Rapid Non-Contact Tension Force Measurements on Stay Cables

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Abstract

Vibration to Tension Force calculations are known in the engineering community through the Taut String Theory. Precise calculations require accurate identification of the natural frequencies and incorporation of correction factors. Additionally, the duration of individual measurements must be optimized to allow for sufficient data aquisition. The work presented in this paper describes a cost effective and optimized approach to determine the tension force in stay cables. This optimization is necessary to accommodate any funding restrictions.

1 Introduction

Cable Stay bridges gain more and more popularity in the structural engineering world. The reasons for the popularity of these structures include the cost efficiency of construction and the pleasing appearance of cable stay bridges. However, the main structural elements holding the deck in place are the stay cables which are in most cases not thoroughly inspected over the lifetime of the bridge. The life span of the structure may be shortened due to corrosion, slippage, settlement of the structure or part of the structure, resulting in load imbalances in the cables. The costs of repairing significant structural damages may exceed the replacement cost, especially when the repairs involve full or partial closure. Therefore, maintaining the structure and fixing the issues when they arise will be most cost efficient in the long run.

Periodic inspection plans for these bridges often include a measurement of the cable tension forces. Traditional methods to measure cable tension forces in stay cables can be classified into 2 main categories:

Lift-off direct measurement by the lift-off method, using hydraulic jacks.

Frequency-based indirect measurement via the natural frequency of the cable.

The natural frequency-based method centers on the Taut String Theory, where the tension force T in a cable of known length L and unit weight w can be calculated from the n-th natural frequency of vibration f_n by

$$T = \frac{4wL^2 f_n^2}{n^2 g} \tag{1}$$

where g is the gravitational constant.

An experimental validation of the determination of tension force through natural frequencies was published in Russell and Lardner [1998].

As the size of bridges presents challenges for accelerometer installation, remote sensing methods have been explored. A Laser Vibrometer is one such device, in which a laser beam is directed at the location of interest and the velocity of the

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vibration is measured[1]. Systems of this kind are capable of conducting measurements from distances of up to 200 m. In 1999, Cunha and Caetano used a laser vibrometer and compared the results to those from cable-mounted accelerometers. Excellent agreement was found. A laser vibrometer was also used successfully by Miyashita and Nagai [2008] and Chen and Petro [2006] to determine the frequency of bridge cables.

This paper describes an application of stay cable tension determination through laser-based vibration measurements. First, the main methods of bridge cable tension assessment are compared. Next, the authors present the results of original research on the importance of measurement duration. Three methods for identifying the natural frequency from the frequency plot are compared for varying measurement durations. Then, a literature review on the effects of sag-extensibility and bending stiffness is conducted, and a system of correction factors is presented.

2 Comparison of Methods

As outlined previously, the primary methods to measure bridge stay cable tension are:

- Lift-off method
- Frequency-based method
  - Using accelerometers mounted on the cable
  - Conducting remote measurements using a laser vibrometer.

The lift-off method has the advantage that each strand can be jacked and its tension load may be measured directly. However, this method requires heavy jacking equipment which is costly to mobilize, install and operate. The anchorage points must be accessible and clear of filling materials. Furthermore, a suitable bearing location for the jacking mechanism is required.

Using accelerometers to conduct frequency-based measurements requires no access to the anchorage points but does require access to the cable surface or sleeve surface. A sufficiently sensitive accelerometer needs to be physically mounted onto the cable at a position where vibrations of multiple modes can be detected and measured (near the third-point for instance). While the sensors themselves are lightweight (usually between 50 and 500 grams), this task typically requires a boom lift for a technician. Then, cabling must be run back to the frequency analyzer. Traffic control, or a bridge closure, may be required if this equipment or cabling interferes with normal operation. Each stay cable may require between 20 to 60 minutes for sensor installation and cabling.

By using non-contact laser based sensors like a Laser Vibrometer, no physical access to the cable is required. This allows the location of the measurement point to be at third or midpoint of the cable, not limited by the reach of the available boom lifts. Set-up time is only a few minutes per location. At a rate of $100-150 per hour per technician on site, reducing the set up time per cable will generate significant cost savings for the client.

