Fresnel Term Approximations for Metals

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ABSTRACT

Colored reflections are governed by the Fresnel term, which can be expressed from the refraction index of the material. This function, especially for metals where the refraction index becomes a complex number, is rather computation intensive. This paper presents an accurate simplification, which can also cope with complex refraction indices. In order to establish the approximation, Schlick's formula is rescaled and the residual error is compensated by a simple rational approximation. The resulting formula can present realistic metals and is simple enough to be implemented on the vertex or pixel shader, and used in games.

Keywords: BRDF, Fresnel term, Schlick's approximation

1. INTRODUCTION

Material models are usually defined by Bidirectional Reflectance Distribution Functions (BRDFs) that describe the chance of reflection for different pairs of incoming and outgoing light directions. Programmable vertex and pixel shaders allow sophisticated material models instead of the simple Phong-Blinn reflection model. Unlike the Phong-Blinn model, these sophisticated models can be physically plausible, that is, they do not violate basic rules of optics, including the Helmholtz symmetry and energy conservation.

2. PHYSICALLY PLAUSIBLE BRDF MODELS

A microfacet based specular BRDF model usually has the following product form [Cook81, He91]:

$$P(\vec{N}\cdot\vec{H})\cdot G(\vec{L},\vec{N},\vec{V})\cdot F(\vec{L}\cdot\vec{H},\lambda),$$

where λ is the wavelength of light, $\vec{N}$ is the surface normal, $\vec{L}$ is the illumination direction, $\vec{V}$ is the viewing direction and $\vec{H}$ is the halfway vector between the illumination and viewing directions.

Microfacet distribution $P(\vec{N}\cdot\vec{H})$ defines the roughness of the surface by describing the density of microfacets that can ideally reflect from the illumination to the viewing direction.

Geometric term $G(\vec{L},\vec{N},\vec{V})$ shows how much of these ideal reflections can actually occur, and is not blocked by another microfacet (called masking or self-shadowing). The geometric term is independent of the material properties, and it causes a general reduction of the specular term for certain illumination and viewing directions. Such reduction should be compensated in the diffuse reflection, since we can assume that photons reflected on the microfacets by multiple times contribute to the *diffuse* (also called *matte*) part. It also means that the matte and specular parts are not independent, as assumed by most of the BRDF models, but are coupled by an appropriate weighting, which depends on the viewing direction [Ashik00, Kele01].

Fresnel term

Finally, Fresnel term $F(\vec{L}\cdot\vec{H},\lambda)$ equals to the probability that a photon is reflected from the microfacet considered as an ideal mirror. According to the law of ideal reflection, the normal of those microfacets that can reflect from illumination direction $\vec{L}$ to viewing direction $\vec{V}$ is exactly the halfway vector $\vec{H}=(\vec{V}+\vec{L})/|\vec{V}+\vec{L}|$. This is why we included the angle of the halfway vector and the light direction in the Fresnel function, which depends on the angle of light incidence.The Fresnel term is the only factor that is wavelength dependent, thus it is the primary source of coloring. That is why the accurate computation of the Fresnel term is so important to present realistic look for materials.

The Fresnel term can be obtained as the solution of the Maxwell equations assuming an ideal planar surface. The formula of an arbitrary polarization can be expressed from two basic solutions, when the oscillation is parallel or perpendicular to the surface.

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In the final result for these cases, the complex refraction index $n + kj$ also plays a crucial role:

$$F_p = \left| \frac{\cos\theta - (n+kj)\cos\theta'}{\cos\theta + (n+kj)\cos\theta'} \right|^2, \quad F_s = \left| \frac{\cos\theta' - (n+kj)\cos\theta}{\cos\theta' + (n+kj)\cos\theta} \right|^2$$

In these formulae, θ is the angle of incidence and θ' is the angle of refraction. Assuming that the light is not polarized, the ratio of the reflected and incident radiance can be expressed by the Pythagoras theorem: $F = \frac{F_s + F_p}{2}$.

