the aggregate vote share of parties ranked third or lower nationally (V_3 to V_N) is below 15 per cent. In fact, there are only a few cases where it is less than 20 per cent. Essentially then the whole bottom third of the double APT area is empty. The strong nationalization of British politics is evident, with the top two national parties contesting (almost) all seats and garnering at least 5-10 per cent support almost everywhere. Thus there are almost no British outcomes on or even close to the APT boundary lines, bar three exceptional cases.[9]

Partly reflecting this effect, there are also only a handful of heartland seats where one of the top two Labour and Conservative parties wins more than two thirds

Figure 11: The seats outcomes in the 2005 general election in Great Britain
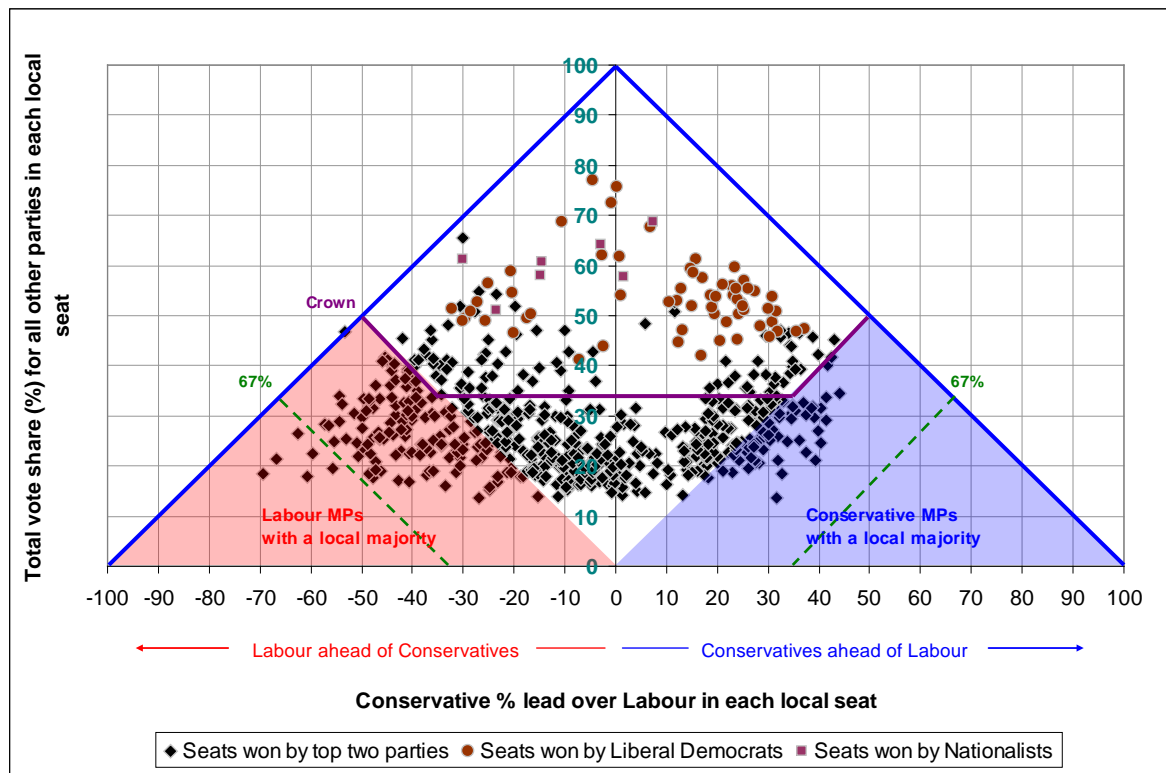
support. The 'invulnerable majority' hypothesis is clearly supported here, since only six seats show V_1 levels above two thirds. This is chiefly because the entire array has risen so far within the double APT, suggesting that different (far more nationalized) mechanisms are at work from those in the Indian case. And in the UK (unlike India) there are still many seats won by the top two parties with leads exceeding 30 per cent over their main rival.