A full investigation of a cable stay structure may include more than 100 cables. To evaluate these cables a simultaneous or sequential measurement can be selected. The simultaneous approach usually involves using many accelerometers at once. Given that enough accelerometers and signal cables are available, and the frequency analyzer has enough input channels (these devices can often support 8, 16 or 32 seperate channels), many measurements can be conducted simultaneously. However, this requires the installation of many accelerometers and signal cables while ensuring that no cable confusion

[1] The principle behind this device is that a laser beam reflected off a moving surface will experience a phase change and will also have its frequency shifted through the Doppler effect. A Laser Vibrometer is in effect an interferometer; in which a single laser beam is split into a measurement beam and reference beam. After reflecting off the vibrating surface, the measurement beam is recombined with the reference beam (which never left the instrument) and the interference pattern is measured.

occurs. Conversely, a laser vibrometer can only measure one point at a time, and therefore the data collection phase becomes a large part of the overall project. Section 3 outlines the implications of choosing an appropriate measurement duration and an experimental verification is performed.

The cost & time expenditures for each method are compared in Table 1. Because the time savings during setup of each measurement location greatly outweigh the extra time spent during measurement, conducting a cable vibration measurement programme using a laser vibrometer is much quicker than with accelerometers.

Overall, contactless laser vibrometer systems require less equipment and less time and are therefore a cost efficient alternative to traditional accelerometer-based and lift-off methods. Furthermore, laser vibrometers are generally quite transportable (an entire system can be wheeled by one technician and checked as airline luggage) and provide similar measurement precision as accelerometer systems.

3 Importance of measurement duration

The number of samples collected during data acquisition is critical for the precise determination of the natural frequency. The natural frequency and harmonics are determined by finding the peaks of the Fast Fourier Transform (FFT), which provides the frequency-domain representation of a time-domain function. For a discrete time series sampled at rate f_a for duration T, the FFT result is also a discrete sequence of the same length (f_aT). This result contains a series of discrete frequency bins uniformly distributed between 0 Hz and f_a. The width of these bins is the frequency resolution f_s, which is independent of the frequency setting of the analyzer, and is simply related to the measurement time T by

$$f_s = \frac{f_a}{f_a T} = \frac{1}{T} \tag{2}$$

As the duration of the measurement increases, the frequency resolution f_s improves, and the natural frequency can be determined to within a smaller interval.

Insufficient measurement durations will result in inadequate frequency resolution of the FFT analysis. However, measurements beyond the minimum required time will extend the time on site and may lead to higher costs for the client. The objective is to minimize data acquisition time whilst still identifying the natural frequency with adequate precision. This section presents three analytical methods to increase the accuracy in identifying the natural frequency from measurements of shorter duration. These methods are then compared using experimental data in order to determine an appropriate measurement length.

The simplest identification method is the First Peak Method, where the first natural frequency is read from the FFT. Given a frequency resolution of f_s, identifying the natural frequency (f_1) via the First Peak Method only gives us the certainty that

$$f_{1_{FP}} - \frac{f_s}{2} < f_{1_{True}} < f_{1_{FP}} + \frac{f_s}{2} \tag{3}$$

3.1 Considering multiple harmonics

Methods that consider multiple use harmonics use the relation of each n-th harmonic to the natural frequency

$$f_n = n f_1 \tag{4}$$

If we consider our certainty of identification of the n-th harmonic, we have the certainty that

$$f_{n_{Measured}} - \frac{f_s}{2} < f_{n_{True}} < f_{n_{Measured}} + \frac{f_s}{2} \tag{5}$$

$$f_{n_{Measured}} - \frac{f_s}{2} < n f_{1_{True}} < f_{n_{Measured}} + \frac{f_s}{2}$$

$$\frac{f_{n_{Measured}} - \frac{f_s}{2}}{n} < f_{1_{True}} < \frac{f_{n_{Measured}} + \frac{f_s}{2}}{n} \tag{6}$$

Table 1: Comparison of cost and time expenditures for measurement methods

		Frequency-based Methods	
Task	**Lift-off Method**	**Accelerometers**	**Laser Vibrometer**
Mobilization & demobilization to/from site	Extensive	Moderate	Quick
Set up/teardown for each cable	Extensive	Extensive (20-60 minutes per location)	Quick (1-3 minutes per location)
Data acquisition	Rapid	Simultaenous at multiple locations; little penalty for long measurement duration	Each point seperately; choice of measurement duration important
Data processing and interpretation	Conversion of gauge pressure to force, rapid.	Natural frequency identification, sag-extensibility and bending stiffness corrections, moderate	

