Calibration of power quality analyzers on total harmonic distortion by standard periodic non-harmonic signals

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***Abstract* — The possibility of using standard periodic non-harmonic signals is considered instead of the traditionally used set of harmonic signals when calibrating by a total harmonic distortion of power quality analyzers. The use of squarewave, triangular, sawtooth signals and a truncated sine wave signal is proposed, which presented in Fourier order contain specific harmonic components. Thus, by selecting a specific standard periodic signal, a corresponding specific reference value of the total harmonic distortion is set. Results of calibration of Fluke 435 power quality analyzer using Metrix CX1651 calibrator are presented, which give an experimental estimate of the ability to calibrate by a total harmonic distortion using standard periodic non-harmonic signals.**

Keywords—power quality analyzers, calibration, total harmonic distortion, standard periodic non-harmonic signals

I. Introduction

Calibration of power quality analyzers (PQA) by total harmonic distortion of voltage and current most often it is performed by the method of the reference signal. When calibrating PQA by total harmonic distortion (THD) a calibrator (standard) generating a sum of at least two or more sinusoidal signals is usually required, the first being the fundamental harmonic and the others being harmonics with frequencies multiples of the fundamental harmonic frequency. The dependences for total harmonic distortion of voltage and current, respectively, are known as follows

$$THD_U = \frac{\sqrt{\sum_{n=2}^{\infty} U_n^2}}{U_1} \quad \text{and} \quad THD_I = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1},$$

where U_1 and I_1 are the effective values of the first (main) harmonic, respectively, of voltage and current, and U_n and I_n are harmonics of the voltage and current, and n is the number of the harmonic.

Often in metrological practice there is no calibrator to create an output signal (voltage or current) that is the sum of two or more sinusoidal signals. In these cases, the idea is proposed to use standard periodic non-harmonic signals, such as squarewave, triangular, sawtooth and truncated sine wave. Each of these standard signals, presented in Fourier series, contains specific harmonic components, i.e. by selecting a specific standard periodic non-harmonic signal, a corresponding specific reference value of the total harmonic distortion can be set for which the analyzer can be calibrated according to THD.

II. Mathematical models of standard periodic non-harmonic signals and their corresponding total harmonic distortion

The mathematical models of standard periodic non-harmonic signals presented in Fourier series and their corresponding total harmonic distortion according to [1] are as follows:

- for periodic squarewave voltage $u_{sq}(t)$, with amplitude U_m, period T and with duty cycle $\mu = 0.5$

$$u_{sq}(t) = \frac{4U_m}{\pi} \sum_{n=1}^{\infty} \frac{\sin \frac{2\pi}{T} t(2n-1)}{(2n-1)} \tag{1}$$

$$THD\left[u_{sq}(t)\right] = \sqrt{\frac{\pi^2}{8} - 1} = 48.34258...\% \tag{2}$$

- for periodic triangular voltage $u_{tr}(t)$ with period T and amplitude U_m

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$$u_{tr}(t)=\frac{8U_m}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin\frac{2\pi}{T}t(2n-1)}{(2n-1)} \quad (3)$$

$$THD\left[u_{tr}(t)\right]=\sqrt{\frac{\pi^4}{96}-1}=12.11529...\% \quad (4)$$

- for periodic sawtooth voltage $u_{sw}(t)$ with period T and amplitude U_m

$$u_{sw}(t)=\frac{2U_m}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin\frac{2\pi}{T}t(2n-1)}{n} \quad (5)$$

$$THD\left[u_{sw}(t)\right]=\sqrt{\frac{\pi^2}{6}-1}=80.30778...\% \quad (6)$$

The mathematical models of the currents are similar when using standard reference signals for current PQA calibration and therefore current calibration will not be considered.

III. Experimental Setup

The experimental setup is presented in the diagram in Figure 1. A Metrix CX1651 calibrator [2] is used as a reference, which can generate standard squarewave, triangular and sawtooth voltages, as well as a voltage truncated sine wave with $THD\left[u_{tr\sin}(t)\right]=13.45\%$ [2]. The points at which the Fluke 435 [3] analyzer is calibrated by THD are set by selecting one of the standard periodic voltages described in paragraph 2.