Our display also distinguishes between seats won by each of the top two parties, and by one of the parties ranked parties ranked third or lower nationally, chiefly the Liberal Democrats and the nationalist parties in Scotland (SNP) and Wales (Plaid Cymru). As expected most of these seats occur in the upper part of the crown. In the lower part of the crown there are a mixture of wins for one of the top two parties and the Liberal Democrats, with a somewhat heavier clustering of seats with Conservative-Liberal Democrat competition noticeable on the right.

This all leaves the bulk of seats with close Conservative-Labour competition squeezed into a crescent shape below the crown, bounded underneath by the 15 per cent level and on the flanks by the 67% vote shares for each of the top two parties.

Inside the crown area itself there are substantial numbers of major party seats. In the Conservative-lead half of the chart, the crescent of district outcomes seems to extend along the APT boundary with a less populated 'hollow' area in the middle. But on the Labour-lead half there is a more even distribution of outcomes.

Again the British case may seem far removed from the other two countries. But we can extend the analysis here to show that this was not always the case, because the crown diagram is also well-adapted to analysing over-time changes in party systems. Figure 12 shows that in 1955, at the height of the two-party system in the UK, the patterns then were far more similar to those for the USA now. A large majority of district outcomes showed only two party competition, and hence lay on the bottom axis itself. Substantial numbers of MPs received support from more than two thirds of local electorates. Only in a minority of exceptional seats did MPs receive less than an absolute majority of local votes, even where there were raised levels of V_3 to V_N levels. At the same time there are also points of continuity with the Indian picture here, especially the rather clear separation between the bulk of two-party-only seats and those where third parties were still competing and garnering votes – essentially in the raised fringe of seats where the Liberal party maintained some hard-core support and could still muster candidates. The most prominent upwards straggling seats in 1955 represented areas in the Scottish and Welsh periphery where the Liberals retained their few MPs.

Looking across the cases presented so far suggests some evolutionary trajectories for plurality rule systems. For the UK the pathway would be as shown in Figure 13. Part (a) here shows a plurality rule system that starts out in a configuration highly consistent with Duverger's Law, with all observations on or close to the bottom axis, and with many districts showing large majority wins for one party or another. Figure 13b suggests that over time the breadth of the core cluster above should tend to shrink, with the 'invulnerable majority' effect leading to some fragmentation of V_1 in 'safe' districts with large social group majorities. This would automatically producing a 'curling up' of the outcomes pattern at the edges into a saucer pattern. Figure 13c shows that as this process progressively affects seats with smaller majorities for one of the two main parties, so the saucer shape could curl over completely across the top of the crown area, leaving perhaps a 'hollow centre'. The aging of a two-party system should also tend to shift outcomes progressively upwards and away from the horizontal axis, as has clearly happened in Great Britain. These