Futhermore, each measurement can be considered to be uniformly distributed between these two bounds, so

$$f_{1_{True}} \sim \text{UNI}\left(\frac{f_{n_{Measured}} - \frac{f_s}{2}}{n}, \frac{f_{n_{Measured}} + \frac{f_s}{2}}{n}\right)$$

with the variance

$$\text{VAR}\left(f_{1_{True}}\right) \stackrel{\text{def}}{=} \sigma_n^2 = \frac{\left(\frac{f_s}{n}\right)^2}{12} = \frac{f_s^2}{12n^2} \tag{7}$$

It is seen that the variance of each measurement is proportional to $^1/_{n^2}$, so higher peaks will have a smaller variance.

If many measurements, each with variance σ_n^2, are combined and averaged, then the variance of this average will be

$$\text{VAR}\left(f_{1_{MP}}\right) = \frac{\sum_{n=1}^{N} \sigma_n^2}{n} \tag{8}$$

By this, the authors propose two methods of peak identification that use multiple peaks to minimize the variance.

3.1.1 Simple Average Method

The Simple Average Method utilizes multiple harmonics in a simple average:

$$f_{1_{SA}} = \frac{1}{N}\sum_{n=1}^{N}\frac{f_n}{n} \tag{9}$$

3.1.2 Variance-Weighted Average Method

In the Variance-Weighted Average Method, each identified peak is taken into account with a weighting that is inversely proportional to its standard deviation (square root of variance).

$$f_{1_{VW}} = \sum_{n=1}^{N} w_n f_n \tag{10}$$

where

$$w_n = \frac{\sum_{n=1}^{N} \sigma_n}{\sigma_n}$$

$$w_n = \frac{\sum_{n=1}^{N}\sqrt{\frac{f_s^2}{12n^2}}}{\sqrt{\frac{f_s^2}{12n^2}}} = \frac{\sum_{n=1}^{N}\frac{1}{n}}{\frac{1}{n}}$$

$$w_n = \frac{n}{\sum_{n=1}^{N} n} \tag{11}$$

3.2 Experimental Validation

The authors conducted three long-duration measurements (102.4 seconds) on each of two side-by-side (and therefore identical) cables. The parameters of these cables are presented in Table 2. This parameter table includes the results of a correction for sag-extensibility and bending stiffness, using the formulas presented in Section 4. As an example, Figure 1 depicts the time series of the first measurement of the East Cable.

In order to validate and compare the performance of the three identification methods on timeseries of different durations, each 102.4 second time series was also truncated to four shorter durations (Table 3) . These durations match those available in many commercial FFT analyzers[2].

Once again using the first measurement of the East Cable as an example, a plot comparing the FFT for each duration is presented in Figure 2. From this plot we can see the peaks are sharper for the longer-duration measurements. Conversely, the peaks are much less defined for shorter-duration measurements, increasing the possibility of misidentification.

3.2.1 Conclusions of Experimental Study

After using the three identification methods on the six time series, the results were plotted in Figure 3. In this figure, the results of the First Peak method are presented as an error bar (in red), thereby showing the certainty range defined by Equation 3.

The following remarks are made:

- The best indicator we have of the “true” natural frequency is the first peak of the longest duration measurement. Therefore, the latter two methods will be considered effective if they can identify the natural frequency as close to this value, using a shorter duration time series.
- The first peak identification method is very erratic for shorter measurement durations (less than 51.2 s).
- The simple average method provides a marked improvement over the first peak method.
- The variance-weighted average method provides an improvement still over the simple average method especially for measurement durations less than 25.6 s.
- It is seen that the latter two methods are very stable for measurement durations 25.6 s and longer, and therefore it is confirmed that 25.6 s is an appropriate choice of measurement duration that provides a good balance of accuracy and efficiency.

4 Sag-extensibility and bending stiffness corrections

In the preceeding sections, the relation of natural frequency to tension force was based on the Taut String Theory, which assumes the cable experiences no sag and has no bending stiffness. In actuality, neither of these conditions hold, and for cable stayed bridges, the cable is also inclined. The causes of these effects are detailed well in Ni et al. [2002].