Using Snell's law, the angle of refraction can be eliminated from the formula, thus the Fresnel term depends on three arguments: *n* and *k* are the real and imaginary parts of the refraction index, respectively, and θ is the angle of incidence for the given microfacet. Expressing the absolute value of the complex numbers, the following form of the Fresnel formula can be obtained [Glass95].

$$F_s = \frac{a^2 + b^2 - 2a\cos\theta + \cos^2\theta}{a^2 + b^2 + 2a\cos\theta + \cos^2\theta}$$

$$F_p = F_s \frac{a^2 + b^2 - 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}{a^2 + b^2 + 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}$$

where *a* and *b* are defined by the following equations:

$$2a^2 = \sqrt{(n^2 - k^2 - \sin^2\theta)^2 + 4n^2k^2} + (n^2 - k^2 - \sin^2\theta)$$

$$2b^2 = \sqrt{(n^2 - k^2 - \sin^2\theta)^2 + 4n^2k^2} - (n^2 - k^2 - \sin^2\theta)$$

The computation of the exact Fresnel is quite expensive even on the graphics hardware. In real time applications we need its approximation, which is much cheaper to evaluate, but is accurate enough not to destroy image quality [NV02]. The main objective of this paper is to propose such Fresnel approximations.

3. PREVIOUS WORK ON FRESNEL TERM APPROXIMATION

For many non-metallic materials, the extinction coefficient k is quite small, which allows us to ignore the imaginary part altogether. The assumption of the extinction coefficient being zero has also been made by Schlick, who has found the following simple rational approximation for the Fresnel term [Schl94]:

$$FSchlick(n, \cos\theta) = \frac{(n-1)^2 + 4n(1-\cos\theta)^5}{(n+1)^2}$$

This formula provides a fairly good approximation if the extinction coefficient is really zero (*k*=0), and n is in the range 1.4...2.2 (outside this range the relative error increases sharply, see Figure 5).

Now let us concentrate on metals or other materials having complex refraction index (*k* > 0) (Figure 1).

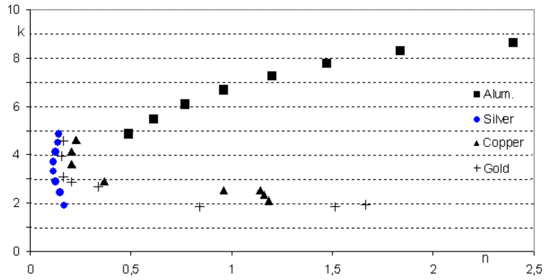

Figure 1. Complex refraction indices for different metals, for different wavelengths (λ=400..800 nm)

For complex refraction indices, the Fresnel term can have various characteristics (Figure 2). For small extinction coefficients, for example, the Fresnel term is a monotonous function of cosθ, with increasing values at cosθ=1 when *k* is increasing. Furthermore, for larger *k* values there exists a local minimum.

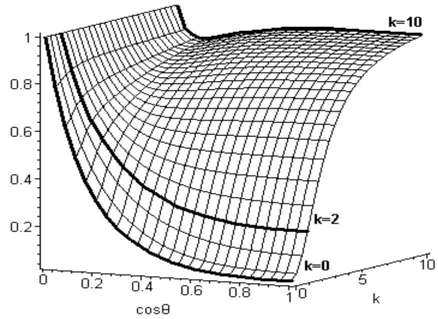

Figure 2. The Fresnel function (*n*=1.5, k=0..10)

If we apply Schlick's formula for metals, the result will be erroneous, with a significantly large error at cosθ=1 (Figure 3 and Figure 4).

In the followings we propose two improvements to significantly reduce the error of the original Schlick's approximation.

Figure 3. A copper ring rendered with the original Fresnel term and with Schlick's approximation

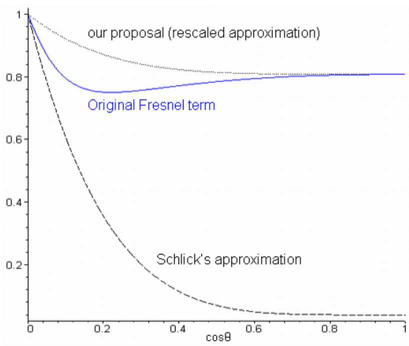

Figure 4. The Fresnel term, Schlick's formula and our new approximation for n=1.5, k=5

4. THE NEW FRESNEL TERM APPROXIMATION FOR METALS

Rescaling the Schlick's function

To reduce the error of the approximation we shall rescale Schlick's formula so that it will obey the value of the original function not only at $\cos\theta=0$, but at $\cos\theta=1$ as well (Figure 4).