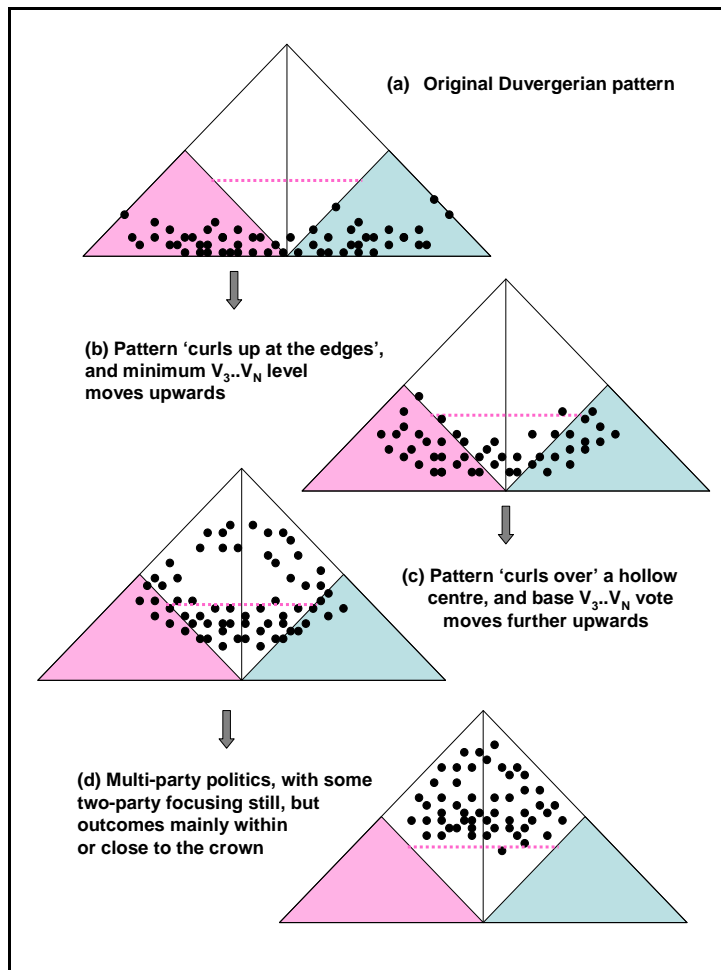**Figure 12: The seats outcomes in the 1955 general election in Great Britain**
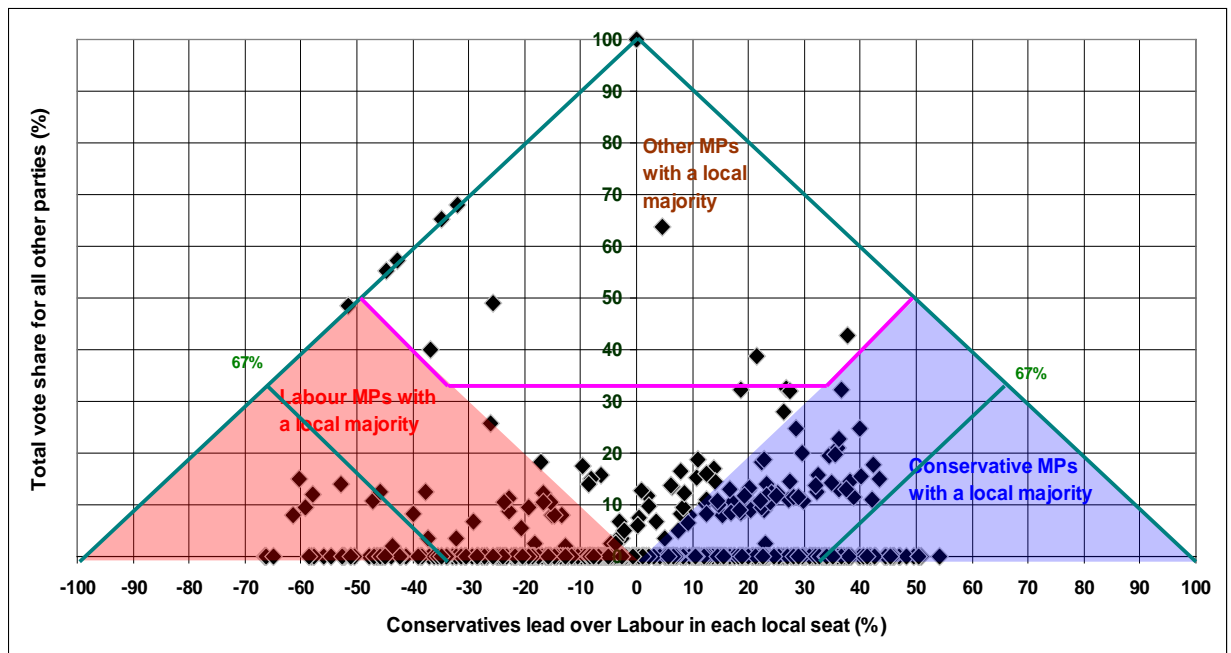
trends may well continue further, as they have in all British elections using proportional representation (on which see Figure 15 below).[10] In this case, Figure 13d would represent the final stage of evolution for the UK, where perhaps some Duvergerian clustering of outcomes chiefly into contests between the top two parties still remains, but the whole distribution has shifted inside the crown area alone and may be hard to distinguish from patterns in multi-party systems (where a similar focusing on the largest parties is also common).