[2] An efficient, and widely used, method for calculating the FFT is the radix-2 Cooley-Tukey algorithm, which works with a time series of 2^N samples, where N is a postive integer. Consequently, many commercial FFT analyzers offer the user a choice of time series lengths that comply with this requirement. Also, to remove the requirement that the user be aware of the effects of aliasing and properly choose a sampling frequency higher than the Nyquist rate, many commercial FFT analyzers have a sampling frequency that is 2.56 times the displayed frequency. Therefore, the user selects a “measurement frequency” based on the highest expected frequency, the analyzer runs at a higher frequency to avoid aliasing, and the user is only returned the portion of the FFT below the “measurement frequency” selected.

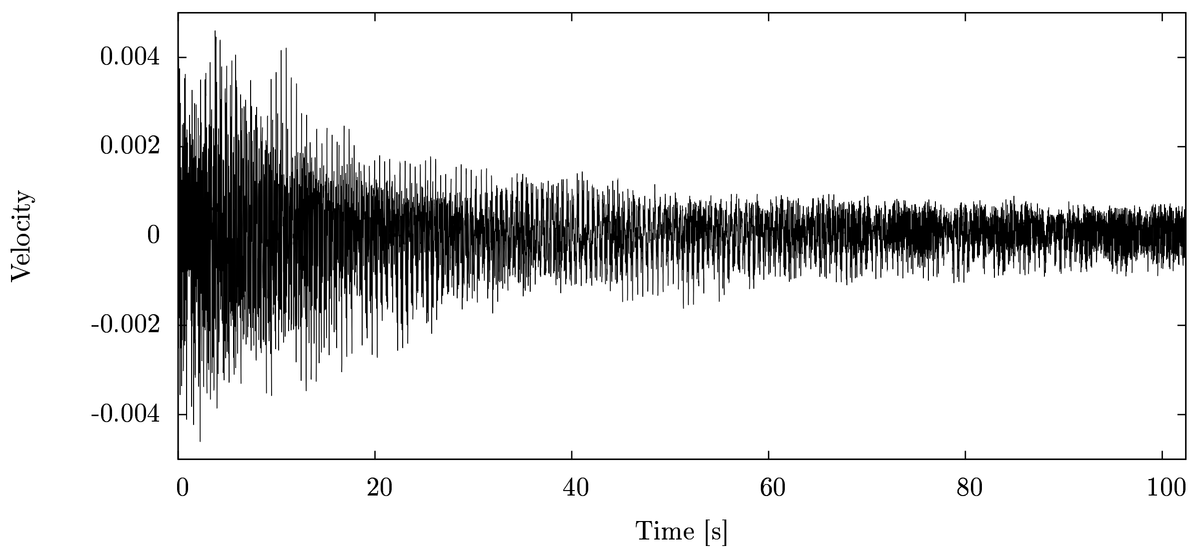

Figure 1: Time-series of East cable, Measurement 1

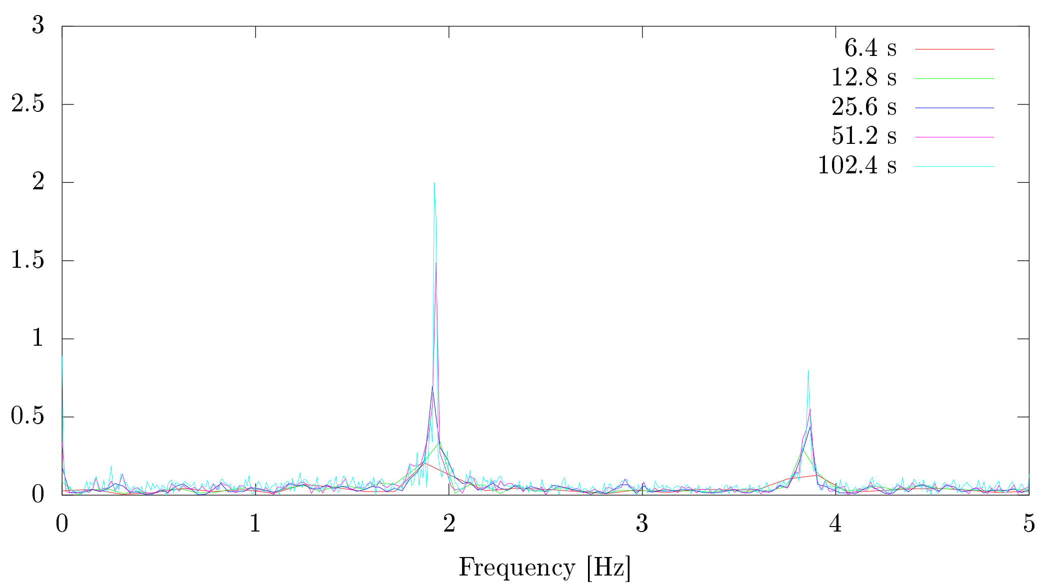

Figure 2: Comparison of 0-5 Hz region of FFT for varying measurement lengths (East Cable, Measurement 1)