To achieve this, we compute the exact and approximated function values at $\cos\theta=1$, as follows:

$$F_1 := 1 - Fresnel(n,k,\cos\theta = 1) = \frac{4n}{(n+1)^2 + k^2}$$

$$S_1 := 1 - FSchlick(n,\cos\theta = 1) = 1 - \frac{(n-1)^2}{(n+1)^2} = \frac{4n}{(n+1)^2}$$

Thus the scaling factor is: $\frac{F_1}{S_1} = \frac{(n+1)^2}{(n+1)^2 + k^2}$.

Now the modified (rescaled) Schlick's formula can be expressed as follows:

$$F^*(n,k,\cos\theta) := 1 - \frac{F_1}{S_1}\left(1 - FSchlick(n,\cos\theta)\right) =$$

$$= 1 - \frac{4n(1-(1-\cos\theta)^5)}{(n+1)^2 + k^2} = \frac{(n-1)^2 + 4n(1-\cos\theta)^5 + k^2}{(n+1)^2 + k^2}$$

The resulting formula is able to deal with complex refraction indices and is simple enough for practical applications:

$$F^*(n,k,\cos\theta) := \frac{(n-1)^2 + 4n(1-\cos\theta)^5 + k^2}{(n+1)^2 + k^2}$$

After examining the relative error of the modified approximation (Figure 5) we can conclude that this simple modification enables us to extend the original Schlick's formula to complex refraction indices without significant increase of the relative error (compare error for k=0 and k>0). Note that for k=0 we get back Schlick's original formula.

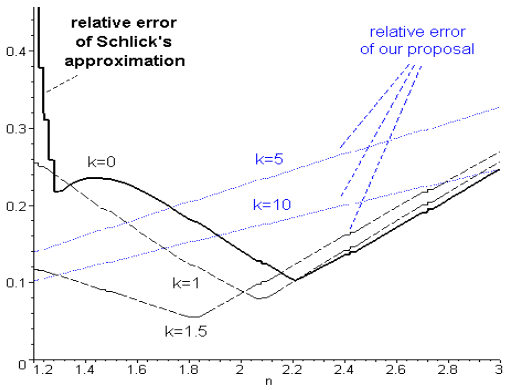

Figure 5. Relative error of the rescaled Schlick's formula for different *n* and *k* values

Error compensation

Since our approximation is unable to follow the local minimum of the original Fresnel function, metals with large (n, k) values (e.g. aluminum) may have a noticeable error. To improve our approximation, we shall compensate the error term – the difference between the original Fresnel function and the rescaled Schlick's formula – with a rational approximation that is simple enough to enable fast calculation and its shape is close to the shape of the error function (Figure 6). After examining the error term we chose the following approximation:

$$-ax(1-x)^{\alpha},$$

where $x=\cos\theta$. The derivative of this expression at x=0 and 1 equals to -a and 0, respectively. The expression has a local minimum at $x = 1/(1+\alpha)$.

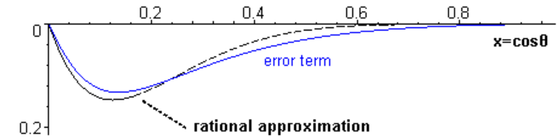

Figure 6. Error term for *n*=1.5, *k*=5 and a rational approximation with parameters *a*=3 and *α*=7

To determine the parameters a and α, we will have to set up two constraints.

Unfortunately, we cannot easily reproduce the local minimum of the error term. However, for most (n, k) values, the local minimum of the error term is located somewhere in the range of 0.1..0.15. (The only exceptions are those (n, k) pairs where both n and k are small, but in these cases – as we have seen earlier – the error term is negligible.) So the rational approximation should obey the value of the original error function at $x=\cos\theta=0.15$ (which is close to its local minimum).

Our second constraint will be the equality of the derivatives at x=0. For large k values – that is where error compensation becomes necessary – we can use the following approximation: $-a = -2n$, where value $-2n$ is a quite good approximation for the derivative of the original Fresnel term at x=0. (The derivative of the modified Schlick's formula is assumed to be zero.) As shown below, the relative error can be reduced below 5% even with this assumption, so there is no need to calculate the exact derivative. Note the dramatic improvement in case of aluminum.

relative error for	rescaling only	approx. derivative	exact derivative
copper	4,9%	**4,1%**	1,9%
gold	5,4%	**5,0%**	5,2%
silver	9,0%	**5,1%**	4,4%
alu	17,5%	**2,6%**	2,0%

Values a and α should be regarded as derived material properties. When the CPU program instructs the GPU to use a different material model, it should compute these parameters and download them to the GPU as uniform variables.