We do not at this stage have the over-time data available to undertake a similar hypothesizing exercise for the USA or India. Tentatively we might think of the American case as a rather exceptional deviation on the evolutionary path, a plurality rule political system that has got frozen into a kind of self-sustaining, super-Duvergerian track, which the analysis above shows displays no invulnerable majority effect. It also has a different kind of 'curl up' effect, where at high levels of support for a majority party locally the other main rival party no longer non-contests quite a few seats, creating 'artificially' larger votes for the only opponents left. But such outcomes remain securely within majority-win area for one or other of the two largest parties.

We might think of India as perhaps having gone from a lop-sided dominant party system with a hegemonic Congress presence on two paths. First, the

Figure 13: Hypothesized ideal-type stages in the evolution of the UK's plurality rule system over time

(a) Original Duvergerian pattern

(b) Pattern 'curls up at the edges', and minimum $V_3..V_N$ level moves upwards

(c) Pattern 'curls over' a hollow centre, and base $V_3..V_N$ vote moves further upwards

(d) Multi-party politics, with some two-party focusing still, but outcomes mainly within or close to the crown

development of two-bloc Congress-BJP competition in a majority of seats (with restricted presence of other parties) retains a strong Duvergerian clustering despite party fragmentation (albeit with no large-majority outcomes). But this occupies a narrow base and no large-majority seats remain. Second, the fragmentation of votes in close to two fifths of seats progressively created a strong upward dispersion of seats outcomes, the top of the mushroom cloud in Figure 10. In a related fashion, in other countries where centrifugal tendencies tear at the support base for the original top two parties, producing regional oppositions, we might expect that there will be a more pronounced 'curling over' of seats distributions than that shown for the UK, as perhaps in Canada at different periods.

The crown diagram (and the double APT chart) are also highly suitable for charting the development of party competition across different regions within a single country. Figure 14 shows the 2010 general election outcomes in Great Britain across four contrasting government standard regions. The top two here are the Conservative-dominated south east; and the Labour-dominated north west (although 2010 was a Tory year). The bottom two here show four-party plus systems with a more marked upward scatter of outcomes because of the presence of nationalist parties and strong rural Liberal Democrat support. Scotland was Labour dominated in 2010, but with a strong SNP and Liberal Democrat presence, while Wales by comparison showed more Conservative strength in some areas.

Finally, the crown diagram can also be applied to other voting systems. For single member district elections using the Alternative Vote, first preferences are charted in an identical manner. For List PR elections, the number of electoral districts is always considerably less, so that the detail shown in the Crown diagram is greatly reduced. But again application is straightforward. Figure 14 shows the outcomes in eleven regional constituencies used in Great Britain across all the European parliament elections under the Blair and Brown governments. The last two sets of outcomes were solidly within the crown area, with the effective number of parties reaching 6.5 or more in every region by 2009. There were marked changes in patterns of behaviour across all the English regions across this time period (whereas Wales and Scotland remained more consistent). In both the later elections two parties to the right of the Conservatives and favouring withdrawal from the EU and stronger immigration controls between them won nearly a quarter of the vote (the UK Independence Party accounting for two thirds of this vote share, and the British National Party a third), while the Greens also did well under List PR.

Figure 14: Regional patterns of party competition in four British regions at the 2010 general election