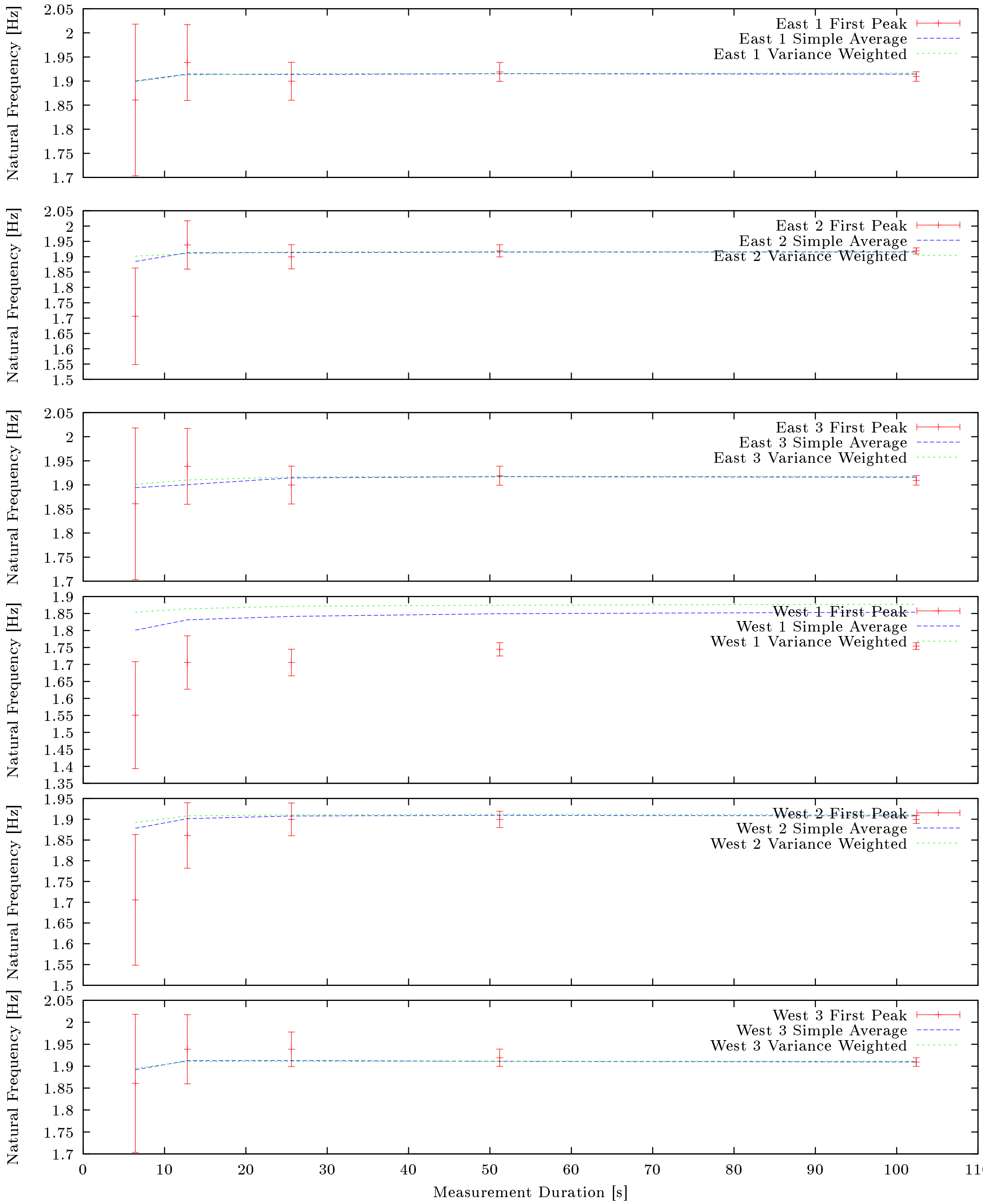

Figure 3: Comparison of identification methods for varying measurement durations

Table 2: Summary of parameters for considered cable

Parameter	Symbol	Value
Cable Length	L	69.233 m
Cable Unit Weight	w	0.236 kN/m
Tension Force (via Taut String Method)	H	1620 kN*
Extensibility	EA	236×10^3kN†
Bending Stiffness	EI	94 kN/m²†
Length after sag (See Eq. 18)	L_e	69.233 m*
Sag-extensibility parameter (See Eq. 12)	λ^2	0.0075*
Bending stiffness parameter (See Eq. 16)	ξ	276
Correction factor for sag-extensibility (See Eq. 14)	α	1.00028*
Correction factor for bending stiffness (See Eq. 15)	β_n	1.007 (1st mode) to 1.011 (8th mode)

* frequency dependent, presented parameter was calculated using the average frequency observed.

† Estimated from tabulated values of bridge stay cables by interpolating diameter.

Table 3: Considered measurement durations

Measurement Duration [s]	Samples	Samples after FFT	Frequency Resolution f_s [Hz]
6.4	1024	400	0.156
12.8	2048	800	0.078
25.6	4096	1600	0.039
51.2	8192	3200	0.020
102.4	16384	6400	0.010

Table 4: Modulus of Elasticity for steel cable

Cable	E, GPa
Helical strand	140
Structural strand	170
Parallel-wire strand	190 to 210

In 1974, Irvine and Caughey considered the problem of sag in a horizontal cable. They introduced a "fundamental cable parameter" of

$$\lambda^2 = \frac{\left(\frac{wL}{H}\right)^2 L}{\frac{HL_e}{EA}} \cos^2\theta \tag{12}$$

The modulus of elasticity for steel cables depends on the rope configuration. Approximate values for each type, compiled by Irvine [1981], are given in Table 4.

Tabatabai et al. [1998] present a correction factor γ that is applied to the measured natural frequency.

$$\gamma = \frac{\omega_n}{\omega_{n_s}} = \alpha\beta_n - 0.24\frac{\mu}{\xi} \tag{13}$$