According to our tests, the proposed simplifications do not reduce image quality, but reduce rendering times to *their half or third*. The following table summarizes the relative computation times:

Fresnel formula	100%
Schlick's formula	28%
Rescaled Schlick's formula	33%
Error compensation	54%

5. CONCLUSIONS

This paper proposed two new approximations of the Fresnel function. Unlike previous approaches, we did not assume that the imaginary part of the refraction index is negligible, thus our model can be applied for a wider range of materials, e.g. metals. Our two approximations differ in accuracy and cost of evaluation. The more accurate approximation (called "error compensation") is worth applying if there is a significant back lighting in the scene. In other cases the rescaled model provides satisfactory results.

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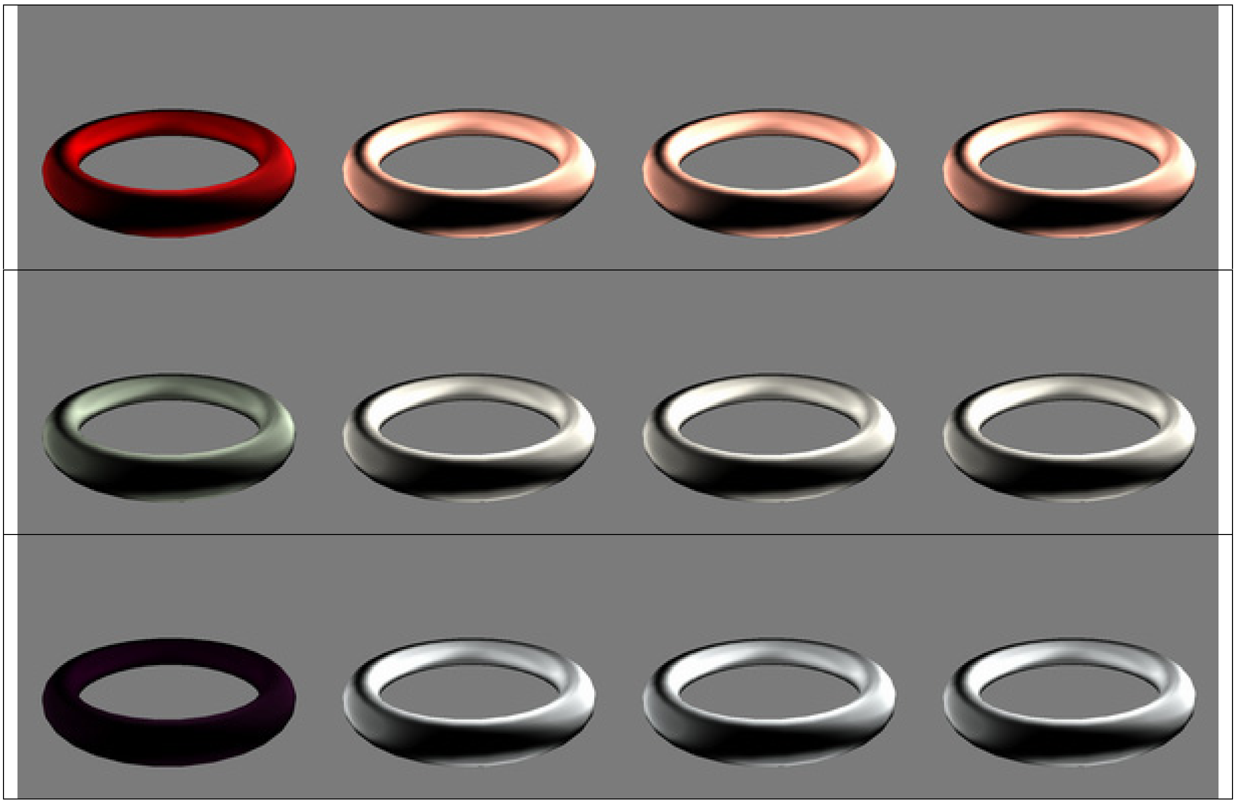

Figure 7. Copper, silver and aluminum rings rendered with different Fresnel approximations.