South-east

North-west

Scotland

Wales
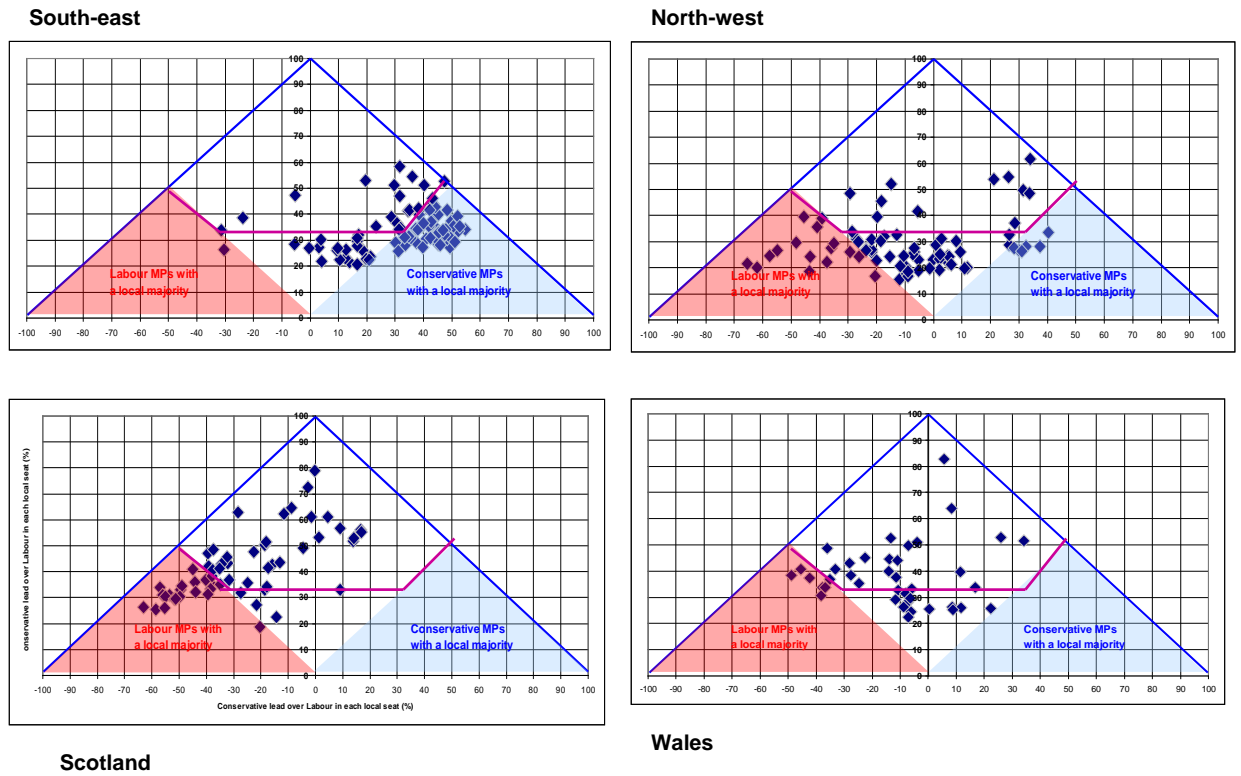
Figure 15: The outcomes of the European Parliament elections in Great Britain in 1999, 2004 and 2009, using regional list PR systems
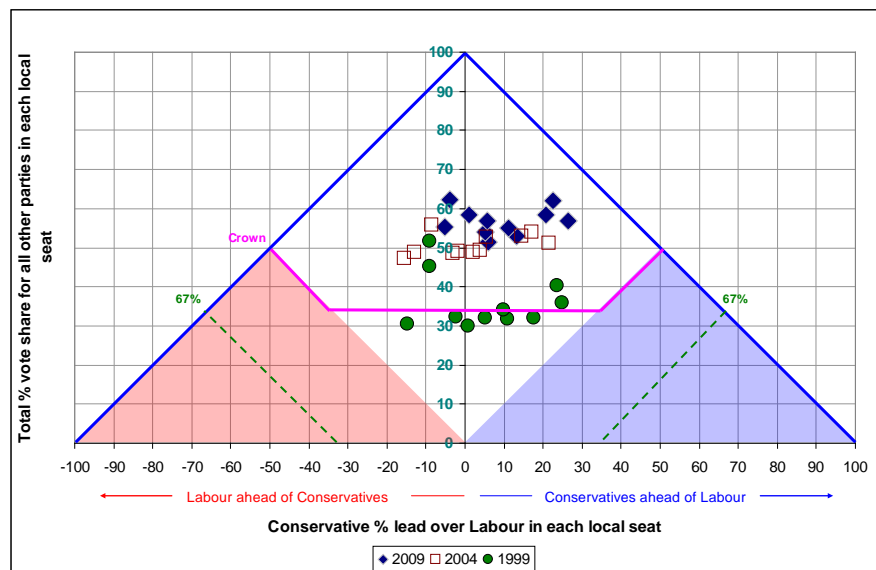
Conclusions