α is the correction factor accounting for the effect of sag-extensibility on the first mode (in-plane) frequency determined by the taut string equation. Correction factor β_n accounts for the effect of bending stiffness on the natural frequency.

Both α and β_n are dependent on the inclination angle of the cable and materials properties:

$$\alpha = 1 + 0.039\mu \tag{14}$$

$$\beta_n = 1 + \frac{2}{\xi} + \frac{4 + \frac{n^2\pi^2}{2}}{\xi^2} \tag{15}$$

with

$$\xi = L\sqrt{\frac{H}{EI}} \tag{16}$$

$$\mu = \begin{cases} \lambda^2 & \text{for n=1 (in-plane)} \\ 0 & \text{for n > 1 (in-plane)} \\ 0 & \text{for all n (out-of-plane)} \end{cases} \tag{17}$$

$$L_e = L\left[1 + \frac{\left(\frac{wL\cos\theta}{H}\right)^2}{8}\right] \tag{18}$$

where

H is the initial estimate of tension force along the chord, determined by Equation 1.

λ^2 is as defined in Equation 12.

For higher modes of vibration ($n > 1$), Equation 13 becomes

$$\frac{\omega_n}{\omega_{n_s}} = \beta_n \tag{19}$$

The bending stiffness and sag-extensibility correction factors in this section should be calculated and applied to each measured peak before utilizing any of the three methods presented previously in Section 3.

5 Conclusion

Stay bridge cable tension measurements are sometimes required to confirm the cables are indeed carrying the designed loads, calculated by the bridge designer. Laser-based vibration measurement systems provide a fast and efficient solution to measure vibration frequencies on stay cables without the need for physical access to the cables. Identifying multiple harmonics allows the natural frequency to be identified to a greater precision by using methods such as a Variance-Weighted Average. Due to the non-contact approach and the long measurement range, these systems allow for fast set up and data acquisition. A fast set up and quick data acquisition results in significant cost savings for the client.

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