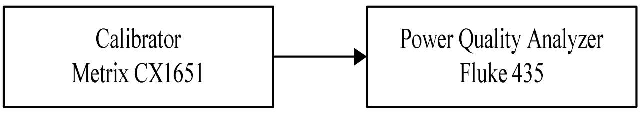

Fig. 1. Scheme of calibration

The procedure for setting the parameters, including the amplitude U_m of each standard voltage and the values of it's THD_U for which the calibration is performed, includes the following steps:

- The frequency of the standard reference voltage is selected, which is usually $f = 50\,Hz$, which is also the frequency of the main (first) voltage harmonic.
- Select the effective value of the voltage of the main (first) harmonic U_1 for which the analyzer will be calibrated. It is usually $U_1 = 230\,V$, but another value can be selected so that it is realizable by the calibrator for each of the standard signals. The voltage U_1 is related to the amplitude U_m of each of the standard signals according to (1), (3) and (5), as follows:

- The amplitude U_m of a periodic squarewave voltage $u_{sq}(t)$ with a duty cycle of 0.5, according to (1) is calculated from the expression $U_m=\frac{\pi}{2\sqrt{2}}U_1$, for selected already effective value of the first harmonic. Thus the calibrator sets the reference value to $THD\left[u_{sq}(t)\right]=48.34258\ \%$, according to (2).
- The amplitude U_m of a periodic triangle voltage $u_{tr}(t)$, according to (3) is calculated from the expression $U_m=\frac{\pi^2}{4\sqrt{2}}U_1$, for selected already effective value of the first harmonic. Thus the calibrator sets the reference value to $THD\left[u_{tr}(t)\right]=12.11529\ \%$, according to (4).
- The amplitude U_m of a periodic sawtooth voltage $u_{sw}(t)$, according to (5) is calculated from the expression $U_m=\frac{\pi}{\sqrt{2}}U_1$, for selected already effective value of the first harmonic. Thus the calibrator sets the reference value to $THD\left[u_{sw}(t)\right]=80.30778\ \%$, according to (6).
- According to the manufacturer [2], the voltage in the form of a truncated sine wave $u_{tr\sin}(t)$ has a value $THD\left[u_{tr\sin}(t)\right]=13.45\ \%$ ·

IV. Calibration of Power Quality Analyzers by Total Harmonic Distortion by Means of Standard Periodic Non-Harmonic Signals

A. Mathematical model

The mathematical model for estimating the actual value of the total harmonic distortions, according to [4, 5] is

$$THD_{U,act}=THD_{U,cal}-\delta THD_{U,s,et}-\delta THD_{U,dr.et}+\delta THD_{U,res.cal} \quad (7)$$

where

- $THD_{U,cal}$ - is the estimate of the measured value of the total harmonic distortion of the specific standard periodic voltage, defined as an average of at least 10 measurements;
- $\delta THD_{U,s.et}$ - correction (deviation) of the set value of the total harmonic distortion from the calibrator [2] for the specific standard periodic voltage. It is determined by the calibration certificate of the calibrator, by the calibration point correction or by its technical documentation, if the calibrator has not been calibrated;
- $\delta THD_{U,dr.et}$ - the value of the drift of the total harmonic distortion of the calibrator.
- $\delta THD_{U,res.cal}$ - correction of the measured value of the total harmonic distortion of voltage, due to the resolution of the calibrated analyzer [3].

B. Determining the estimates of the input values of the mathematical model and the actual value of THD

The Fluke 435 analyzer is calibrated by the total harmonic distortion of voltage THD_U by setting a reference value of the parameter THD_U, i.e. by selecting the appropriate standard (squarewave , triangular, sawtooth and truncated sine wave) voltage from the Metrix CX1651 calibrator. Calibration points on THD_U of the corresponding standard signal is set at the same or at least similar value of the first harmonic, which in this case is achievable for $U_1 = 90\ V$. Based on dependences (1), (3) and (5) for the first harmonic, i.e. $n = 1$ for the corresponding standard voltage are calculated respective amplitudes U_m at each of the standard voltages. In this case the THD_U calibration is performed as follows:

- Set a squarewave voltage $u_{sq}(t)$ with a duty cycle of 0.5 and an amplitude $U_m = 99.9649\ V$, calculated from expression (1).
- Set a triangular voltage $u_{tr}(t)$ with amplitude $U_m = 150.0244\ V$, determined by expression (3).
- Set a sawtooth voltage $u_{sw}(t)$ with amplitude $U_m = 199.9207\ V$ determined by expression (5).
- Set a truncated sine wave $u_{tr\sin}(t)$ with amplitude $U_m = 109.7325\ V$, which is experimentally determined on the basis of the analyzer reading to achieve a value of the first harmonic of 90 V.

For each standard voltage, i.e. for each calibration point k measurements are performed on THD_U, the index i indicates the number of the current measured value. The obtained k measured values of $THD_{U,cal,i}$ for the four standard voltages are reported through the software of analyzer and are presented in Table 1.

The estimate of the measured value of the total harmonic distortion, reported by the analyzer for each standard voltage is determined by the expression

$$THD_{U,cal} = \frac{1}{k}\sum_{i=1}^{k} THD_{U,cal,i} \tag{8}$$

The estimation of the deviation of the set value $\delta THD_{U,s.et}$ of the calibrator for the corresponding standard voltage is determined by its specification [2] by the specified maximum error value, which in this case is $\delta_{THDu,et} = 0.05\ \%$.

The evaluation of the drift $\delta THD_{U,dr.et}$ the total harmonic distortion of the calibrator (in %) has a zero value, i.e. $\delta THD_{U,dr.et} = 0$ due to the deterministic spectrum of the standard stresses used, determining a specific numerical value of the total harmonic distortion.

Estimation of the analyzer resolution deviation $\delta THD_{U,res.cal}$ is a quantity with a uniform distribution within the limits of the value of the least significant digit of the analyzer by the total harmonic distortion and therefore this estimate is $\delta THD_{U,res.cal} = 0$.

The estimate of the actual value of THD_U according to the mathematical model (7) the species is obtained

$$THD_{U,act} = THD_{U,cal} - \delta THD_{U,s,et} \tag{9}$$

C. Determination of the standard uncertainty of the input quantities

The standard uncertainty of the estimation of the measured values of the total harmonic distortions for each standard voltage $u\left(THD_{U,cal}\right)$ [4] is determined by the expression

$$u\left(THD_{U,cal}\right) = \sqrt{\frac{\sum_{i=1}^{k}\left(THD_{U.cal,i} - THD_{U,cal}\right)^2}{k(k-1)}} \tag{10}$$

The standard uncertainty of the estimation of the deviation of the set value of the calibrator $u\left(\delta THD_{U,s,et}\right)$ for each standard voltage is determined by its specification as ½ from the maximum value of the uncertainty of the total harmonic distortions indicated in [2] 0.3%, i.e., $u\left(\delta THD_{U,s,et}\right) = 0.15\ \%$.

The standard uncertainty of the analyzer resolution deviation estimate $u\left(\delta THD_{U,res.cal}\right)$ is determined by the value of the lowest voltage analyzer bit a = 0.0001 %, from the expression

$$u\left(\delta THD_{U,res.cal}\right) = \frac{a}{2\sqrt{3}} = 0.002887\ \% \tag{11}$$

Table 1

Calibration point THD_U	$THD_{U,cal,1}$	$THD_{U,cal,2}$	$THD_{U,cal,3}$	$THD_{U,cal,4}$	$THD_{U,cal,5}$	$THD_{U,cal,6}$	$THD_{U,cal,7}$	$THD_{U,cal,8}$	$THD_{U,cal,9}$	$THD_{U,cal,10}$
%	**%**	**%**	**%**	**%**	**%**	**%**	**%**	**%**	**%**	**%**
48.34258	46.9925	46.9978	46.9980	46.9990	46.9975	47.0008	47.0003	46.9928	46.9975	47.0028
12.11529	12.0900	12.0900	12.0898	12.0903	12.0900	12.0900	12.0900	12.0900	12.0880	12.0895
80.30778	78.6973	78.6978	78.7003	78.6983	78.6995	78.6955	78.6988	78.7008	78.6968	78.7003
13.45	13.4308	13.4308	13.4310	13.4303	13.4300	13.4310	13.4313	13.4353	13.4310	13.4323