Within plurality rule election systems, the district-level comparison of general election outcomes has been increasingly central in the debate about competing Duvergerian expectations and opposing mechanisms favouring party fragmentation (such as the 'invulnerable majority' effect). A few previous analysts have cumulated election data covering many separate district contests (some 58,000 results in the case of Chhibber and Kollman, 2004, Ch. 2), yet confined their analysis to means and standard deviations, presenting little of the richness of outcomes patterning that can be accessed by better methods of charting. We have demonstrated that it is very helpful to visually chart both

- a reductionist picture of V_1V_2 competition in races with different numbers of party in competition, shown in the ECS view, and giving insights into the aggregated, layer-cake nature of general election outcomes; and

- information about how district outcomes relate to the most important basis for political alignments and how the holistic patterns vary across election outcome, shown in the crown diagram.

A careful analyst will always consider both displays.[11]

In substantive terms, Duvergerian expectations of strong two-party focussing in plurality rule systems extend to a psychological effect during the campaign period for voters not to support candidates standing from non-top two (or non-major) parties (Prediction B in Table 1 above). Our analysis demonstrates that this prediction is broadly borne out for the USA, but that it is now widely inapplicable in both India and Great Britain, albeit in different ways. India demonstrates strong Duvergerian conformity in a large sub-set of districts, but extensive vertical scattering of results within most ECS areas and the crown. The British party system shows a strongly uniform, nationwide expansion of third and subsequent party voting, with results clustering just below and into the crown. While the USA shows no signs at all of the opposing 'invulnerable majority' effect, both India and the UK demonstrate strong conformity with the Dickson and Scheve rational choice prediction, albeit in different ways. The small increases in the vertical scatter of outcomes across ECS areas in the UK, and stronger effects in India, also suggest that when small parties compete and attract enough votes to make changes in the numbers of observable parties, they may

help to trigger much larger changes in the comparative situations of all parties locally, by changing the feasible competition space.[12]

Charting tools have been very important in scientific progress because they allow users to access information about a large number of data points simultaneously, and in a fashion that is especially useful for pattern-finding and generating hypotheses. Visual representations of multiple outcomes in general elections are especially useful in suggesting a range of initial synoptic characterizations of party systems, grounded in large amounts of information about multiple district outcomes. By expressing more of the richness of data and information available to us, they can provide the exploratory seed beds for hypothesis formulation (as Figure 13 demonstrates). Legitimate charting tools applied to rich information can also help to illuminate aspects of complex, multi-dimensional data arrangements that are otherwise hard to capture using single statistical indices or even bundles of complementing indices. Hence better visual representations often serve as a key jumping-off point for developing appropriate data-clustering analyses (Gan et al, 2007; Fielding, 2007).

However, visual patterns alone can be misleading. So in the next stage of development it will also be essential to sort out statistical methods attuned to the analysis of this rich field of information, capable of rigorously distinguishing between random and patterned (non-entropic) effects, and able to characterize successfully the information content conveyed by any patterning. Our focus here on competition space defined in terms of V_1V_2 slots is also itself contestable, representing a limit view, the simplest possible characterization of the spaces within which feasible outcomes can be expected to occur. At an opposing limit, taking account of every possible combination of vote shares, we could chart outcomes set against all the 'non-equivalent distributions' of party support for each V_1V_2 slot in the ECS area, that is all outcomes feasible on an equi-probability basis, as we do in further work. In between these two poles there are also many other possible positions. Resolving these conceptual and methodological issues will be key for deriving appropriate null hypotheses of where we should expect to see outcomes distributed across the ECS area, and hence of the extent of non-entropic patterning that we actually observe, both in plurality rule elections and in other, proportional systems.

NOTES

1. In fact, it might be possible to use a simplex representation somewhat effectively in multi-party conditions in plurality rule elections where there are three clear leading parties and a range of other parties sharing a minority vote share (say below 10 per cent) between them, as in the British general election of 2010 (Dunleavy, 2010b). Essentially the three axes are here recalibrated to run from 0 to 100-x% where x is the aggregate share of the fourth and subsequent parties, whose vote shares are not shown explicitly. This still lets the diagram focus effectively on the fluctuations of support between the top three main party contenders.

2. Because Nagayama's original paper was in Japanese, and has not been translated, the key English sources for this attribution is Reed (2001 pp. 216-7). See also Reed (1990) and (2003).

3. Of course, as more parties enter the system the maximum vote share for the smallest party in the system is 100 per cent divided by the number of parties. This limit shrinks, albeit at a slower and slower rate, to 33.3 per cent for V_3 with three parties, 25 per cent for V_4 with 4 parties, 20 per cent for V_5 with five parties, , and so on.

4. In addition, the 'all possibilities triangle' (APT) label is not a neologism or departure from conventional usage for its own sake, but also serves two further key roles: (i) It alerts users to the fact that the orientation and axis labelling originally used by Nagyama are both changed in the analysis below. And (ii) The 'all possibilities' label usefully stresses key continuities between the analysis of electoral outcomes here and a wide range of parallel ways of analysing political competition using the APT, such as power index approaches, discussed in related work.

5. Occasionally analysts may lose sight of the causal link running from an increased number of observable parties to ENP. For instance, Chhibber and Kollman (2004, p, 46) present a graph with ENP as the horizontal axis (independent) variable apparently

influencing average third party votes shares on the vertical axis. The correct influence is the other way around, showing that V_3 average vote shares above 20 per cent are compatible with any level of ENP above 2.75.

6. For more background on the 2005 general election outcomes see Dunleavy and Margetts (2005).

7. In the USA the dominance of Democrat-Republican competition fits fairly easily into a left-right frame, as does the historicalbeit now almost hollowed-out) Labour-Conservative predominance in Westminster politics. Both these countries (in different ways) also show a strong focusing of political life directly on the top two parties. Indian politics now illustrates an interesting pattern found in other modern democracies (such as Italy and perhaps France), where two main *blocs* of linked parties (more permanent than coalitions) regularly compete, each bloc having a main party at its core but several other components (or in India's case many other components). The Congress bloc versus BJP bloc dimension primarily has many ethnic group, caste politics and secularist-versus-Hindu components, but has some left-right connotations also. Positing a clear top dimension of here is simply a convenient simplification, and it should be crystal clear that we are not prejudging the substantive content of the dimension in each case. Our approach adapts to any other form of content for the main dimension, and to the single dimension itself being a strongly composite one or essentially just an analytic construct.

8. The compositions of the two main blocs are as follows. The core party of the Congress bloc is the Indian National Congress, and allied bloc parties are Arunachal Congress, Lok Jan Shakti Party, Rashtriya Janata Dal (Laloo), Nationalist Congress, Jharkhand Mukti Morcha, Telengana Raya Samiti, Pattali Makkal Katchi, Dravida Munnetra Kazhagam, Marumalarchi Dravida Munnetra Kazhagam, Congress Independent, and other Congress party allies. The core party of the BJP bloc is Bharatiya Janata Party/Jan Sangh, and the allied bloc parties are Telugu Desam/Telugu Desam (NAIDU), Janata Dal/Janata Party, Shivsena, Shiromani Akali Dal/Shiromani Akali Dal (Badal), All India Anna Dravida Munnetra Kazhagam, Janata Dal/Janata Party, Trinamul Congress, BJP-supported Independent, and other BJP party allies. For additional background, see Diwakar (2006, and 2007), Sridharen (1997), Chhibber and Murali (2006).

9. These are the Speaker's seat (where main opposition parties did not stand, but smaller parties did); and two seats where Labour failed to stand a candidate through errors.

10. See also Dunleavy and Margetts (2001) and Dunleavy (2005).

11. Because the crown diagram aggregates third and subsequent party vote shares, it is important to bear in mind that (unlike an ENP_{votes} score) it does not differentiate between situations such as (40, 30, 30) versus (40, 30, 5, 5, 5, 5, 5, 5). However, in ECS terms the first situation is in the plot for districts with 3 observable parties, and the second situation in the plot for 8 observable parties.

12. For useful analytic perspectives on new parties, see Hug, 1996 and 2001; Dewan, 2007; and Rabinowitz et al, 1991. For empirical cases see Sikk (2005), Szajkowski (1991) and Bélanger (2004).

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