Table 2

Calibration point THD_U	$THD_{U,cal}$	$\delta THD_{U,s,et}$	$THD_{U,act}$	$u(THD_{U,cal})$	$u(\delta THD_{U,s,et})$	$u(THD_{U,act})$	$U(THD_{U,act})$
%	%	%	%	%	%	%	%
48.34258	46.9979	0.02417	46.97373	0.00102	0.07050	0.07050	0.14101
12.11529	12.0898	0.00606	12.08369	0.00020	0.01813	0.01814	0.03627
80.30778	78.6985	0.04015	78.65835	0.00054	0.11805	0.11805	0.23610
13.45000	13.4314	0.00673	13.42463	0.00047	0.02015	0.02010	0.04031

Table 3

Calibration point THD_U	Standard voltage	$THD_{U,act}$	$U(THD_{U,act})$	$U(THD_{U,act})_{rel}$	$\delta(THD_{U,act})$
%	-	%	%	-	%
48.34258	$u_{sq}(t)$	46.9737	0.14101	0.30019	-1.36888
12.11529	$u_{tr}(t)$	12.0837	0.03627	0.30017	-0.03159
80.30778	$u_{sw}(t)$	78.6583	0.23610	0.30016	-1.64948
13.45000	$u_{tr\sin}(t)$	13.4246	0.04031	0.30023	-0.0254

D. Determination of the combined standard uncertainty of the input quantities

The combined standard uncertainty is determined by the components of the standard uncertainty of the estimates of all inputs (for sensitivity coefficients for inputs equal to one) [4] from the expression

$$u(THD_{U,act}) = \sqrt{u(\delta THD_{U,cal})^2 + u(\delta THD_{U,s,et})^2 + u(\delta THD_{U,res.cal})^2} \quad (12)$$

E. Determination of extended uncertainty

Extended uncertainty is determined [4] by the expression

$$U(THD_{U,act}) = k.u(THD_{U,act}) \quad (13)$$

where $k = 2$ is the coverage multiplier with a probability of 95% approximately normal distribution law.

The values of the estimates of the input values, of the actual value of the total harmonic distortion and of the estimates of the standard, combined and extended uncertainty for each standard voltage are presented in Table 2.

F. Announced results of calibration of Fluke 435 analyzer by total harmonic distortion of voltage

The results of the calibration by the total harmonic distortion of voltage for each standard voltage are presented in Table 3 by the estimates of the actual value $THD_{U,act}$, their expanded uncertainty $U(THD_{U,act})$, the relative extended uncertainty $U(THD_{U,act})_{rel} = \frac{U(THD_{U,act})}{THD_{U,act}}$ and the deviation from the calibration point $\delta(THD_{U,act}) = THD_{U,act} - THD_U$.

From the results obtained in Table 3 it follows:

- The values of the relative extended uncertainty at the calibration points $U(THD_{U,act})_{rel}$ are very close, which determines the same metrological capabilities of the standard signals for the purposes of calibration of PQA by THD.
- Deviation values at the calibration point $\delta(THD_{U,act})$ are multiplicative in character and are negative due to frequency spectrum limitations up to the 50th harmonic of the calibrated Fluke 435 analyzer.

V. Conclusion

1. Through standard periodic non-harmonic signals can be set with high accuracy fixed reference values of the total harmonic distortion.
2. In case of calibrator is not possible to generate two or more sinusoidal signals, then calibration of the PQA by total harmonic distortion, through standard periodic non-harmonic signals it is possible.
3. Metrological results obtained when calibrating a Fluke 435 analyzer with a Metrix CX1651 calibrator by total harmonic distortions demonstrate the applicability of standard periodic non-harmonic signals for calibration of PQA by total harmonic distortions in metrological practice.

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